Efficient Reliability and Sensitivity Analysis of Complex Systems and Networks with Imprecise Probability

Thesis submitted in accordance with the requirements of

the University of Liverpool for the degree of Doctor in Philosophy

by

Geng Feng

May 2017

Declaration

I hereby declare that except where specific reference is made to the work of others, the contents of this dissertation are original and have not been submitted in whole or in part for consideration for any other degree or qualification in this, or any other university. This dissertation is my own work and contains nothing which is the outcome of work done in collaboration with others, except as specified in the text and Acknowledgements. This dissertation has 145 pages, 60 figures, 23 tables and 48062 words.

Geng Feng
May 2017

Acknowledgement

My main appreciation and first greatest acknowledgements to my supervisors Professor Michael Beer and Dr Edoardo Patelli. Michael Beer's ultimate support and guidance have been crucial to expand my academic horizon and take my research forwards step by step. Also, his gracious and generous personality is an example for me to learn from and makes my doctoral experience rather unique. I am very grateful to him.

Edoardo Patelli has given me constant and specific guidance during my PhD studies. I appreciate his high standards, his knowledge of research methodologies and his talented ability for software. His willingness and invaluable suggestions highly improve my dissertation. The years working with him have made my doctorate very prolific, enjoyable and unique, so I appreciate him from my heart.

I am extremely grateful to Professor Frank P.A. Coolen, who has pointed out a clear academic direction for me in the research ocean. His strong mathematical and systematic knowledge impresses me a lot, and he has taught me patiently and without reservation. It is really hard to find the words to express my gratitude and appreciation for him.

I very much acknowledge Dr Sean Reed and Dr Marco de Angelis's help, as their understanding of computational analysis and software development helped me a lot as a PhD researcher. Many sincere thanks to my close friend Hindolo George-Williams for his inreplaceable help and efficient cooperation during my PhD. Special thanks to Professor David Percy and Dr Anas Batou for their useful comments on this thesis.

I am also pretty grateful to the Institute for Risk and Uncertainty, which provided the perfect and friendly environment for my research. My deep appreciation goes out to Professor Scott Ferson, Dr Louis Aslett, Dr David Opeyemi, Dr Raphael Moura, Dr Oscar Nieto-Cerezo, Dr Matteo Broggi, Ms Roshini Prasad, Uchenna Oparaji and other Institute members who played essential roles during my study.

My special thanks go to my parents who give me everything a son could wish for. Words cannot express how grateful I am to my mother and father for all of the sacrifices that they have made on my behalf. I want also to acknowledge my closest supporters, my beloved wife Danqing Li, for her meticulous care and constant encouragement, and my son Zeyou Feng, who lightens my life with his smiles and ever present love.

Abstract

Complex systems and networks, such as grid systems and transportation networks, are backbones of our society, so performing RAMS (Reliability, Availability, Maintainability, and Safety) analysis on them is essential. The complex system consists of multiple component types, which is time consuming to analyse by using cut sets or system signatures methods. Analytical solutions (when available) are always preferable than simulation methods since the computational time is in general negligible. However, analytical solutions are not always available or are restricted to particular cases. For instance, if there exist imprecisions within the components' failure time distributions, or empirical distribution of components failure times are used, no analytical methods can be used without resorting to some degree of simplification or approximation. In real applications, there sometimes exist common cause failures within the complex systems, which make the components' independence assumption invalid.

In this dissertation, the concept of survival signature is used for performing reliability analysis on complex systems and realistic networks with multiple types of components. It opens a new pathway for a structured approach with high computational efficiency based on a complete probabilistic description of the system. An efficient algorithm for evaluating the survival signature of a complex system bases on binary decision diagrams is introduced in the thesis.

In addition, the proposed novel survival signature-based simulation techniques can be applied to any systems irrespectively of the probability distribution for the component failure time used. Hence, the advantage of the simulation methods compared to the analytical methods is not on the computational times of the analysis, but on the possibility to analyse any kind of systems without introducing simplifications or unjustified assumptions. The thesis extends survival signature analysis for application to repairable systems reliability as well as illustrates imprecise probability methods for modelling uncertainty in lifetime distribution specifications.

Based on the above methodologies, this dissertation proposes applications for calculation of importance measures and performing sensitivity analysis. To be specific, the novel methodologies are based on the survival signature and allow to identify the most

critical component or components set at different survival times of the system. The imprecision, which is caused by limited data or incomplete information on the system, is taken into consideration when performing a sensitivity analysis and calculating the component importance index.

In order to modify the above methods to analyse systems with components that are subject to common cause failures, α-factor models are presented in this dissertation. The approaches are based on the survival signature and can be applied to complex systems with multiple component types. Furthermore, the imprecision and uncertainty within the α-factor parameters or component failure distribution parameters is considered as well.

Numerical examples are presented in each chapter to show the applicability and efficiency of the proposed methodologies for reliability and sensitivity analysis on complex systems and networks with imprecise probability.

List of Publications

Journal Papers:

- Feng G, Patelli E, Beer M, and Coolen F PA. Imprecise System Reliability and Component Importance based on Survival Signature, *Reliability Engineering & System Safety*, 2016, 150, 116-125
- Patelli E, Feng G, Coolen F PA, and Coolen-Maturi T. Simulation Methods for System Reliability Using the Survival Signature, *Reliability Engineering & System Safety*, 2017, 167, 327-337

Conference Papers:

- Feng G, Patelli E, and Beer M. Reliability Analysis of Systems Based on Survival Signature, International Conference on Applications of Statistics and Probability in Civil Engineering, 2015
- Feng G, Patelli E, and Beer M. Survival Signature-based Sensitivity Analysis of Systems with Epistemic Uncertainties, European Safety and Reliability Conference, 2015
- Feng G, Patelli E, and Beer M. Reliability Analysis of Complex Systems with Uncertainties by Monte Carlo Simulation Method, Asian-Pacific Symposium on Structural Reliability and its Application, 2016
- Feng G, Patelli E, Beer M, and Coolen F PA. Component Importance Measures for Complex Repairable System, European Safety and Reliability Conference, 2016
- Patelli E, Feng G. Efficient Simulation Approaches for Reliability Analysis of Large Systems. International Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems, 2016
- Beer M, Feng G, Patelli E, Broggi M, and Coolen F PA. Reliability Assessment of Systems with Limited Information, International Symposium on Reliability Engineering and Risk Management, 2016

- Feng G, Reed S, Patelli E, Beer M, and Coolen F PA. Efficient Reliability and Uncertainty Assessment on Lifeline Networks Using the Survival Signature, International Conference on Uncertainty Quantification in Computation Science and Engineering, 2017
- Feng G, Patelli E, Beer M, and Coolen F PA. Reliability Analysis on Complex Systems with Common Cause Failures, International Conference on Structural Safety and Reliability, 2017

Contents

List of Figures

List of Tables

Chapter 1

Introduction

1.1 Overview

Reliability engineering deals with the construction and study of reliable systems. The first example of reliability calculation and estimation can be found in [1], which studied the probability for humans surviving to different ages. At the onset of World War II, with statistics theory and mass production well established, reliability engineering was ready to emerge [2].

Weibull proposed a statistical distribution function of wide applicability in [3], which is a standard tool for reliability applications. Birnbaum firstly put forward the importance measure in [4], which can be used to rank components in a system according to how important they are. Barlow and Proschan proposed the mathematical theory for reliability [5], which is one of the standard texts in the reliability engineering field.

Nowadays reliability engineering is used in a wide range of applications in complex systems and networks, which are series of components interconnected by communication paths. The analysis of these systems becomes more and more important as they are the backbones of our societies. Examples include the Internet, social networks of individuals or businesses, transportation networks, power plant systems, aircraft and space flights, metabolic networks, and many others. Since the breakdown of a system may causes catastrophic effects, it is essential to be able to assess the reliability and availability of these systems.

Uncertainty is an unavoidable component affecting the behaviour of systems and more so with respect to their limits of operation. In spite of how much dedicated effort is put into improving the understanding of systems, components and processes through the collection of representative data, the appropriate characterisation, representation, propagation and interpretation of uncertainty will remain a fundamental element of the reliability analysis of any complex systems [6].

By studying the survival function of the complex systems and networks, the engineers can know the performance of them at different times. By using component importance measures, it is possible to draw conclusions about which component or components set is the most important to the whole system. By researching on the configuration and the lifetime of components, experts can design for reliability of the complex networks and systems. By considering uncertainty within the system, insights into the analysis outcomes can be produced which can be used meaningfully by decision-makers.

1.2 Problem Statement

The study of the reliability of complex systems, particularly systems with structures that cannot be sequentially reduced by considering alternative series and parallel subsystems, is a subject which has attracted much attention in the literature and which is of obvious importance in many applications [7]. A system is a collection of components whose proper function leads to the coordinated functioning of the system. In reliability analysis, it is therefore important to model the relationship between various items as well as the reliability of the individual components, to determine the reliability of the system as a whole.

Traditionally, the reliability analysis of systems is performed adopting different well-known tools such as reliability block diagrams, fault tree and success tree methods, failure mode and effect analysis, and master logic diagrams [8]. The main limitation to applying these traditional approaches to large complex systems is the complex and tedious calculations to find minimal path sets and cut sets.

In recent years, the system signature has been recognised as a useful tool to quantify the reliability of systems consisting of independent and identically distributed (iid) or exchangeable components with respect to their random failure times [9] [10]. It can be said that such systems only have "components of one type". The system signature enables full separation of the system structure from the component probabilistic failure time distribution when deriving the system failure time distribution.

However, attempting to generalise the system signature to systems with more than one component type is not really possible as it requires the computation of the probabilities of different order statistics of the different failure time distributions involved [11], which tends to be intractable.

Most complex systems, such as automobiles, communication systems, aircraft, aircraft engine controllers, printers, medical diagnostics systems, helicopters, train locomotives, etc., are repaired and are not replaced when they fail [12]. To be specific, a complex repairable system is a system that can be restored to an operating condition following a failure [13]. Similarly, the repairable components are those that are not replaced following the occurrence of a failure; rather, they are repaired and put into operation again. Therefore, it is essential to perform reliability analysis on complex repairable systems in the real application area.

Component importance measurement allows to quantify the importance of system components and identify the most "critical" component. It is a useful tool to find weaknesses in systems and to prioritise reliability improvement activities. Birnbaum [4] proposed in 1969 a measure to find the reliability importance of a component, which is obtained by partial differentiation of the system reliability with respect to the given component reliability. An improvement or decline in reliability of the component with the highest importance will cause the greatest increase or decrease in system reliability. Several other importance measures have been introduced [14]. Improvement potential, risk achievement worth, risk reduction worth, criticality importance and Fussell-Vesely's measure were all reviewed in Ref. [15] [16] [17] [18]. To conduct reliability importance of components in a complex system, Wang et al. [19] introduced and presented failure criticality index, restore criticality index and operational criticality index. Zio et al. [20] [21] presented generalised importance measures based on Monte Carlo simulation. The component importance measures can determine which components are more important to the system, which may suggest the most efficient way to prevent system failures.

However, the traditional importance measures mainly focus on non-repairable systems, and mainly concern reliability importance of individual components. In many practical situations it is of interest to evaluate the importance of a set of components instead of just an individual component.

Some of the importance measures can be computed through analytical methods, but limited to systems with few components. Traditional simulation methods provide no easy way to compute component importance [19]. In addition, in the case of imprecision in component failures, the simulation approaches become intractable.

As an intrinsic feature, practical systems involve uncertainties to a significant extent. Since the reliability and performance of systems are directly affected by uncertainty, a quantitative assessment of uncertainty is widely recognised as an important task in practical engineering [22]. The obvious pathway to a realistic and powerful analysis of systems is a probabilistic approach.

Most existing models assume that there are precise parameter values available, so the quantification of uncertainty is mostly done by the use of precise probabilities [23]. However, due to lack of perfect knowledge, imprecision within the component failure times or their distribution parameters can not be ignored. Hence, the sensitivity analysis for the whole system is affected by the epistemic uncertainty [24].

In order to deal with the uncertainty, the Dempster-Shafer approach to represent uncertainty was articulated by Dempster [25] and Shafer et al. [26]. Troffaes et al. [27] presented a robust Bayesian approach to modelling epistemic uncertainty in common-cause failure models. Tonon [28] used random set theory to propagate epistemic uncertainty through a mechanical system. Helton et al. [29] combined sensitivity analysis with evidence theory to represent epistemic uncertainty. Fuzzy set theory is also proposed to deal with uncertainty in [30] [31]. An integrated framework to deal with scarce data, aleatory and epistemic uncertainties is presented by Patelli et al. [32], and OpenCossan is an efficient tool to perform uncertainty management of large finite element models [33].

On top of the above methods, Williamson and Downs [34] introduced interval-type bounds on cumulative distribution functions, which is called "probability boxes" or "p-boxes" for short. The use of p-boxes in risk analysis offers many significant advantages over traditional probabilistic approaches because it provides convenient and comprehensive ways to handle several of the most practical serious problems faced by analysts [35]. For example, Karanki et al. [36] expressed uncertainty analysis based on p-boxes in probabilistic safety assessment. Evidential networks for reliability analysis and performance evaluation of systems with imprecise knowledge was introduced by Simon and Weber [37]. In order to make a quantification of margins and uncertainties, Sentz and Ferson [38] presented probabilistic bounding analysis (PBA), which also can be used to perform the sensitivity analysis of systems. This approach represents the uncertainty about a probability distribution by a set of cumulative distribution functions lying entirely within a pair of bounding distributions [39].

Dependence among failures might affect considerably the reliability of a system, and it is a relationship that causes multiple components to fail simultaneously, which exists widely in complex systems [40]. Dependence represents a common feature of component failures that needs to be modelled appropriately for a realistic analysis of systems and networks. The proper consideration and modelling of CCFs are essential in complex systems reliability analysis as they may have a large effect on the systems' overall functionality. Often the assumption that the component failures are independent is considered in classical reliability analysis on systems. However, CCFs make this simplification not realistic. What is more, CCFs have been shown to decrease the reliability and availability of complex systems [41]. Therefore, common cause failures are extremely important in reliability assessment and must be given adequate treatment to minimise overestimation of systems' performances.

A number of parametric models have been developed for common cause failures over the last decades. For instance, Rasmuson and Kelly [42] reviewed the basic concepts of modelling CCFs in reliability and risk studies. One of the most commonly used single parameter models defined by Fleming [43] is called the β-factor model, which is the first parameter model applied to common cause failures in risk and reliability analysis. Then, he generalised the β-factor model to a multiple Greek letter model in 1986 [44]. The α-factor model originally proposed by Mosleh et al. [45] develops CCFs from a set of failure ratios and the total component failure rate.

Recently, based on the α-factor model, Kelly and Atwood [46] presented a method for developing Dirichlet prior distributions that have specified marginal means. A robust Bayesian approach to modelling epistemic uncertainty in the imprecise Dirichlet model has been discussed by Troffaes et al. [27]. Coolen and Coolen-Maturi [47] present a non-parametric predictive inference for system reliability following common cause failures of components but limited to systems with exchangeable or a single type of components. Here, the work presented in Ref [47] is extended to perform reliability analysis on complex systems by considering CCFs among components belonging to different types; the proposed approach is based on the survival signature [48].

This dissertation mainly uses the survival signature methodology, which is associated with a survival analysis of systems [49]. Survival analysis has important applications in biology, medicine, insurance, reliability engineering, demography, sociology, economics, etc. In engineering, survival analysis is typically referred to as reliability analysis, and the survival function is then called reliability function. This survival function or reliability function quantifies the survival probability of a system at a certain point in time.

System signature has been recognised as an important tool to quantify the reliability of systems. However, the use of the system signature is associated with the assumption that all components in the system are of the same type.

Generally, in reliability problems for large-scale real-world systems and networks, simulation tools are required in order to provide reliability metrics. It is therefore important to put forward the efficient simulation methods which only use the survival signature. These methodologies do not need to use the entire structure function of the systems, which will provide a new insight into the system reliability.

Parameter uncertainties and imprecisions are generally epistemic in nature due to the lack of knowledge or data, or the unknown relationship between components (e.g., poor understanding of accident initiating events or coupled physics phenomena, lack of data to characterise experiment processes, random errors in measuring and analytic devices), all of them make it difficult to characterise probabilistically the failure time of components. Since the reliability and performance of systems are directly affected by uncertainties and imprecisions, a quantitative assessment of uncertainty is widely recognised as an important task in engineering [50].

Simulation approaches are used to investigate large and complex systems and to obtain numerical solutions where analytical solutions are not available. In particular, simulation methods allow the explicit consideration of the effect of uncertainty and imprecision on the system under investigation, providing a powerful tool for risk analysis, which allows better decision making under uncertainty. Simulation methods can be used to identify problems before implementation, evaluate ideas, identify inefficiencies and understand why observed events occur.

The use of simulation methods for system reliability has many attractive features. Generally, they can be used for the sensitivity analysis of multi-criteria decision models [51], to optimise models with rare events [52] and to perform multi-attribute decision making [53].

Most of the current simulation methods are based on Monte Carlo simulation and structure function. By generating the state evolution of each component, the structure function is computed to determine the state of the system. However, the calculation of the structure function usually requires the calculation of all the cut-sets and for large systems it is a challenging and error-prone task (see e.g. [54]). Moreover, the structure function is in a Boolean format and can only be used to identify a specific output of the system. Of course, more structure function can be used to match all the possible status of the system, at the cost of significantly increasing the time required by the analysis. Instead, the survival signature is a summary of the structure function, that is sufficient for basic reliability inferences (e.g. determining the system reliability function). In particular, for very large scale systems and networks, storing only the survival signature and not the entire structure function is clearly advantageous.

In a nutshell, in practical cases there are five specific challenges that need to be addressed to obtain realistic results.

- First, the complexity of the system needs to be reflected in the numerical model. This goes far beyond a model based on a set of components with simple connections between them. For instance, there may be several different types of components in the same system. The variety of the components in the large size of real-life systems makes it difficult to predict the system lifetime and reliability, especially when they exit uncertainty characteristics.

- Second, when modelling failure time data, there is a distinction that needs to be made between one-time-use (or non-repairable) and multiple-time-use (or repairable) systems. When a non-repairable system fails, engineers simply replace it with a new system of the same type. In real engineering applications, however, there will be always repairable complex systems to be analysed.

- Third, the analysis of repairable systems leads to component importance measures, which are essential to find out the most “critical” component or components set within the repairable system.

- Fourth, when quantifying the system survival probability, it should be evident that the components will not always fail independently. A single common cause can affect many components at the same time, which is a common character for complex engineering systems.

- Fifth, the available information for the quantitative specification of the uncertainties associated with the components is often limited and appears as incomplete information, limited sampling data, ignorance, measurement errors and so forth. Therefore, it is important to consider imprecision during the whole system reliability analysis period.

1.3 Aims and Objectives

The present work contributes towards a solution to the above challenges and the aim of the research project is to perform efficient reliability and sensitivity analysis on complex systems and networks with imprecise probability. The proposed approaches can be used in large systems with multiple component types, which exist widely in the reality. Also, it opens up a new perspective to reliability and component importance measures of such kind systems, as well as considering the common cause failures within the system.

All in all, the main objectives of the research project can be generalised as:

- To propose a general method to analyse the complex systems and networks. Efficient simulation approaches based on survival signature are used to estimate the reliability of systems. This is very important when considering large systems, since they can only be analysed by means of simulation. The proposed simulation approaches are generally applicable to any system configuration.

- To consider the reliability of systems with repairable components. An algorithm based on the survival signature is introduced to analyse the repairable systems. This method is efficient as it is based on the survival signature instead of estimating all the cut sets of the system, and Monte Carlo simulation is used to generate the repairable components' transition times.

- To take the indeterminancy and vagueness into consideration when analysing the network system. The proposed approach in the thesis allows us explicitly to include imprecision and vagueness in the characterisation of the uncertainties of system components. The imprecision characterises indeterminacy in the specification of the probabilistic model. That is, an entire set of plausible probabilistic models is specified using set-values (herein, interval-valued) descriptors for the description of the probabilistic model. The cardinality of the set-valued descriptors reflects the magnitude of the imprecision and, hence, the amount and quality of information that would be needed in order to specify a single probabilistic model with sufficient confidence.

- To rank the importance degree of components in precise and imprecise systems. For this purpose, a component importance measure is implemented to identify the most "critical" components of a system taking into account the imprecision in their characterization. Specifically, a new component importance measure is introduced as the relative importance index (RI). Through simulation methods based on survival signature, upper and lower bounds of the survival function of the system or relative importance index can be efficiently obtained. On this basis, the survival function of the system and the importance degree of components can be quantified.

- To analyse the reliability of complex systems with common cause failures. The standard α-factor model is extended to a general α-factor model. Both models are based on the survival signature, and allow us to distinguish between the total failure rate of a component and the common cause failures modelled by α-factor parameters.

1.4 Structure of Thesis

This thesis is organised such that each chapter addresses one main inference problem, and is related to papers that have been published. The theoretical background of the thesis is briefly introduced in Chapter 2. Subsequently, Chapter 3 performs non-repairable system reliability analysis by survival signature-based analytical method and simulation method respectively. Then an efficient simulation method is proposed to analyse the repairable systems in Chapter 4. In Chapter 5, some novel component importance measures are presented. After that, common cause failures within the complex systems are studied in Chapter 6. Finally, Chapter 7 closes the thesis with conclusions and suggestions for future work, to summarise the presented work and indicate directions for potential future developments.

Chapter 2

Theoretical Background

2.1 Introduction

A system is a collection of components, and the reliability of a system can be defined as the probability that the system functions as provided by the given state of its components [55].

In this Chapter, the definitions of state vector $\underline{x}$, structure function $\phi(\underline{x})$, the minimum path sets P and the minimum cut sets C are discussed. All of these help people to understand the reliability of coherent systems. However, when the number of components increases, the application of structure function tends to be of limited use.

For complex systems and networks with large numbers of components, the system signature has been introduced recently to simplify quantification of reliability for systems and networks. The main disadvantage of system signature, however, is the strict assumption that all the components within the system are of the same type, which is not applicable to most real world systems. In order to overcome the limitation of the system signature, survival signature is presented to perform reliability analysis on systems with multiple component types. The usefulness of the tools in system reliability is discussed in this Chapter.

2.2 State Vector and Structure Function

Component reliability has may connotations. In general, it refers to a component's capacity to perform an intended function. The better the component performs its intended function, the more reliable it is. The systems and networks are series of components interconnected by communication paths. Therefore, the reliability of systems and networks is dependent on the performance of their components. Now we consider the definitions of component state vector and system structure function, as well as the relationship between them.

2.2.1 Component State Vector

The state vector gives the state of each component in the system.

Suppose there is one system formed by m components. Let the state vector of components be $\underline{x} = (x_1, x_2, ..., x_m) \in \{0, 1\}^m$ with $x_i = 1$ if the ith component is in working state and $x_i = 0$ if not.

2.2.2 System Structure Function

The structure function of a system gives the overall state of the system. To be specific, the structure function indicates whether the system as a whole works or not.

Let $\phi = \phi(\underline{x}) : \{0, 1\}^m \to \{0, 1\}$ define the system structure function, i.e., the system status based on all possible $\underline{x}$. ϕ is 1 if the system functions for its corresponding components state vector $\underline{x}$ and 0 if not.

2.2.3 Relationship between State Vector and Structure Function

Now let us study the structural relationship between a system (structure function) and its components (state vector).

Series System: A system that is working if and only if all the components are functioning is called a series system, which can be illustrated by the reliability block diagram in Figure 2.1.

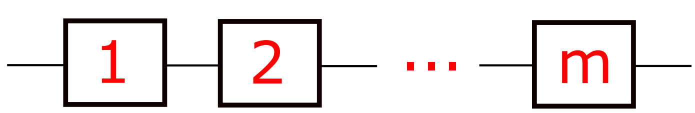

Figure 2.1: Series system with m components.

The structure function for this system is given by

$$\phi(\underline{x}) = x_1 \cdot x_2 \cdot ... \cdot x_m = \prod_{i=1}^{m} x_i \tag{2.1}$$

Parallel System: A system that is working if and only if at least one component is functioning is called a series system. The corresponding block diagram is shown in Figure 2.2.

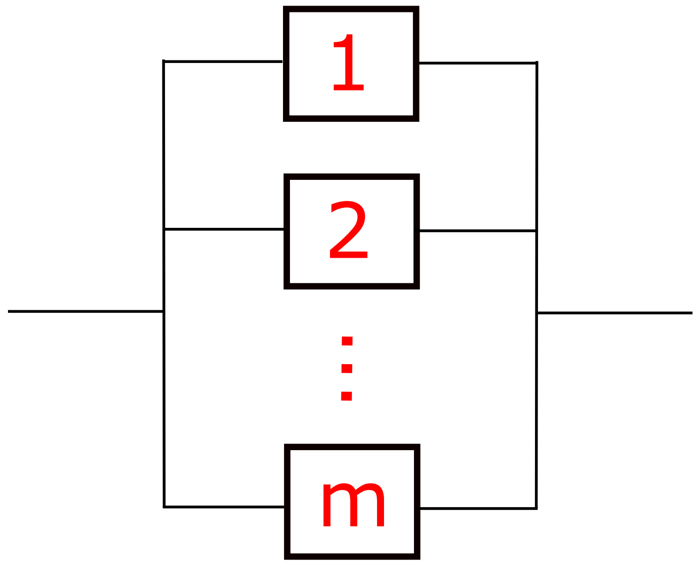

Figure 2.2: Parallel system with m components.

The system structure function is given by

$$\phi(\underline{x}) = 1 - (1 - x_1)(1 - x_2)...(1 - x_m) = 1 - \prod_{i=1}^{m}(1 - x_i) \tag{2.2}$$

Coherent System: A system is coherent if the structure function $\phi(\underline{x})$ is not decreasing in any of the components of x. The system functioning, therefore, cannot be improved by worse performance of one or more of its components. The coherent system has another characteristic that $\phi(\underline{0}) = 0$ and $\phi(\underline{1}) = 1$, so the system fails if all its components fail and it functions if all its components function. We mainly focus on coherent systems in this thesis, as it is reasonable for most of the real world systems and networks.

2.3 Computing System Reliability

Assessment of the reliability of a system from its components is one of the most important aspects of reliability engineering. In reliability analysis, it is essential to model the relationship within components to determine the reliability of the system and network as a whole.

2.3.1 Simple System

Once the system structure function $\phi(\underline{x})$ is known, the reliability of the system, which is also called survival function, can be calculated. Let p_i be the reliability of the component i, while R is the corresponding reliability of the system, and assume that components function independently.

Reliability of a Series System: For a series structure the system functioning means that all the components function, hence

$$\begin{aligned} R = P(\phi(\underline{x}) = 1) = P(\prod_{i=1}^{m} x_i = 1) = P(x_1 = 1, x_2 = 1, ..., x_m = 1) \\ = \prod_{i=1}^{m} P(x_i = 1) = \prod_{i=1}^{m} p_i \end{aligned} \quad (2.3)$$

Reliability of a Parallel System: Similarly, the reliability of a parallel structure system is given by

$$R = 1 - \prod_{i=1}^{m}(1 - p_i) \quad (2.4)$$

2.3.2 Parallel-Series System

In the engineering world, most practical systems are not series or parallel, but exhibit some hybrid combination of the two. This kind of system is called a parallel-series system, an example of which can be seen in Figure 2.3.

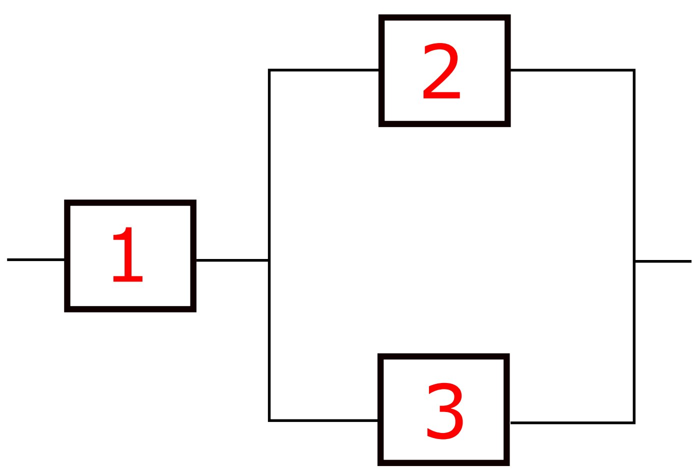

Figure 2.3: Complex parallel-series system.

Reliability of a Complex Parallel-Series System: A complex parallel-series system can be analysed by separating it into the simple parallel and series parts and then calculating the survival function for each part individually.

For example, the reliability of the parallel-series system in Figure 2.3 is given by

$$R = p_1 \cdot (1 - (1 - p_2)(1 - p_3)) = p_1p_2 + p_1p_3 - p_1p_2p_3 \tag{2.5}$$

2.3.3 Non-Parallel-Series System

Another type of complex system is one that is neither series nor parallel, nor parallel-series. Figure 2.4 shows an example of such a system.

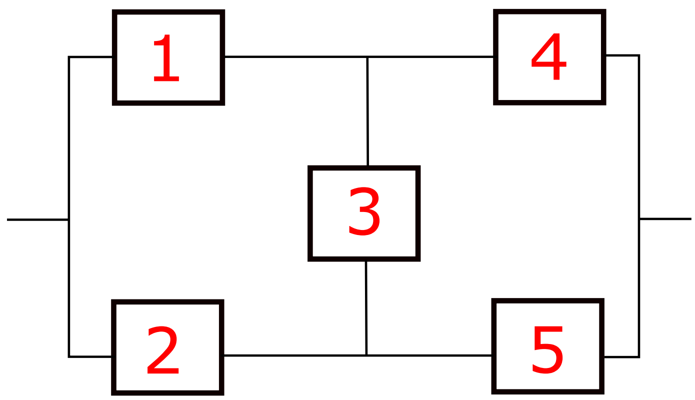

Figure 2.4: Complex non-parallel-series system.

Reliability of a Complex Non-Parallel-Series System: Minimum path set method and minimum cut set method are commonly used in reliability analysis for complex non-parallel-series systems.

For a coherent system, a set of components P is called as a path set if the system functions whenever all the components in the set P work. A minimum path is a set of components that comprise a path, but the removal of any one component will cause the resulting set to not be a path [56].

The minimum path sets of the complex system in Figure 2.4 are $P_1 = \{1, 4\}$, $P_2 = \{2, 5\}$, $P_3 = \{1, 3, 5\}$ and $P_4 = \{2, 3, 4\}$.

If a system has n minimum path sets denoted by $P_1, P_2, ..., P_n$, then the system reliability is obtained from

$$P[system\ success] = P[P_1 \bigcup P_2 \bigcup ... \bigcup P_n] \tag{2.6}$$

Therefore, the reliability function of the system in Figure 2.4 can be calculated as

$$\begin{aligned}P[system\ success] &= P[(x_1^w \bigcap x_4^w) \bigcup (x_2^w \bigcap x_5^w) \bigcup \\ &\quad (x_1^w \bigcap x_3^w \bigcap x_5^w) \bigcup (x_2^w \bigcap x_3^w \bigcap x_4^w) \\ &= [p_1p_4 + p_2p_5 + p_1p_3p_5 + p_2p_3p_4] \\ &\quad -[p_1p_2p_4p_5 + p_1p_3p_4p_5 + p_1p_2p_3p_4 \\ &\quad +p_1p_2p_3p_5 + p_2p_3p_4p_5 + p_1p_2p_3p_4p_5] \\ &\quad +[p_1p_2p_3p_4p_5 + p_1p_2p_3p_4p_5 + p_1p_2p_3p_4p_5 \\ &\quad +p_1p_2p_3p_4p_5] - [p_1p_2p_3p_4p_5]\end{aligned} \quad (2.7)$$

where p_i means the probability that component i is working.

Similarly, a set of components C is called as a cut set if the system fails whenever all the components in the set C fail. While a minimum cut is a set of components that comprise a cut, but the removal of any one component from the set causes the resulting set to not be a cut [56].

So the minimum cut sets of the complex system in Figure 2.4 are $C_1 = \{1, 2\}$, $C_2 = \{4, 5\}$, $C_3 = \{1, 3, 5\}$ and $C_4 = \{2, 3, 4\}$.

System reliability can also be determined through the minimum cut sets. Suppose there is a system with n minimum cut sets which denoted by $C_1, C_2, ..., C_n$, then the system reliability is given by

$$P[system\ failure] = P[C_1 \bigcup C_2 \bigcup ... \bigcup C_n] \quad (2.8)$$

Thus, the reliability function of the system in Figure 2.4 can be calculated as

$$\begin{aligned}P[system\ failure] &= P[(x_1^f \bigcap x_2^f) \bigcup (x_4^f \bigcap x_5^f) \bigcup \\ &\quad (x_1^f \bigcap x_3^f \bigcap x_5^f) \bigcup (x_2^f \bigcap x_3^f \bigcap x_4^f) \\ &= [q_1q_2 + q_4q_5 + q_1q_3q_5 + q_2q_3q_4] \\ &\quad -[q_1q_2q_4q_5 + q_1q_2q_3q_5 + q_1q_2q_3q_4 \\ &\quad +q_1q_3q_4q_5 + q_2q_3q_4q_5 + q_1q_2q_3q_4q_5] \\ &\quad +[q_1q_2q_3q_4q_5 + q_1q_2q_3q_4q_5 + q_1q_2q_3q_4q_5 \\ &\quad +q_1q_2q_3q_4q_5] - [q_1q_2q_3q_4q_5]\end{aligned} \quad (2.9)$$

where q_i means the probability that component i fails.

2.4 Signature

The above subsections have discussed component state vector $\underline{x}$, system structure function $\phi(\underline{x})$, minimum path set P and minimum cut set C. All of these help people to understand the characteristics of coherent systems and system reliability.

However, for complex systems with many components, the computation of structure function becomes algebraically cumbersome. In recent decades, system signature has been proven to be a powerful tool for quantification of reliability on a coherent system.

2.4.1 System Signature

The definition of system signature is given by Samaniego in [9]. Suppose that there is a coherent system with m components, and assume that the failure times of all the components are independent and identically distributed (iid). The system signature S is defined as a m-dimensional vector whose ith component s_i represents the probability that the ith component failure causes the system to fail.

Let $T_s > 0$ be the random failure time of the system and $T_{i:m}$ be the ith order statistic of the m component failure times for $i = 1, 2, ..., m$, where $T_{1:m} < T_{2:m} < ... < T_{m:m}$. So $T_s = T_{i:m}$ means that the system fails at the moment of the ith component failure.

The signature of the ith component can be expressed by

$$s_i = \frac{n_i}{m!} \tag{2.10}$$

where $i = 1, 2, ..., m$, n_i is the number of orderings for which the ith component failure causes system failure, $s_i \in [0, 1]$ and $\sum_{i=1}^{m} s_i = 1$.

For instance, the system signature of the series system in Figure 2.1 is $S = (1, 0, ..., 0)$, while for the parallel system in Figure 2.2, the system signature is $S = (0, 0, ..., 1)$. For the complex system in Figure 2.3, the ordered component failure times can be seen in Table 2.1.

Therefore, the signature of this system is $S = (\frac{2}{6}, \frac{4}{6}, 0) = (\frac{1}{3}, \frac{2}{3}, 0)$.

Note that the system signature only relies on the permutation of the $m!$ ordered component failure times instead of depending on the failure time distribution. Therefore, the biggest advantage of the system signature is the separation between the system structure

Table 2.1: Ordered component failure times for the system in Figure 2.3.

ordered component failure times	order statistic equal to system failure time T_s
$T_1 < T_2 < T_3$	$T_{1:3}$
$T_1 < T_3 < T_2$	$T_{1:3}$
$T_2 < T_1 < T_3$	$T_{2:3}$
$T_2 < T_3 < T_1$	$T_{2:3}$
$T_3 < T_1 < T_2$	$T_{2:3}$
$T_3 < T_2 < T_1$	$T_{2:3}$

and the components' failure time distribution. However, the disadvantage of system signature is also clear as it can only be used under the iid assumption, which means all the components within the system have to be the same type.

2.4.2 Survival Signature

In order to overcome the limitations of the system signature, Coolen and Coolen-Maturi [48] proposed the survival signature as an improved concept, which does not rely on the restriction to one component type anymore. Specifically, the characteristics of the components do not need to be independently and identically distributed (iid). In the case of a single component type, the survival signature is closely related to the system signature.

Recent developments have opened up a pathway to perform a survival analysis using the concept of survival signature even for relatively complex systems. Coolen et al. [11] have shown how the survival signature can be derived from the signatures of two subsystems in both series and parallel configurations, and they developed a non-parametric predictive inference scheme for system reliability using the survival signature [57]. Aslett developed a Reliability Theory package which was used to calculate the survival signature [58] and analysed system reliability within the Bayesian framework of statistics [59]. Feng et al. [60] deals with the imprecision within the system by analytical and numerical ways respectively, and new component importance measures are presented in this paper. Patelli et al. [61] [62] proposed efficient simulation approaches, which were based on survival signature for reliability analysis on a large system. An imprecise Bayesian non-parametric approach by using sets of priors to system reliability with multiple types of components is developed by Walter et al. [63]. Coolen and Coolen-Maturi [64] linked the (imprecise) probabilistic structure function to the survival signature.

Survival Signature for Single Component Type: The survival signature for a system with just one type of components, which can be denoted by $\Phi(l)$, for $l = 1, 2, ..., m$ is the probability that a system functions given that there are exactly l of its components functioning. It can be expressed as

$$\Phi(l) = P(system\ functions \mid l\ components\ are\ working) \tag{2.11}$$

For a coherent system with m components, it follows that $\Phi(0) = 0$ and $\Phi(m) = 1$. If exactly l components function, then in the state vector $\underline{x}$, there are precisely l of the x_i with $x_i = 1$, and all the remaining $x_i = 0$. For instance, if the system has m components overall, then there are $\binom{m}{l}$ different state vectors representing the system in this state. Letting S_l be the set of these state vectors, it can be known that $|S_l| = \binom{m}{l}$.

The survival signature for a system with one type of independent and identically distributed components can be expressed by

$$\Phi(l) = \frac{\sum_{x \in S_l} \phi(\underline{x})}{|S_l|} = \binom{m}{l}^{-1} \sum_{x \in S_l} \phi(\underline{x}) \tag{2.12}$$

It can be seen from the above equation that the survival signature is the sum of the structure functions for all relevant state vectors, divided by the number of such state vectors.

In fact, for a system with m components of a single type, the survival signature is similar in nature to the system signature. Coolen et al [48] derived a relationship between survival signature and system signature as follows

$$\Phi(l) = \sum_{j=m-l+1}^{m} s_j \tag{2.13}$$

where $\Phi(l)$ is the survival signature with exactly l functioning components, while s_i is the system signature for the component that fails at the ith stochastic ordering.

Example: Take the system in Figure 2.3 for instance, assume that all the three components are in the same type. In the case that $l = 2$, the survival signature can be obtained by Equation 2.12 as $\Phi(2) = \binom{3}{2}^{-1} \times 2 = \frac{2}{3}$.

As calculated before, the signature of this parallel-series system is $S = (\frac{1}{3}, \frac{2}{3}, 0)$. According to Equation 2.13, $\Phi(2) = \sum_2^3 s_i = s_2 + s_3 = \frac{2}{3} + 0 = \frac{2}{3}$, which is the same result as before.

Survival Signature for Multiple Component Types: Now consider a system with $K \geq 2$ types of m components, with m_k indicating the number of components of each type and $\sum_{k=1}^{K} m_k = m$. It is assumed that the failure times of the same component type are independently and identically distributed (iid) or exchangeable. The components of the same type can be grouped together because of the arbitrary ordering of the components in the state vector, which means that a state vector can be written as $\underline{x} = (\underline{x}^1, \underline{x}^2, ..., \underline{x}^K)$, with $\underline{x}^k = (x_1^k, x_2^k, ..., x_{m_k}^k)$ representing the states of the components of type k.

Coolen et al. [48] introduced the survival signature for such a system, denoted by $\Phi(l_1, l_2, ..., l_K)$, with $l_k = 0, 1, ..., m_k$ for $k = 1, 2, ..., K$, which is defined to be the probability that the system functions given that l_k of its m_k components of type k work, for each $k \in \{1, 2, ..., K\}$. There are $\binom{m_k}{l_k}$ state vectors $\underline{x}^k$ with precisely l_k components x_i^k equal to 1, so with $\sum_{i=1}^{m_k} x_i^k = l_k$ $(k = 1, 2, ..., K)$, and $S_{l_1,l_2,...,l_K}$ denotes the set of all state vectors for the whole system.

Assuming that the random failure times of components of the different types are fully independent, and in addition the components are exchangeable within the same component types, the survival signature can be rewritten as

$$\Phi(l_1, ..., l_K) = [\prod_{k=1}^{K} \binom{m_k}{l_k}^{-1}] \times \sum_{\underline{x} \in S_{l_1,l_2,...,l_K}} \phi(\underline{x}) \tag{2.14}$$

Example: Consider a typical complex bridge system shown in Figure 2.5, which consists of $m = 6$ components belonging to $K = 2$ different types, with $m_1 = 3$ and $m_2 = 3$. Therefore, the survival signature $\Phi(l_1, l_2)$ must be specified for all $l_1 \in \{0, 1, 2, 3\}$ and $l_2 \in \{0, 1, 2, 3\}$. The state vector of the system is $\underline{x} = (x_1^1, x_2^1, x_3^1, x_1^2, x_2^2, x_3^2)$.

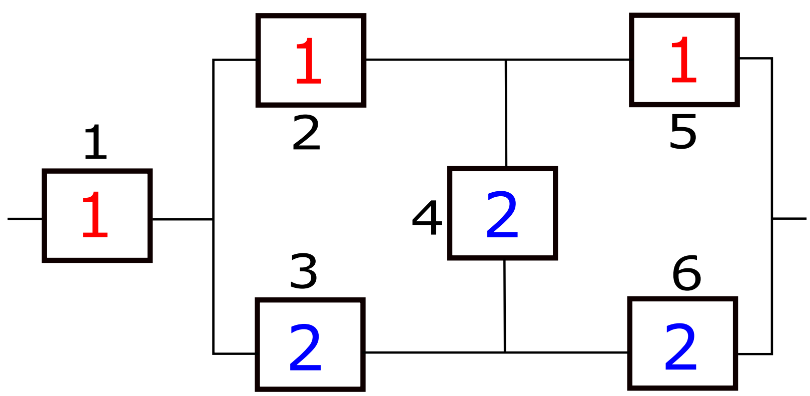

Figure 2.5: A typical complex bridge system with two types of components: the number outside the box is the component index, while the number inside the box represents the component type.

Now let us calculate the survival signature of $\Phi(1, 2)$ in detail. According to the definition of survival signature, $\Phi(1, 2)$ is the probability that the system works if precisely 1 component of type one and 2 components of type two function, which means considering the state vectors with $x_1^1 + x_2^1 + x_3^1 = 1$ and $x_1^2 + x_2^2 + x_3^2 = 2$. There are altogether $\binom{3}{1}\binom{3}{2} = 9$ such state vectors. However, only one of them can make the system work.

To be specific, only $\underline{x} = (1,0,1,0,0,1)$, which means component 1, component 3 and component 6 function, can lead the system to works.

Since there is an assumption that the components within the same type are independent and identically distributed, while the components belonging to different types are independent, all these 9 state vectors are equally likely to occur, so $\Phi(1,2) = \frac{1}{9}$. The other results of survival signature can be calculated in a similar way, and can be seen in Table 2.2.

Table 2.2: Survival signature of the system in Figure 2.5

l_1	l_2	$\Phi(l_1, l_2)$
0	0	0
0	1	0
0	2	0
0	3	0
1	0	0
1	1	0
1	2	1/9
1	3	1/3
2	0	0
2	1	0
2	2	4/9
2	3	2/3
3	0	1
3	1	1
3	2	1
3	3	1

2.5 Numerical Tools

2.5.1 R Package

Recently, the "ReliabilityTheory" R package proposed by Aslett [58] [65] makes it convenient to calculate the survival signature for a complex system with multiple component types.

This package is an enumerative algorithm for which each possible state vector is evaluated in turn. Since there are altogether 2^m possible component state vectors for the system, the computational expense of this approach grows exponentially as m increases. Thus, it becomes time consuming or infeasible to calculate the survival signature for large scale complex systems.

In summary, a new method is required to calculate the survival signature of large and complex real world systems and networks.

2.5.2 An Efficient Algorithm for Calculating Survival Signature of Complex Systems

The algorithm collaborates with Reed in this part is based on the work of Reed in [66]. The state vector count or survival signature values for a system, such as a lifeline network, can be represented by a multidimensional array. The values stored at index $(l_1, ..., l_K)$ of the array stores the value corresponding to $l_1, ..., l_K$ components of types 1 to K surviving. However, computing these arrays using enumerative methods becomes quickly infeasible since the number of state vectors to consider is equal to 2^m and therefore the computational complexity grows exponentially with the number of components in the network. An efficient algorithm for computing the multidimensional array representation of the survival signature for a system, based on the use of the reduced ordered binary decision diagrams (BDD) data structure, is proposed.

A BDD [67] is a data structure in the form of a rooted directed acyclic graph which can be used to compactly represent and efficiently manipulate a Boolean function. It is based upon Shannon decomposition theory [68]. The Shannon decomposition of a Boolean function f on Boolean variable x_i is defined as $f = x_i \wedge f_{x_i=1} + \overline{x_i} \wedge f_{x_i=0}$ where $f_{x_i=v}$ is f evaluated with $x_i = v$. Each BDD contains two terminal nodes that represent the Boolean constant values 1 and 0, whilst each non-terminal node represents a subfunction g, is labelled with a Boolean variable v and has two outgoing edges. By applying a total ordering on the m Boolean variables for function f by mapping them to the integers $x_0, ..., x_{m-1}$, and applying the Shannon decomposition recursively to f, it can be represented as a binary tree with $m + 1$ levels. Note that the chosen ordering can have a significant influence on the size of the BDD [69]. Each intermediate node, referred to as an if-then-else (ite) node, at level $l \in \{0, ..., m - 1\}$ (where the root node is at level 0 and the nodes at level $m-1$ are adjacent to the terminal nodes) represents a Boolean function g on variables $x_l, x_{l+1}, ..., x_{m-1}$. It is labelled with variable x_l and has two out edges called 1-edge and 0-edge linking to nodes labelled with variables higher in the ordering. 1-edge corresponds to $x_l = 1$ and links to the node representing $g_{x_l=1}$, whist 0-edge corresponds to $x_l = 0$ and links to the node representing $g_{x_l=0}$. In addition, the following two reduction rules are applied. Firstly, the isomorphic subgraphs are merged; and secondly, any node whose two children are isomorphic is eliminated.

Complement edges [70] are an extension to standard BDDs that reduce memory size and the computation time. A complement edge is an ordinary edge that is marked to indicate that the connected child node (at a higher level) has to be interpreted as the complement of its Boolean function. The use of complement edges is limited to the 0-edges to ensure canonicity.

The BDD representing the system structure function for a network can be computed in various ways, e.g. from its cut-sets or network decomposition based methods [71]. In order to show the implementation of the approach, a simple network with 4 nodes and 4 edges is considered and shown in Figure 2.6.

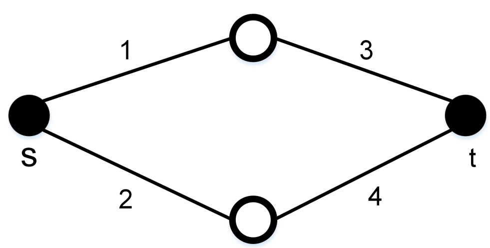

Figure 2.6: A simple network with 4 nodes and 4 edges.

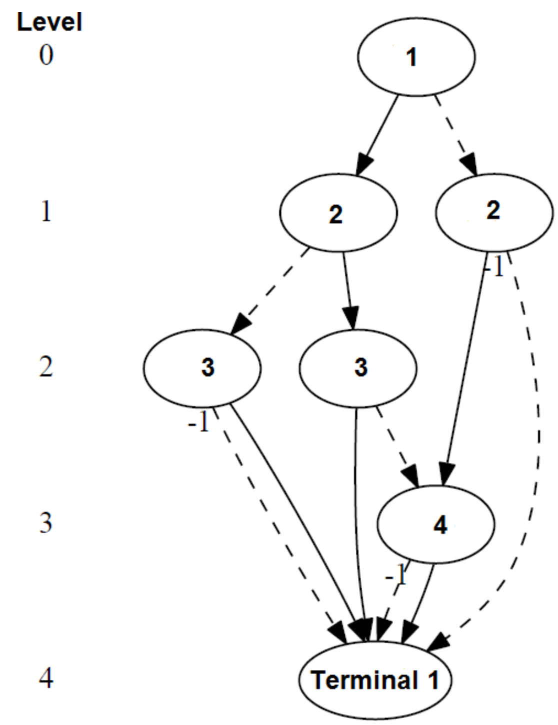

Figure 2.7: BDD for the simple network from Figure 2.6.

The corresponding BDD representing the structure function of this network, where the dashed edges represent 0-edges (marked with -1 if complemented) and solid edges represent 1-edges, is shown in Figure 2.7. The survival signature from a BDD representation of the system structure function for a network can then be calculated through the iterative algorithm described by Figure 2.8.

The number of operations performed during the execution of the algorithm grows approximately linearly with the number of nodes in the BDD. In general, the BDD representation of the structure function for a network has far fewer nodes than 2^m nodes. It is therefore far more computationally efficient than using enumerative algorithms.

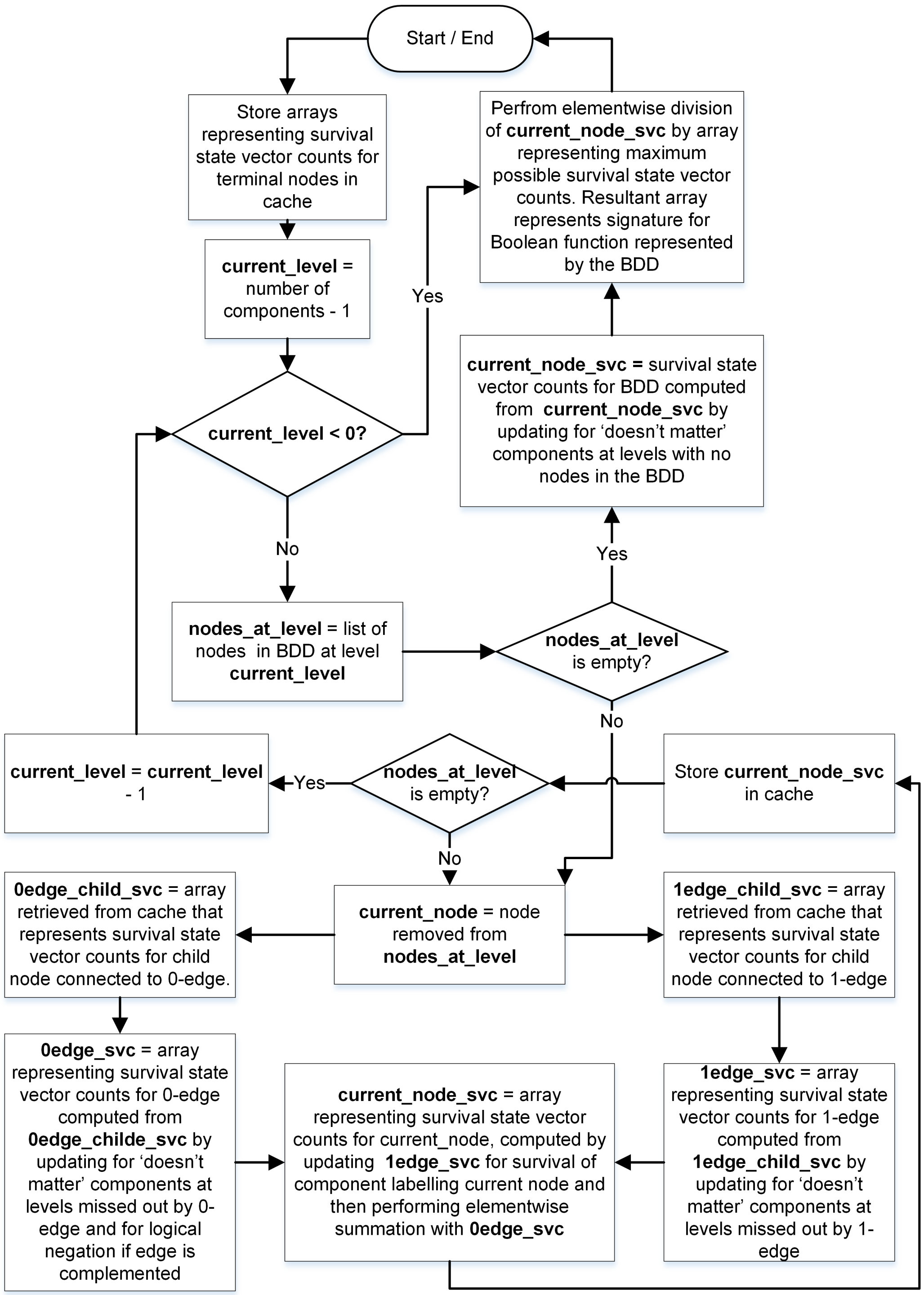

Figure 2.8: Algorithm for computing signature from the BDD representation of a system structure function.

2.5.3 OpenCossan

The OpenCossan engine is an invaluable tool for uncertainty quantification and management [72]. All the algorithms and methods have been coded in a Matlab toolbox allowing numerical analysis, reliability analysis, simulation, sensitivity, optimization, robust design and much more.

This thesis uses OpenCossan codes for uncertainty quantification and reliability analysis on the complex systems.

Chapter 3

Complex System Reliability Analysis Based on Survival Signature

3.1 Introduction

Most systems consist of multiple types of components in reality, contravening the iid assumption of system signature. It would be difficult to use system signature to assess the reliability of this kind of systems, because it is hard to find rankings of order statistics of component failure times of different types. Therefore, survival signature is recognised as a better method to perform reliability analysis on complex systems and networks with multiple component types.

In this Chapter, a reliability approach based on the survival signature is proposed to analyse systems and networks with multiple types of components. The proposed approach allows us to include explicitly imprecision and vagueness in the characterisation of the uncertainties of system components. The imprecision characterises indeterminacy in the specification of the probabilistic model. That is, an entire set of plausible probabilistic models is specified using set-values (herein, interval-valued) descriptors for the description of the probabilistic model. The cardinality of the set-valued descriptors reflects the magnitude of imprecision and, hence, the amount and quality of information that would be needed in order to specify a single probabilistic model with sufficient confidence.

In Section 3.2, the reliability assessment on systems with multiple component types is discussed. The method is based on the survival signature, which not only holds the merits of system signature, but is suitable for analysing large and complex systems and networks. Then, Section 3.3 takes uncertainty within the system into account. Both analytical methods and simulation methods are used to deal with the imprecision. After that, two survival signature-based Monte Carlo simulation methods are proposed to evaluate the system reliability in an efficient way in Section 3.4. The proposed approaches of the improved survival signature are demonstrated by some examples in Section 3.5.

3.2 Reliability Assessment on Complex System with Multiple Component Types

Now let us quantify reliability for a system with multiple component types. Assume that $C_k(t) \in \{0, 1, ..., m_k\}$ denotes the number of type k components working at time t. Assume that the components of the same type have a known CDF, $F_k(t)$ for type k. Moreover, the failure times of different component types are assumed independent. Then:

$$\begin{aligned} P(\bigcap_{k=1}^{K}\{C_k(t) = l_k\}) &= \prod_{k=1}^{K} P(C_k(t) = l_k) \\ &= \prod_{k=1}^{K} \binom{m_k}{l_k} [F_k(t)]^{m_k - l_k} [1 - F_k(t)]^{l_k} \end{aligned} \tag{3.1}$$

where $C_k(t) = l_k$ means that at time t, precisely l_k components of type k are working. Hence, the survival function of the system with K types of components becomes:

$$\begin{aligned} P(T_s > t) &= \sum_{l_1=0}^{m_1} ... \sum_{l_K=0}^{m_K} \Phi(l_1, ..., l_K) P(\bigcap_{k=1}^{K}\{C_k(t) = l_k\}) \\ &= \sum_{l_1=0}^{m_1} ... \sum_{l_K=0}^{m_K} \Phi(l_1, ..., l_K) \prod_{k=1}^{K} \binom{m_k}{l_k} [F_k(t)]^{m_k - l_k} [1 - F_k(t)]^{l_k} \end{aligned} \tag{3.2}$$

It is obvious from Equation 3.2 that the survival signature can separate the structure of the system from the failure time distribution of its components, which is the main advantage of the system signature. To be specific, the survival signature part $\Phi(l_1, ..., l_K)$ takes the structure of the system into consideration, which is how the state of the components influence the system performance. The part of $\prod_{k=1}^{K} P(C_k(t) = l_k)$ only depends on $F_k(t)$, which is the lifetime distribution of the components of type k.

In addition, the survival signature only needs to be calculated once for any system, which is similar to the system signature for systems with only single type of components.

It is easily seen that survival signature is closely related with system signature. For a special case of a system with only one type ($K = 1$) of components, the survival signature and Samaniego's signature are directly linked to each other through Equation 2.13. However, the latter cannot be easily generalised for systems with multiple types ($K \geq 2$) of components.

This implies that all attractive properties of the system signature also hold for the method using the survival signature, also the survival signature is easy to apply for systems with multiple types of components, and one could argue it is much easier to interpret than the system signature. Furthermore, the difficulty of finding the probabilities of rankings of order statistics from different probability distributions can be avoided, which is indeed a simplification of computation. Finally, the quite simple survival signature (in particular for large systems with only relatively few different component types) and its monotonicity for coherent systems provides clear advantages to work towards approximations of the system reliability metrics. This does not limit the applicability of the survival signature to non-coherent systems (for example, electricity distribution network or part of the electronic equipment of safety features).

3.3 Generalised Probabilistic Description of the Failure Times of Components

Reliability analysis of complex systems requires the probabilistic characterisation of all the possible component transitions. This usually requires a large data-set that is not always available. In fact, it might not be possible to unequivocally characterise some component transitions due to lack of data or ambiguity. To avoid the inclusion of subjective knowledge or expert opinions, the imprecision and vagueness of the data can be treated by using concepts of imprecise probabilities.

Imprecise probability combines probabilistic and set theoretical components in a unified construct (see e.g. [73] [74] [75]). It allows a rational treatment of the information of possibly different forms without ignoring significant information, and without introducing unwarranted assumptions. For instance, if only few data points are available it might be difficult to identity the parameters and the form of a distribution. An unknown value of a (deterministic) parameter that is often modelled using a uniform distribution based on the principle of maximum entropy should be modelled as an interval and not as a distribution [76]. In the analysis, imprecise probabilities combine, without mixing, randomness and imprecision. Randomness and imprecisions are considered simultaneously but viewed separately at any time during the analysis and in the results. The probabilistic analysis is carried out conditional on the elements from the sets, which leads eventually to sets of probabilistic results, see e.g. [77].

Considering the imprecision in the component parameters will lead to bounds of survival function of the systems and it can therefore be seen as a conservative analysis, in the sense that it does not make any additional hypothesis with regard to the available information. In some instances analytical methods will not be appropriate means to analyse a system. Again, simulation methods based on survival signature can be adopted to study systems considering parameter imprecision. A naive approach consists in adopting a double loop sampling where the outer loop is used to sample realization in the epistemic space. In other words, each realization in the epistemic space defines a new probabilistic model that needs to be solved adopting the simulation methods proposed above. Then the envelopment of the system reliability is identified.

3.3.1 Introduction of Probability Box

As stated in the previous section, the probability of the failure of each component is described by the CDF, $F_k(t)$. However, it is not always possible to fully characterise the probabilistic behaviour of components due to ignorance or incomplete knowledge. This lack of knowledge comes from many sources: in-adequate understanding of the underlying processes, imprecise evaluation of the related characteristics, or incomplete knowledge of the phenomena. These problems can be tackled by resorting to generalised probabilistic methods, such as imprecise probabilities, see e.g. [78] [79] [80]. The main problem of generalised probabilistic methods is the computational cost associated with their evaluation. In fact, these approaches required multiple probabilistic model evaluations, and often use global optimization procedures. Efficient numerical methods have been developed and made available in powerful toolboxes such as OpenCossan software [81] [82].

The generalised probabilistic model makes the uncertainty quantification a rather challenging task in terms of computational cost, and the challenge comes mainly from computing the lower and upper bounds of the quantities of interest. Let $\underline{F}$ and $\overline{F}$ be non-decreasing functions mapping the real line $\Re$ into [0,1] and $\underline{F}(x) \leq \overline{F}(x)$ for all $x \in \Re$. Let $[\underline{F}, \overline{F}]$ denote a set of the non-decreasing functions F on the real line such that $\underline{F}(x) \leq F(x) \leq \overline{F}(x)$. When the functions $\underline{F}$ and $\overline{F}$ circumscribe an imprecisely known probability distribution, $[\underline{F}, \overline{F}]$ is called a "probability box" or "p-box" [83]. Using the framework of imprecise probabilities in form of a p-box (see Ref [84]), the lower and upper CDF for the failure times of components of type k are denoted by $\underline{F}_k(t)$ and $\overline{F}_k(t)$, respectively. The lower and upper CDF bounds can be obtained by calculating the range of all distributions that have parameters within some intervals. For some distribution families, only two CDFs need to be computed to enclose the p-box. For most distribution families, however, four or more crossing CDFs need to be computed to define a p-box, see Ref [85].

For instance, let us assume that X has a Lognormal distribution with imprecise parameters ([0.5,0.6], [0.05,0.1]). The p-box of event X is calculated by taking all combinations of (0.5, 0.05), (0.6, 0.05), (0.5, 0.1) and (0.6, 0.1) into account. Figure 3.1 reflects all these combinations of distributions for event X, while Figure 3.2 shows the p-box for event X.

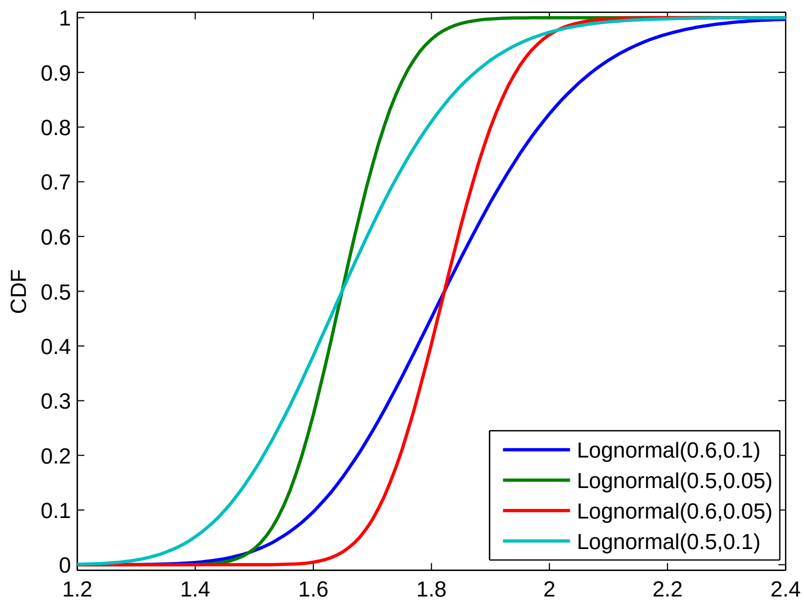

Figure 3.1: All combinations of distributions for event X.

3.3.2 Analytical Method to Deal With Imprecision Within Components' Failure Times

Lower and upper bounds of the survival function for a system consisting of multiple types of components can be calculated analytically based on Coolens works for non-parametric predictive inference in [57]. $C_k(t)$ denotes the number of k type components working at time t, and it is assumed that the components cannot be repaired or replaced. The lower survival function is:

$$\underline{P}(T_S > t) = \sum_{l_1=0}^{m_1} ... \sum_{l_K=0}^{m_K} \Phi(l_1, ..., l_K) \prod_{k=1}^{K} \overline{D}(C_k(t) = l_k) \tag{3.3}$$

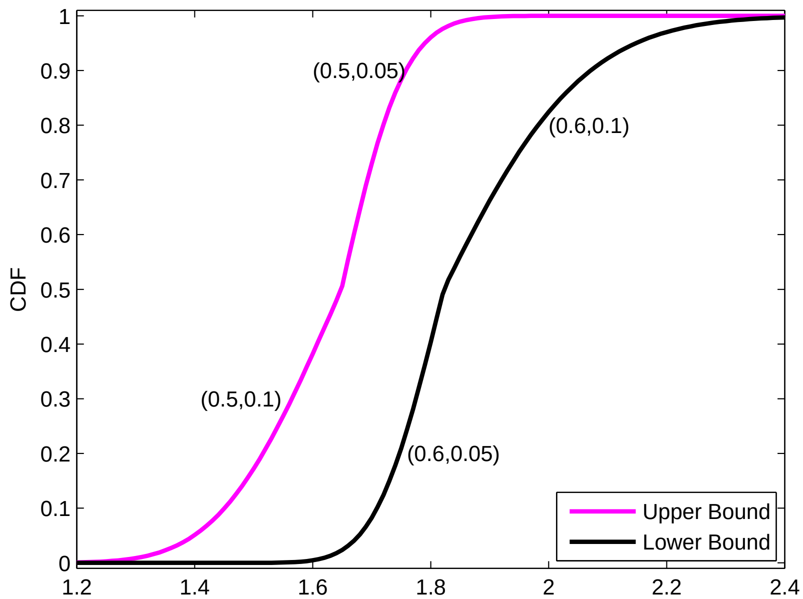

Figure 3.2: P-box for event X.

where

$$\overline{D}(C_k(t) = l_k) = \overline{P}(C_k(t) \leq l_k) - \overline{P}(C_k(t) \leq l_k - 1) \tag{3.4}$$

while the corresponding upper bound of the survival function is:

$$\overline{P}(T_S > t) = \sum_{l_1=0}^{m_1} ... \sum_{l_K=0}^{m_K} \Phi(l_1, ..., l_K) \prod_{k=1}^{K} \underline{D}(C_k(t) = l_k) \tag{3.5}$$

where

$$\underline{D}(C_k(t) = l_k) = \underline{P}(C_k(t) \leq l_k) - \underline{P}(C_k(t) \leq l_k - 1) \tag{3.6}$$

For a system with m components in one type, C_t is represented by a binomial distribution, with $C_t \sim Binomial(m, 1 - F(t))$. According to stochastic dominance theory [86], C_t increases as $(1 - F(t))$ increases.

For a parametric distribution, the CDF of a components failure time can be expressed by $F(t \mid \theta)$, with $\theta \in \Theta$ (e.g. parameter $\theta \in [\underline{\theta}, \overline{\theta}]$). Therefore, there will be a $\underline{\theta} \in \Theta$ leading to $F(t \mid \underline{\theta}) = \underline{F}(t)$ and a $\overline{\theta} \in \Theta$ leading to $F(t \mid \overline{\theta}) = \overline{F}(t)$, which holds for all t.

Here, take an exponential distribution with parameter $\lambda \in [\lambda_1, \lambda_2]$ as an example. It is known that $\underline{F}(t) = F(t \mid \lambda_1) = 1 - e^{-\lambda_1 t}$ and $\overline{F}(t) = F(t \mid \lambda_2) = 1 - e^{-\lambda_2 t}$. C_t increases as $(1 - F(t))$ increases, so $\underline{P}(C_t \leq l) = \sum_{u=0}^{l} \binom{m}{u} (1 - e^{-\lambda_2 t})^{m-u} (e^{-\lambda_2 t})^u$ and $\overline{P}(C_t \leq l) = \sum_{u=0}^{l} \binom{m}{u} (1 - e^{-\lambda_1 t})^{m-u} (e^{-\lambda_1 t})^u$.

For a system with one type of components, the lower bound of the survival function for the system at time t becomes:

$$\underline{P}(T_S > t) = \sum_{l=0}^{m} \Phi(l) \binom{m}{l} (1 - e^{-\lambda_1 t})^{m-l} (e^{-\lambda_1 t})^l \tag{3.7}$$

and the corresponding upper bound of the survival function becomes:

$$\overline{P}(T_S > t) = \sum_{l=0}^{m} \Phi(l) \binom{m}{l} (1 - e^{-\lambda_2 t})^{m-l} (e^{-\lambda_2 t})^l \tag{3.8}$$

For a system composed of $K \geq 2$ types of components, with parameter $\lambda^k \in [\lambda_1^k, \lambda_2^k]$, the lower bound of the survival function for the system at time t is:

$$\underline{P}(T_S > t) = \sum_{l_1=0}^{m_1} ... \sum_{l_K=0}^{m_K} \Phi(l_1, ..., l_K) \prod_{k=1}^{K} \binom{m_k}{l_k} [1 - e^{-\lambda_1^k t}]^{m_k - l_k} [e^{-\lambda_1^k t}]^{l_k} \tag{3.9}$$

The corresponding upper bound of the survival function becomes:

$$\overline{P}(T_S > t) = \sum_{l_1=0}^{m_1} ... \sum_{l_K=0}^{m_K} \Phi(l_1, ..., l_K) \prod_{k=1}^{K} \binom{m_k}{l_k} [1 - e^{-\lambda_2^k t}]^{m_k - l_k} [e^{-\lambda_2^k t}]^{l_k} \tag{3.10}$$

To illustrate the method presented in this Section, the lower and upper bounds of survival function for the typical complex system in Figure 3.3 are calculated.

The system has six components which belonging to two types. Results of survival signature of the system can be calculated through "ReliabilityTheory" R package and can be seen in Table 3.1.

The failure times of the two component types are according to the exponential distribution, with interval parameters $\lambda_1 \in [0.4, 1.2]$ and $\lambda_2 \in [1.3, 2.1]$, respectively. This leads to lower and upper bounds of survival functions of the system as seen in Figure 3.6.

For other distribution types, like Weibull distribution or gamma distribution, if the shape parameter is fixed, the upper and lower bounds of survival function can be deduced

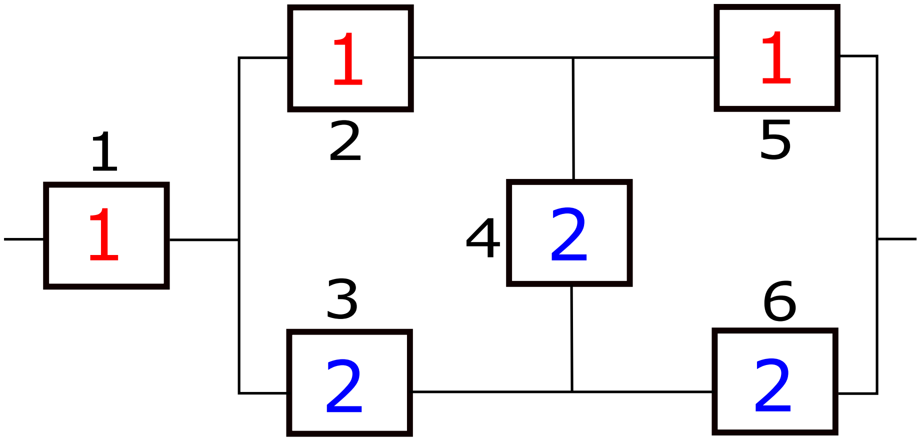

Figure 3.3: System with two types of components.

Table 3.1: Survival signature of the bridge system of Figure 3.3

l_1	l_2	$\Phi(l_1, l_2)$
0	$[0, 1, 2, 3]$	0
$[1, 2]$	$[0, 1]$	0
1	2	1/9
1	3	1/3
2	2	4/9
2	3	2/3
3	$[0, 1, 2, 3]$	1

in a similar way as shown for the exponential distribution type. However, if shape parameter is in an interval, finding the lower bound of survival function reduces to an optimisation problem over one variable (shape parameter) only. Also, if all the parameters have interval values, by means of simulation method is a replacement to calculate the probability bounds of the survival function.

3.3.3 Imprecise System

In the real engineering application, due to confidential contracts, specific configuration of part of the lifeline network might also not be known exactly and considered as a "grey box", which leads to the use of imprecise survival signature. Bounds of the survival function become

$$\underline{P}(T_s > t) = \sum_{l_1=0}^{m_1} ... \sum_{l_K=0}^{m_K} (\underline{\Phi}(l_1, ..., l_K))P(\bigcap_{k=1}^{K} \{C_k(t) = l_k\}) \tag{3.11}$$

and

$$\overline{P}(T_s > t) = \sum_{l_1=0}^{m_1} ... \sum_{l_K=0}^{m_K} (\overline{\Phi}(l_1, ..., l_K))P(\bigcap_{k=1}^{K} \{C_k(t) = l_k\}) \tag{3.12}$$

3.4 Proposed Simulation Methods

The analytical approach involves the determination of a mathematical expression which describes the reliability of the system, expressed in terms of the reliabilities of the components. In other words, in the analytical approach, the system performance is obtained analytically from each component's failure distribution using probability theory. Therefore, analytical solutions offer great power and speed in system analysis [87].

However, when the system is complicated and large, it maybe difficult to derive an analytical formulation for the system reliability, which makes this process quite time consuming. What is more, if the system is repairable or other maintainability information is taken into account, system reliability analysis through an analytical approach will become very difficult and restrictive. Thus, a simulation method is straightforwardly used to deal with these complicated conditions.

The survival signature presented in the previous section can be adopted in a Monte Carlo based simulation method to estimate the system reliability in a simple and efficient way. A possible system evolution is simulated by generating random events (i.e. the random transition such as failure times of the system components) and then estimating the status of the system based on the survival signature (Equation 3.1). Then, counting the occurrence number of a specific condition (e.g. counting how many times the system is in working status), it is possible to estimate the reliability of the system. In this Section, two Monte Carlo simulation methods adopting the survival signature are presented. To be specific, the novel Algorithms 1 and 2 are used to estimate the reliability of non-repairable systems.

3.4.1 Algorithm 1

The first simulation method is based on the realisations of failure events of the system's components. For each failure event the status of the system is generated based on the probability that the system is working knowing that a specific number of components are working. Such probability is given by the survival signature as defined in Equation (2.14). The survival signature is computed only once before starting the Monte Carlo simulation.

Suppose there is a system with C components, K component types and m_k components of type k. Hence, $C = \sum_{k=1}^{K} m_k$. We assume that components of type k have the same failure time distribution and that there is no repair opportunity for the components. The reliability of the system can be estimated adopting the following procedure:

Step 0. Initialise variables and counters (i.e. V_r);

Step 1. Sample the failure times for each component, f_i, for $i = 1, 2, \ldots, C$. The failure time of a component of type k is obtained by sampling from the corresponding CDF F_k;

Step 2. Order the sequence of failure times $t_i \leq t_{i+1}$ for $i = 1, 2, \ldots, M$. Hence, t_1 represents the first failure of a system component, t_2 represents the second failure and so on;

Step 3. At each failure time, it is easy to calculate the number of components working for each component type: $C_k(t_i)$;

Step 4. Evaluate the survival signature which applies immediately after the corresponding failure indicated as $\Phi_{t_i} \equiv \Phi(C_1(t_i), C_2(t_i), \ldots, C_K(t_i))$;

Step 5. Draw from a Bernoulli distribution with probability $1 - \Phi_{t_1}$ the system status X_1 at time t_1, if $X_1 = 1$ the system fails;

Step 6. If the system does not fail at t_1, then consider t_2. The probability that the system functions at time t_2 is $\Phi_{t_2}/\Phi_{t_1} = q_2$, given that it has survived at time t_1. So the system status at time t_2, X_2, is drawn from a Bernoulli distribution with the probability $1 - q_2$;

Step 7. Repeat Step 6 to process other failure times, setting $i = i + 1$;

Step 8. Store the status of the system over the time, as follows: $Vr(j) = Vr(j) + 1 \quad \forall j : j \cdot dt < t_f$ where t_f is the system failure time and dt represents the discretisation time.

The above procedure is repeated for N samples and the estimate of the survival function is obtained by averaging the vector collecting the status of the system over the number of samples: $P(T_s > t) \approx \frac{Vr(t)}{N}$.

A pseudo-algorithm of the simulation method is shown in Algorithm 1.

This method simulates one system failure time in each run (Steps 1-7). It should be noted that with the assumption that the system fails if no component functions, this implies that there is an i^*, less than or equal to C, such that $q_{i^*} = 0$. Hence the system fails certainly at this t_{i^*} if it has not failed before.

Algorithm 1

Require: N: Number of simulations; *dt*: Discretisation time; F_k: CDF failure times, $Vc = [m_1, m_2, \ldots, m_k]$: Number of components per type; Nt: number of discretisation steps.

```
Set V_r(1 : Nt) = 0                                   ▷ Ititialise counter
Set C = sum(V_c)                                      ▷ Compute total number of components
Set Φ ← Survival signature                            ▷ Compute the survival signature
for n ← 1 : N do                                      ▷ loop over number of samples
    for k ← 1 : K do                                  ▷ loop over number of component type
        for j = 1 : m_k do                            ▷ loop over number of components
            Mf(j, k) ~ F_k                            ▷ Sample failure time component j of type k
        end for
    end for
    [Vt, Vi] = sort(Mf)                               ▷ Reorder transition times (Vt)
                                                      ▷ Return component index vector (Vi)
    Φ_Old = 1                                         ▷ Initialise variables
    for m ← 1 : C do                                  ▷ loop over number of components
        Vc(Vi(m)) = Vc(Vi(m)) − 1                     ▷ Update number working components
        Φ_Now ← Φ(Vc)
        q ← Φ_Now/Φ_Old
        if rand(1) < q ← then                         ▷ system working
            Φ_Old = Φ_New
        else
            for all j : j · dt < Vi(m) do
                Vr(m) = Vr(m) + 1                     ▷ Update counter
            end for
            Break                                     ▷ Process next sample
        end if
    end for
    Vr = Vr/N                                         ▷ Normalise counter
end for
```

3.4.2 Algorithm 2

It is possible to estimate the system reliability without the necessity to sample the system status at each component failure time. The idea is to interpret the survival signature as a normalised "production capability" of the system defined by the Equation 2.14. For instance, if all the components are working, the system output is 1. If all components are in failure status, the system output is 0. Hence, instead of sampling the system state at each failure time, the survival signature is evaluated to collect the "production level of the system", i.e. the survival signature is evaluated immediately after each sampled component failure time and collected in proper counters.

This can be obtained adopting the Algorithm 2 derived from the approach proposed in [54] to estimated the production availability of an offshore installation requiring the derivation of the complete status of the system (based on the structural function and cut-sets). Here, a novel algorithm is proposed to estimate the reliability adopting the survival signature and hence avoiding the tedious calculation of all the system status.

The reliability of the system can be estimated by modifying steps 5-7 of Algorithm 1 as follows:

Step 5'. Compute the production level of the system by evaluating the survival signature at each time of interest Φ_{t_i}. The probability that the system survives at time t_1 is Φ_{t_1};

Step 6'. Collect the value of the survival signature in the vector Vr representing the survival function as follows: $Vr(j) = Vr(j) + \Phi_{t_i} \quad \forall j : j \cdot dt < t_i$ where dt represents the discretisation time.

The above procedure is repeated for N samples and the reliability of the system is computed by averaging the values of the survival signature ($P(T_s > t) \approx \frac{1}{N} Vr(t)$). A pseudo-algorithm of the simulation method for non-repairable components is shown in Algorithm 2.

The uses of the survival signature makes this approach extremely efficient since it does not require to sample the system output at each component transition time (i.e. component failures). The flow chart of the simulation methods proposed for estimating the reliability of non-repairable systems is shown in Figure 3.4.

For each Monte Carlo simulation, this method generates a random grid of time points at which to evaluate the survival signature representing the survival probability of the system at those times. Finally, the survival function is obtained by directly averaging the survival signature over the time.

Algorithm 2 follows the productivity idea, which gives each run a possible survival function while Algorithm 1 gives a single system failure time in each run. Therefore,

Algorithm 2

Require: N: Number of simulations; *dt*: Discretisation time; F_k: CDF failure times, $Vc = [m_1, m_2, \ldots, m_k]$: Number of components per type; Nt: number of discretisation steps.

```
Set V_r(1 : Nt) = 0                        ▷ Ititialise counter
Set C = sum(V_c)                           ▷ Compute total number of components
Set Φ ← Survival signature                 ▷ Compute the survival signature
for n ← 1 : N do                           ▷ loop over number of samples
    for k ← 1 : K do                       ▷ loop over number of component type
        for j = 1 : m_k do                 ▷ loop over number of components
            Mf(j, k) ~ F_k                 ▷ Sample failure time component j of type k
        end for
    end for
    [Vt, Vi] = sort(Mf)                    ▷ Reorder transition times (Vt)
                                           ▷ Return component index vector (Vi)
    z = 1                                  ▷ Initialise index
    for m ← 1 : C do                       ▷ loop over number of components
        Ck(Vi(m)) = Ck(Vi(m)) − 1          ▷ Update number working components
        while z · dt ≤ Vt(m) do
            Vr(z) = Vr(z) + Φ(V_c)         ▷ Update counter
            z ← z + 1                      ▷ Update index
        end while
    end for
    Vr = Vr/N                              ▷ Normalise counter
end for
```

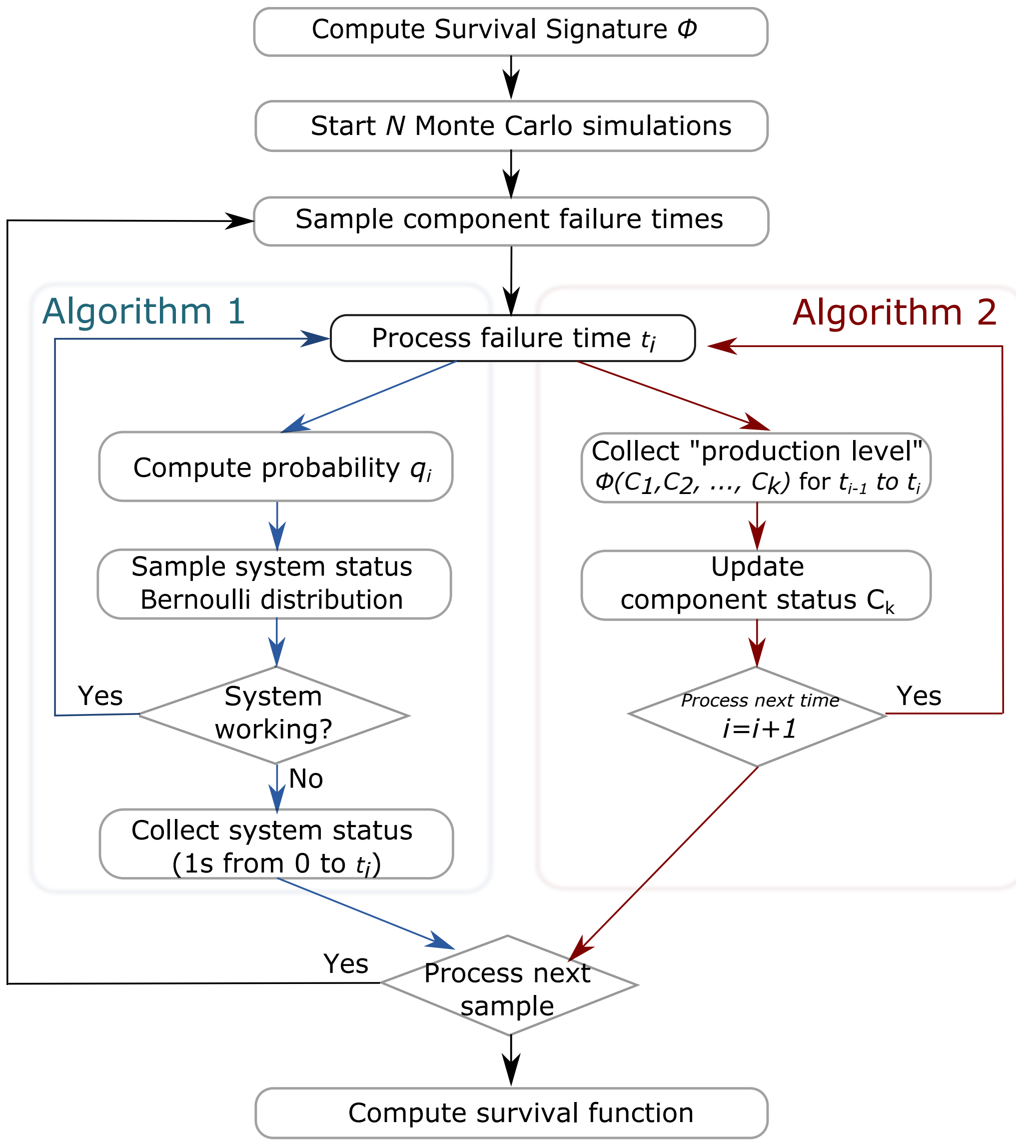

Figure 3.4: Flow chart of Algorithms 1-2.

Algorithm 1 is useful for inference where one explicitly wants the simulated system failure times, whilst Algorithm 2 is efficient for inference on the system survival function.

It can be shown that the variance of the survival function estimator at each time of interest obeys the following formula [88]:

$$Var[Vr(t)] \approx \frac{1}{N}\left(\overline{Vr^2(t)} - \overline{Vr(t)}^2\right) \tag{3.13}$$

where N represents the number of samples and $\overline{Vr^2(t)}$ the mean of the squared values of the survival function at time t and $\overline{Vr(t)}^2$ the square of the mean values of the survival function at time t. Also, in Equation 3.13 it is common to substitute $N-1$ in place of N although the correction is negligible because $N \gg 1$. The Algorithm 2 tends to lead to better estimates of the system reliability when compared to Algorithm 1, as detailed in Section 3.5 and shown in Figure 3.10.

3.4.3 Simulation Method to Deal With Imprecision Within Components' Failure Times

Let us use the system in Figure 3.3 as an example to illustrate the simulation method. The survival signature represents the probability that the system works given that the number of components of each type that are working. The system in Figure 3.3 is equivalent to a system composed by two components that can be in four types of status (status 0 to status 3) as shown in Table 3.1. Each status represents the number of working components.

The method used to simulate the survival function is derived from the production level approach proposed in [54]. The simulation approach requires the following steps:

Step 1. Sampling the transition times of the first component type, hence a sequence of transition times t_1, t_2, t_3 and t_4 can be obtained;

Step 2. Repeating the procedure of step (1) for the component type 2, which will obtain 4 additional transition times;

Step 3. Reordering all the transition times of $(t_1, t_2, ..., t_8)$;

Step 4. For each time interval the probability that the system functions can be computed based on survival signature;

Step 5. Repeating the steps (1) to (4) for n system histories and averaging the obtained results;

Step 6. The system probability of survival over the time t is obtained by averaging the values of the survival functions.

The above simulation procedures are used for components without imprecision. A general schematic diagram for one simulation sample can be seen in Figure 3.5.

The probabilistic uncertainty and imprecision in component parameters are challenging phenomena in reliability analysis of complex systems. To solve the parameter epistemic imprecision within components, it is just needed to add an optimization loop around the survival signature-based simulation method which are proposed above. In other words,

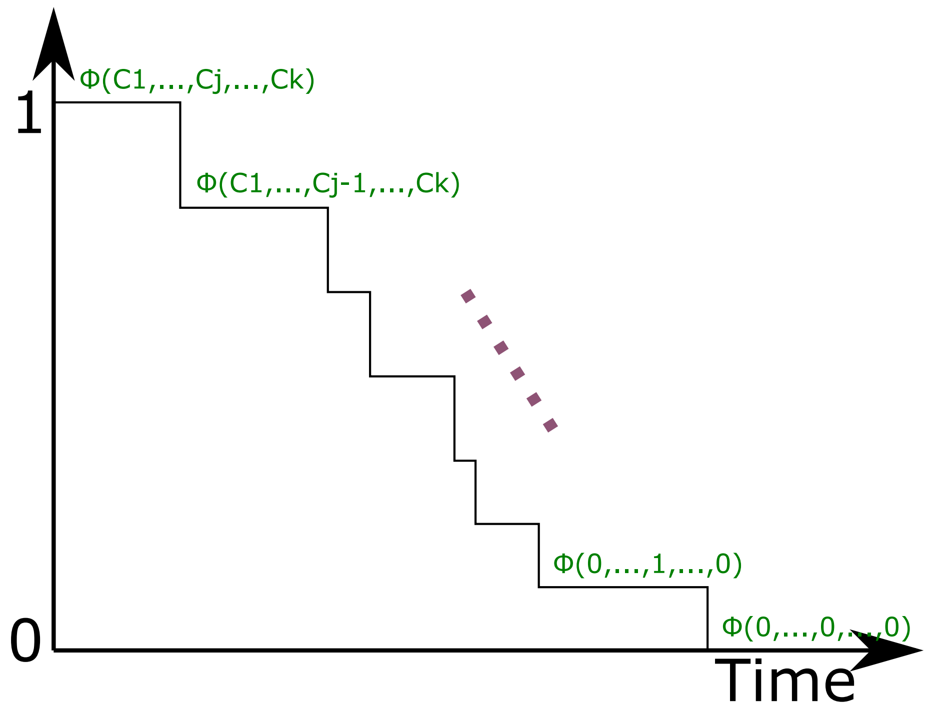

Figure 3.5: A general schematic diagram for one simulation sample.

it can be done by adding a simple Monte Carlo loop and sampling the values of component parameters from uniform distributions.

The double loop sampling involves two layers of sampling: the outer loop is called the parameter loop since it concerns sampling different values for the set of distribution parameters for all of the uncertain quantities; while the inner loop goes by the name of probability loop because it involves sampling from precise probability distribution functions. As a matter of fact, double loop sampling implicates sampling from an analytical distribution whose parameters have been generated by sampling.

Figure 3.6 shows the lower and upper bounds of survival function obtained by simulation method and compared with the analytical solution, and shows excellent agreement.

Therefore, this simulation method not only has the advantage of survival signature to handle complex systems reliability problems, but can recur to Monte Carlo simulation to deal with the uncertainties within the systems. Furthermore, the simulation method can be used for analysing any systems with general imprecision. Suppose components failure times of type 1 and type 2 obey Weibull distribution and gamma distribution, respectively. Their imprecise parameters can be seen in Table 3.2.

It is difficult to determine the bounds of survival function by an analytical method. However, this problem can be tackled through a simulation method. The results are shown in Figure 3.7.

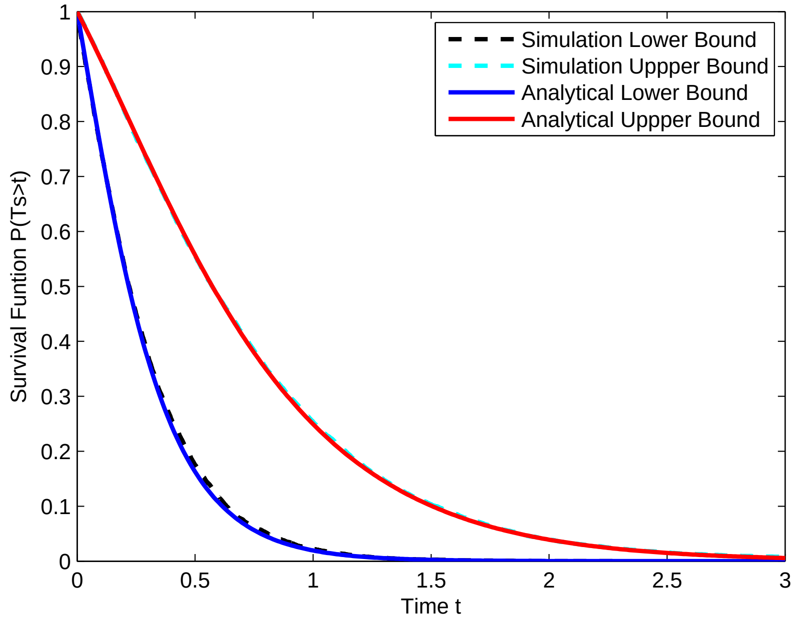

Figure 3.6: Lower and upper bounds of the survival function obtained by simulation and analytical method.

Table 3.2: Imprecise distribution parameters of components in a system

Component type	Distribution type	Parameters (α, β)
1	Weibull	([1.2,1.8], [2.3,2.9])
2	Gamma	([0.8,1.6], [1.3,2.1])

3.5 Numerical Examples

3.5.1 Circuit Bridge System

The purpose of this numerical example is to verify the proposed two algorithms since for this simple problem analytical solutions are available. The system configuration is represented in Figure 3.3, $k = 1, 2$. The circuit bridge system comprises six components, which belong to two types. It has no series section or parallel section which can enable simplification. The survival signature can easily be computed either manually or using the R-package "ReliabilityTheory". The values of the survival signature are reported in Table 3.1, where l_1 and l_2 indicate the numbers of working components of type $k = 1$ and $k = 2$, respectively and $\Phi(l_1, l_2)$ is the survival signature of the bridge system.

In this example the failure times of both component types 1 and 2 obey exponential distributions with parameters $\lambda_1 = 0.8$ and $\lambda_2 = 1.5$, respectively, i.e. the components have a constant mean time to failure. It is also assumed that the component once failed

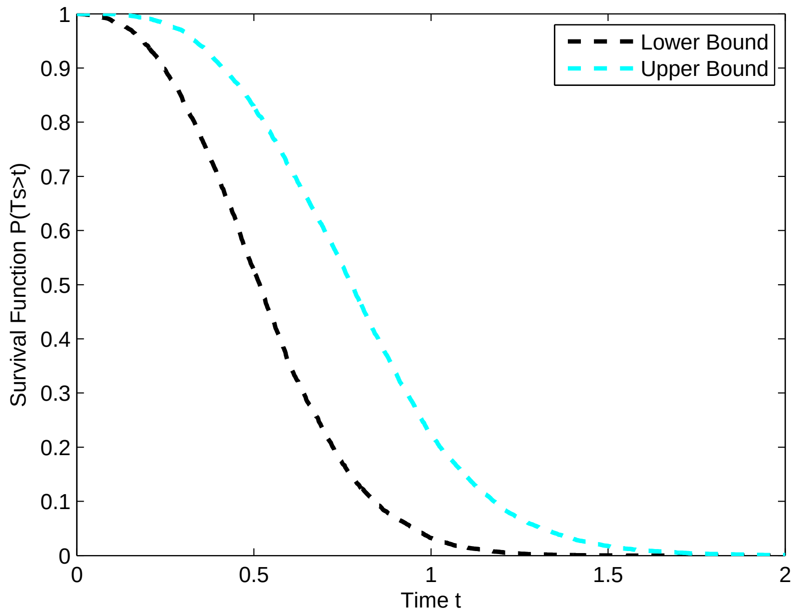

Figure 3.7: Lower and upper bounds of survival function by simulation method.

can not be repaired.

The survival function of the bridge system is then calculated by means of Algorithms 1 and 2. The resulting functions are then compared with the analytical solution. The survival function can be obtained from Equation 3.2:

$$P(T_S > t) = \sum_{l_1=0}^{3} \sum_{l_2=0}^{3} \Phi(l_1, l_2) \binom{3}{l_1} [1 - e^{-0.8t}]^{3-l_1} [e^{-0.8t}]^{l_1} \times \binom{3}{l_2} [1 - e^{-1.5t}]^{3-l_2} [e^{-1.5t}]^{l_2} \tag{3.14}$$

The results of the reliability analysis are shown in Figure 3.8, which shows near perfect agreement of the simulation methods with the analytical solution. The Monte Carlo simulation has been performed using $N = 5000$ samples and a discretisation time $dt = 0.0015$. The discretization time is only required to collect the numerical results (i.e. survival function) although the simulation of the system is continuous with respect to the time.

Figure 3.9 shows an example of system evolution as a function of time with associated number of working components C_k.

In order to show the efficiency of the proposed algorithm, the evolution of the variance of the estimators as a function of number of samples has been computed and shown in Figure 3.10. Algorithm 2 shows a smaller variance compared to Algorithm 1, in particular

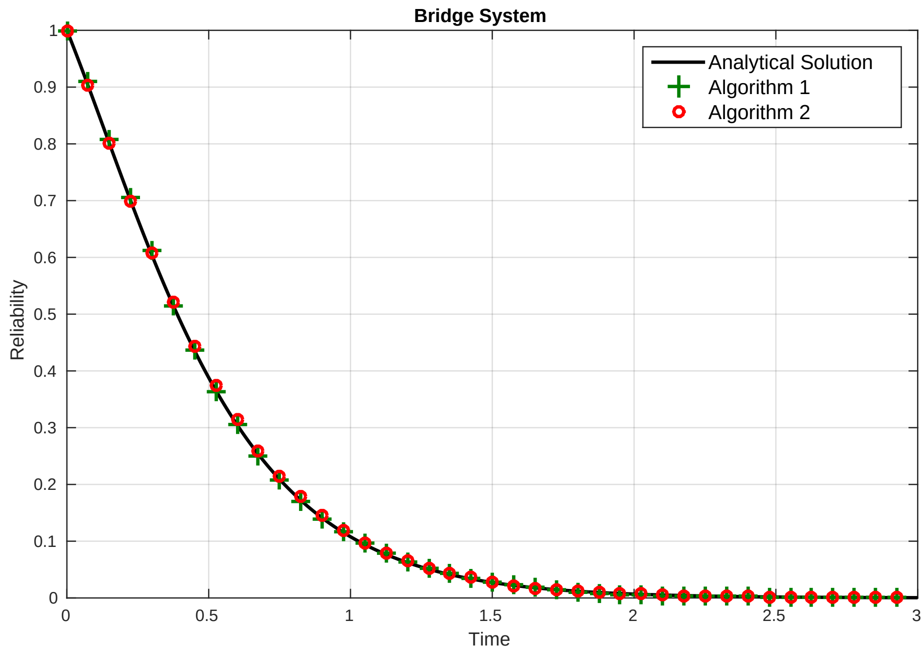

Figure 3.8: Survival function of the bridge system calculated by two simulation methods and analytical method, respectively.

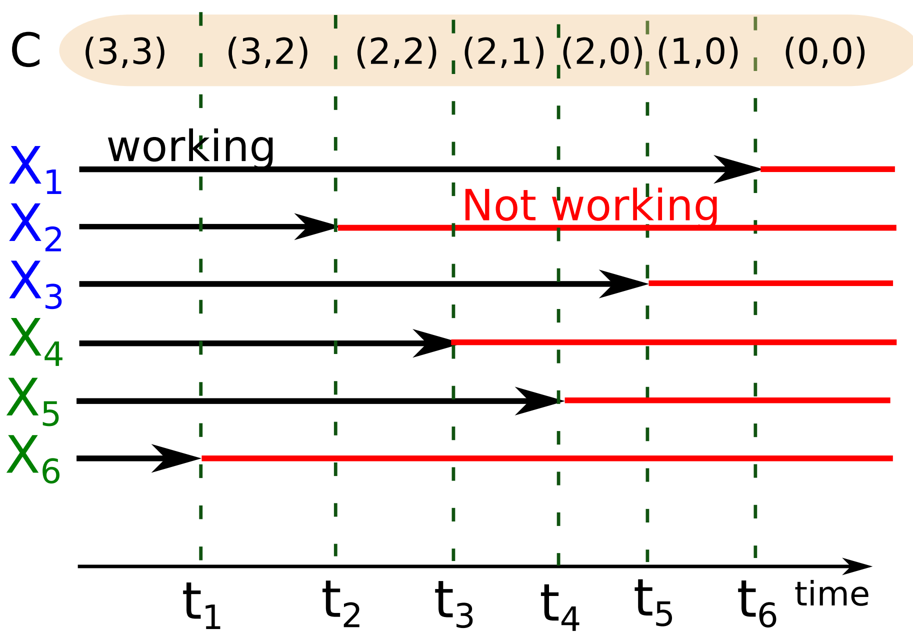

Figure 3.9: Example of a realization of the number of working components C_k as a function of time.

when small sample sizes are used.

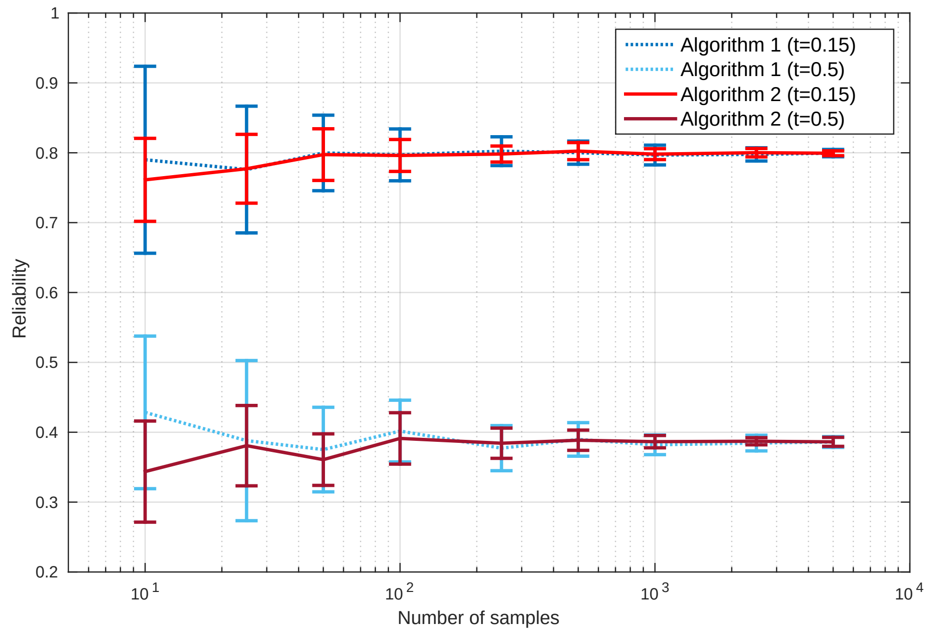

Figure 3.10: Standard deviation of the estimator of the survival function as a function of the number of samples.

3.5.2 Hydroelectric Power Plant System

In this Section, a survival analysis of a real world hydroelectric power plant based on survival signature is conducted. The system is schematically shown in Figure 3.11 and its reliability block diagram is illustrated in Figure 3.12.

It can be modelled as a complex system comprising the following main twelve components: (1) control gate (CG), which is built on the inside of the dam, the water from the reservoir is released and controlled through the gate; (2) two butterfly valves ($BV1$,$BV2$), which can transport and control the water flow; (3) two turbines ($T1$,$T2$), where the flowing waters kinetic energy is transformed into mechanical energy; (4) three circuit breakers ($CB1$,$CB2$,$CB3$), which are used to protect the hydro power plant system; (5) two generators ($G1$,$G2$), which produce alternating current by moving electrons; and (6) two transformers ($TX1$,$TX2$), which inside the powerhouse take the alternating current and convert it to higher-voltage current.

Two cases are presented in the following part: Case A presents the survival analysis with the fully probability model; Case B considers imprecision within the model.

Case A: It is assumed that all components of the same type have the same failure time distribution. Failure type and distribution parameters are listed in Table 3.3.

Let l_1, l_2, l_3, l_4, l_5 and l_6 denote CG, BV, T, G, CB and TX, respectively. Table 3.4 shows the survival signature of the hydro power plant, whereby the rows with values $\Phi(l_1, l_2, l_3, l_4, l_5, l_6) = 0$ are omitted.

Table 3.3: Failure types and distribution parameters of components in a hydro power plant

Component name	Distribution type	Precise Parameters	Imprecise Parameters
CG	Weibull	(1.3,1.8)	([1.2,1.5], [1.5,2.1])
BV	Weibull	(1.2,2.3)	([1.0,1.6], [2.1,2.5])
T	Exponential	0.8	[0.4,1.2]
G	Weibull	(1.6,2.6)	([1.3,1.8], [2.3,2.9])
CB	Gamma	(1.3,3.0)	([1.2,1.4], [2.8,3.3])
TX	Gamma	(0.6,1.1)	([0.3,0.8], [1.0,1.3])

Table 3.4: Survival signature of a hydro power plant in Figure 3.11; rows with $\Phi(l_1, l_2, l_3, l_4, l_5, l_6) = 0$ are omitted

l_1	l_2	l_3	l_4	l_5	l_6	$\Phi(l_1, l_2, l_3, l_4, l_5, l_6)$
1	1	1	1	2	[1,2]	1/12
1	1	1	2	2	[1,2]	1/6
1	1	2	1	2	[1,2]	1/6
1	2	1	1	2	[1,2]	1/6
1	1	1	1	3	[1,2]	1/4
1	1	2	2	2	[1,2]	1/3
1	2	1	2	2	[1,2]	1/3
1	2	2	1	2	[1,2]	1/3
1	1	1	2	3	[1,2]	1/2
1	1	2	1	3	[1,2]	1/2
1	2	1	1	3	[1,2]	1/2
1	2	2	2	2	[1,2]	2/3
1	1	2	2	3	[1,2]	1
1	2	1	2	3	[1,2]	1
1	2	2	[1,2]	3	[1,2]	1

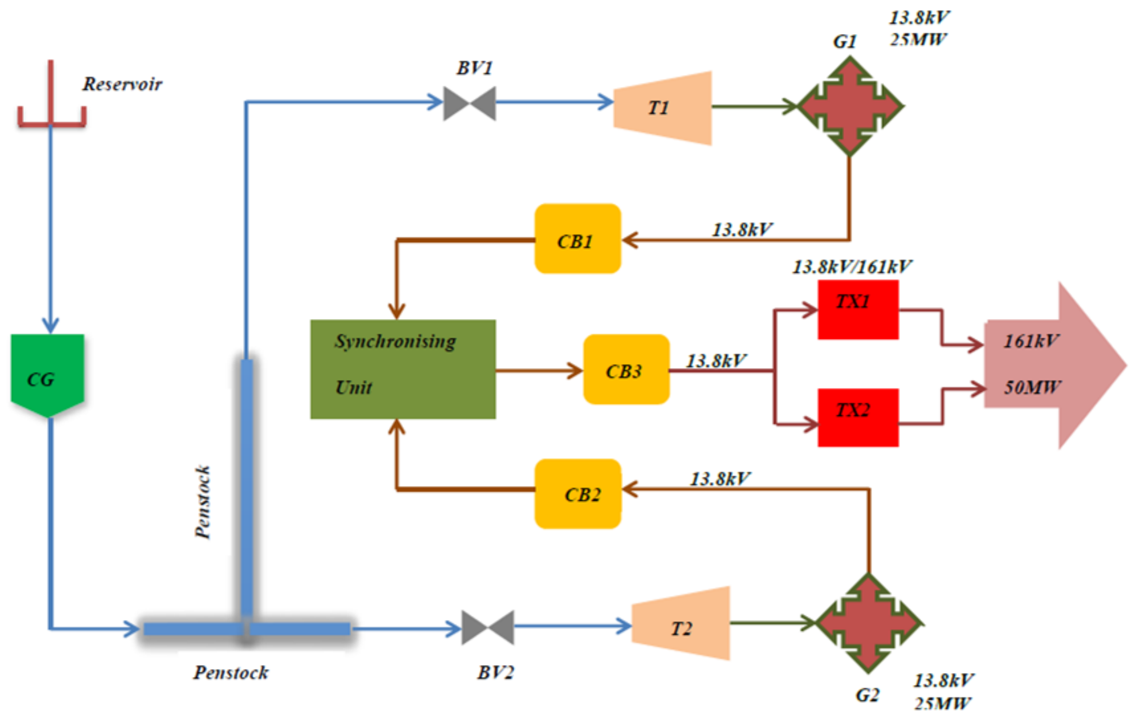

Figure 3.11: Schematic diagram of a hydroelectric power plant system.

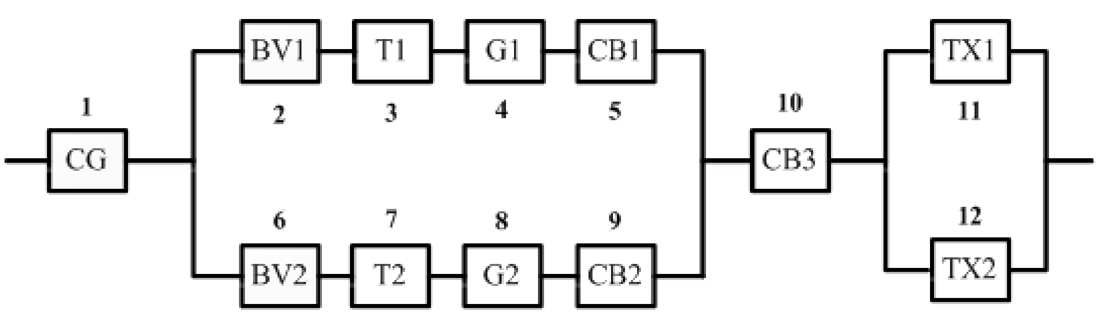

Figure 3.12: Reliability block diagram of a hydroelectric power plant system.

The survival signature can now be used as follows. There are $m_1 = 1$, $m_2 = m_3 = m_4 = m_6 = 2$ and $m_5 = 3$ components of each type. The survival signature must consider combinations for all $l_1 \in \{0, 1\}$, $l_2, l_3, l_4, l_6 \in \{0, 1, 2\}$ and $l_5 \in \{0, 1, 2, 3\}$, and the state vector is $\underline{x} = (x_1^1, x_1^2, x_2^2, x_1^3, x_2^3, x_1^4, x_2^4, x_1^5, x_2^5, x_3^5, x_1^6, x_2^6)$. Now consider $\Phi(1, 1, 1, 2, 2, 1)$ for example. This covers all possible vectors $\underline{x}$ with $x_1^1 = 1$, $x_1^2 + x_2^2 = 1$, $x_1^3 + x_2^3 = 1$, $x_1^4 + x_2^4 = 2$, $x_1^5 + x_2^5 + x_3^5 = 2$ and $x_1^6 + x_2^6 = 1$. There are 24 such vectors, but only four of these can make the system function. Due to the *iid* assumption of the failure times of components of the same type, and due to independence between components of different types, all these 24 vectors have equal probability to occur, hence

$\Phi(1,1,1,2,2,1) = 4/24 = 1/6$.

The survival function of the hydroelectric power plant system with twelve components of six types is shown in Figure 3.13.

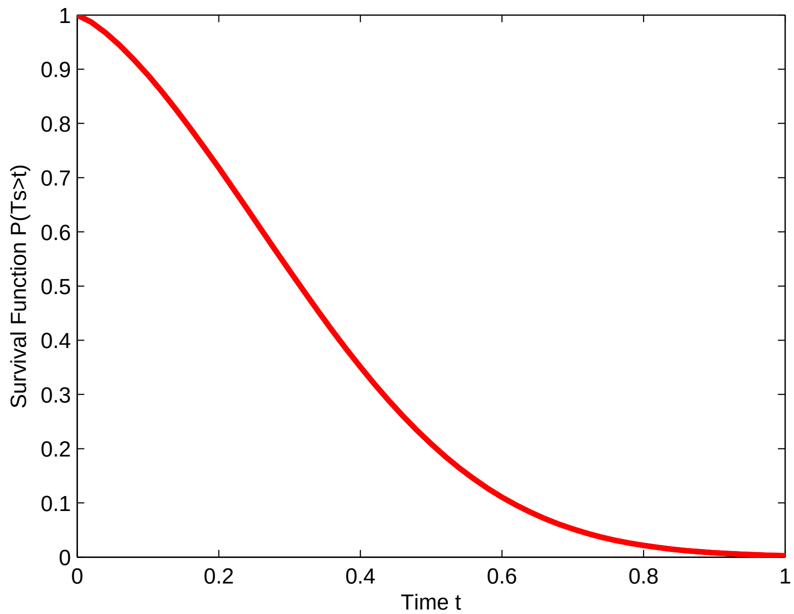

Figure 3.13: Survival function of a hydroelectric power plant system along with survival functions for the individual components.

Case B: The investigation from CASE A is now extended by considering imprecision in the description of the probabilistic model for the failure characterization of the system components. Intervals are used to describe the imprecision in the failure time distribution as shown in Table 3.3.

The upper and lower bounds of the parameters reflect the ideal and the worst cases of the performance of the components, respectively. The range of the parameters represents epistemic uncertainty, which results from expert assessments of the component performance. This modelling leads to upper and lower survival functions of the hydro power plant system reflecting the epistemic uncertainties as a range between the curves, see Figure 3.14. The imprecision from the input is translated into imprecision of the output.

3.5.3 Grey System

In order to illustrate the efficiency and the applicability of the proposed simulation approaches a complex system composed by 8 components of three types is analysed. The component failure types and distribution parameters are shown in Table 3.5, again affected by imprecision.

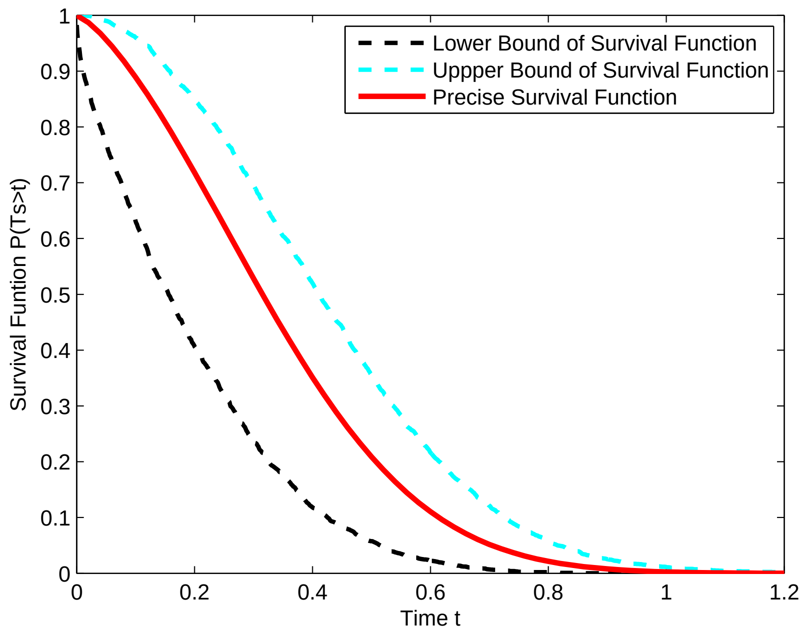

Figure 3.14: Upper, lower and precise survival functions of the hydroelectric power plant system.

Table 3.5: Components failure types and distribution parameters for the system in Figure 3.15

Component type	Distribution	Parameters
1	Weibull	$([1.6, 1.8], [3.3, 3.9])$
2	Exponential	$([2.1, 2.5])$
3	Weibull	$([3.1, 3.3], [2.3, 2.7])$

In addition, it is assumed that the exact configuration of part of the system is unknown as shown in Figure 3.15, i.e. it might be composed by an additional component of type 1 or two components of type 2 connected in parallel.

However, the system can still be described using the survival signature although affected by imprecision as shown in Table 3.6. For instance, if 2 components of type one and 1 component of type three are working the system can be either in a failing state or working with a probability of 0.5 (if the unknown part of the system is composed by an additional component of type one).

The upper and lower bounds of survival function for the system with imprecision both in the survival signature and on the component distribution parameters are shown in Figure 3.16.

This example shows the flexibility and the applicability of the simulation approaches proposed for the analysing of systems affected by imprecision where no analytical solu-

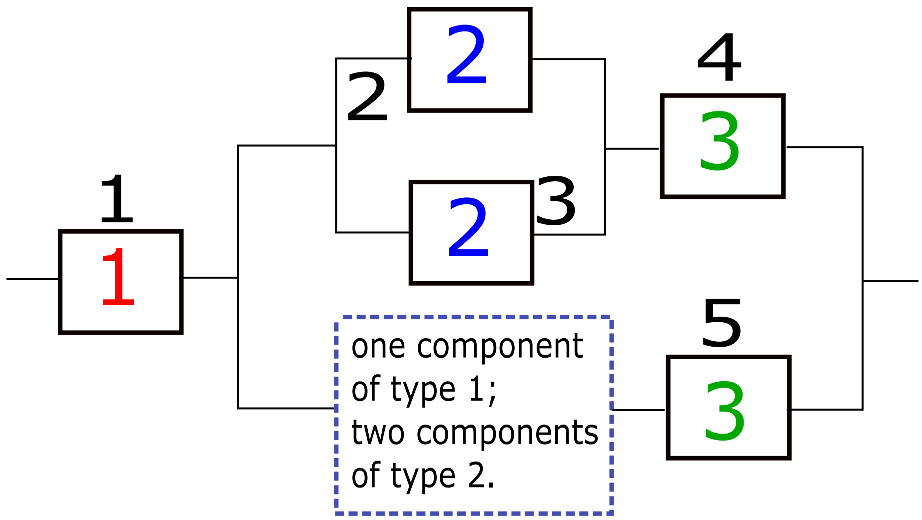

Figure 3.15: Grey system composed by 8 components of three types with imprecision of the exact system configuration.

Table 3.6: Imprecise survival signature of the system of Figure 3.15, $\Phi(l_1, l_2, l_3) = 0$ and $\Phi(l_1, l_2, l_3) = 1$ for both lower and upper bounds are omitted.

l_1	l_2	l_3	$[\underline{\Phi}(l_1, l_2, l_3), \overline{\Phi}(l_1, l_2, l_3)]$
1	1	1	[1/8,1/8]
1	1	2	[1/4,1/4]
1	2	1	[1/5,1/4]
1	2	2	[3/7,1/2]
1	3	1	[1/4,3/8]
1	3	2	[1/2,1/2]
1	4	1	[1/4,1/2]
1	4	2	[1/2,1/2]
2	0	1	[0,1/2]
2	0	2	[0,1]
2	1	1	[1/4,3/4]
2	1	2	[1/2,1]
2	2	1	[1/2,1]
2	3	1	[3/4,1]

tions are available.

3.5.4 Complex Lifeline Network

In order to show the efficiency of the proposed algorithm in Section 2.5.2, a complex repairable network is analysed. Figure 3.17 shows a lifeline network of 17 nodes and 32 edges. The source is the node s and the sink is the node t. All the nodes are assumed to be

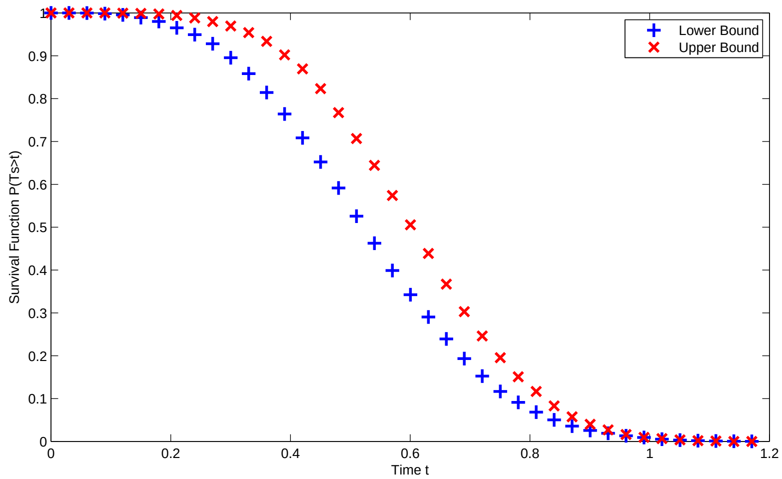

Figure 3.16: Upper and lower bounds of survival function for the system in Figure 3.15.

perfectly reliable in the network.

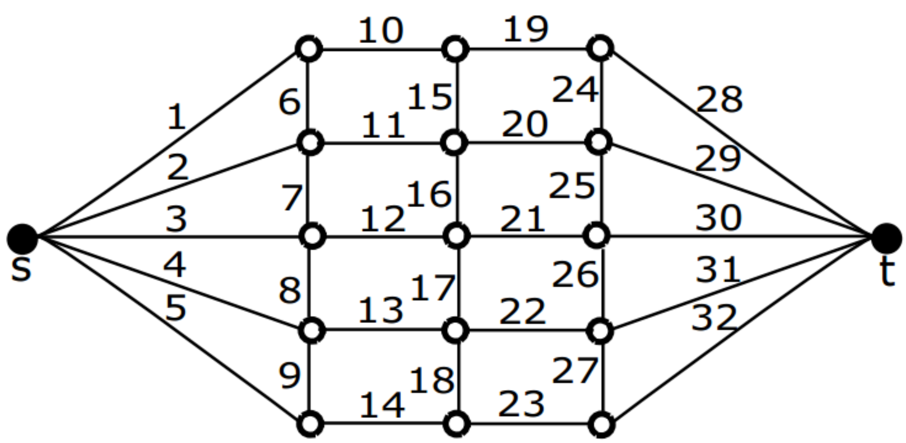

Figure 3.17: A lifeline network with 17 nodes and 32 edges.

Three cases are considered. The first case is used to compare results between the former improved recursive decomposition method and the presented survival signature-based method. The proposed approach is extended to analyse a complex network with multiple component types in the second case. For the third case, imprecision is taken into consideration.

Case 1: Network with Single Type of Components

Reliability analysis on the network shown in Figure 3.17 considering only one type of components has been studied by Liu and Li in [89]. In this Case, there is an assumption that the edges of the network are independent and identically distributed. All edges are undirected edges (which means all edges are connected by nodes). Let all edges' reliability be 0.9 (Case I), 0.8 (Case II), 0.2 (Case III) and 0.1 (Case IV).

The network reliability calculated by the improved recursive decomposition algorithm is 0.999930 (Case I), 0.996522 (Case II), 0.017194 (Case III) and 0.000777 (Case IV), respectively.

By using the efficient algorithm which is proposed in this thesis, the survival signature of the complex network can be calculated in 28.07 seconds, and the results can be seen in Table 3.7.

Table 3.7: Survival signature of the network in Figure 3.17.

l	$\Phi(l)$	l	$\Phi(l)$	l	$\Phi(l)$	l	$\Phi(l)$	l	$\Phi(l)$	l	$\Phi(l)$
0	0	1	0	2	0	3	0	4	0.00014	5	0.00081
6	0.00285	7	0.0077	8	0.01765	9	0.03597	10	0.06683	11	0.00014
12	0.18409	13	0.27635	14	0.38916	15	0.51445	16	0.63944	17	0.75075
18	0.18409	19	0.90414	20	0.94679	21	0.97271	22	0.98719	23	0.99458
24	0.99799	25	0.99938	26	0.99985	27	0.99998	28	1	29	1
30	1	31	1	32	1						

In all four cases, the network reliabilities calculated through the survival signature-based reliability method given by Equation 3.2 are identical to those calculated using the method from Liu and Li. However, the survival signature-based method only needs to calculate the survival signature of the network once and store the results, so it is efficient to calculate the network reliability for more cases. Furthermore, the proposed method is powerful at dealing with the complex networks with multiple component types and components with time varying distributions.

Case 2: Network with Multiple Types of Components

Assume that the edges within the network are belonging to three types instead of one single type. To be specific, edges 1, 2, 3, 4, 5, 28, 29, 30, 31 and 32 are in type one with reliability 0.9; edges 6, 7, 8, 9, 15, 16, 17, 18, 24, 25, 26 and 27 are in type two with reliability 0.8; edges 10, 11, 12, 13, 14, 19, 20, 21, 22 and 23 are in type three with reliability 0.2.

In order to estimate the network reliability, the survival signature of this network can be calculated by the proposed algorithm in 23.78 seconds, and the results can be seen in Table 7.1 in the Appendix. Then, the reliability of the network is 0.3746931 by using Equation 3.2.

It can be seen from the above examples that the network reliability is time independent, because we assume the edge reliability values are stable as time goes. In the real engineering world, however, the failure times of edges are according to different distribution types (i.e., Exponential, Weibull, Gamma or Lognormal distribution) sometimes. All of these distribution are time dependent, and will lead to the network reliability values being time varying.

Now let us assume that the failure times of edges type one are according to the Exponential distribution with parameter $\lambda = 0.12$. Similarly, type two $\sim$ Weibull(0.4,4.2) and type three $\sim$ Normal(0.7,0.02).

The survival signature remains the same as the network does not change its configuration. The survival function of the network is shown in Figure 3.18. It can be seen that the survival function is time varying. Thus, it is easy to determine the network reliability at each time.

Case 3: Imprecise Network Reliability

The available information for the quantitative specification of the uncertainties associated with the components is often limited and appears as incomplete information, limited sampling data, ignorance, measurement errors and so forth. Therefore, it is important to consider imprecision during the whole system reliability analysis period.

The first case is that due to lack of data or limited knowledge, there are not always precise data for edges failure time distributions. For instance, Table 3.5.4 shows the failure types and imprecise distribution parameters of edges in the network.

Table 3.8: Failure types and imprecise distribution parameters of edges in the network of Figure 3.17.

Edge Type	Distribution Type	Imprecise Parameters λ or (α, β)
1	Exponential	[0.08, 0.18]
2	Weibull	([0.3, 0.6], [3.8, 4.6])
3	Normal	([0.5, 0.8], [0.01, 0.03])

The lower and upper bounds of survival function of the network can be estimated by means of a double loop approach. Figure 3.18 shows the interval of the survival function.

The second case concerns “grey box” in the lifeline network. Since it is difficult to know the exact network configuration, there exists imprecision within the network survival signature. For example, there is a “grey box” after node t of the network in Figure 3.17. We do not know the precise configuration of this shaded box, but know that this part consists of three edges which belong to each type respectively. To be specific, edge 33 belongs to one, while edge 34 is in type two, and edge 35 of type three in the “grey box”, which can be seen in Figure 3.19.

The unknown configuration of this part in the network leads to imprecise survival signature, which can be calculated through the proposed algorithm. Based on Equations 3.11 and 3.12, the lower and upper bounds of the survival function can be calculated, which can be seen in Figure 3.20.

From the above two cases, we can conclude that imprecision either within the components failure time distribution parameters or in the survival signature can lead to survival function intervals of the complex network.

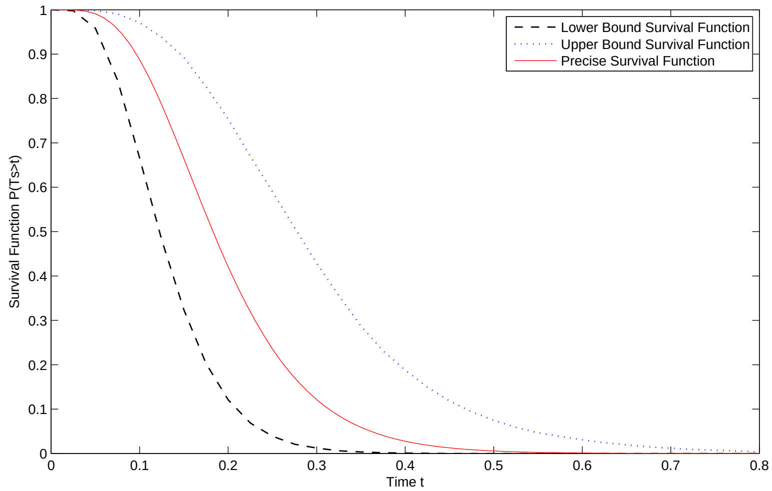

Figure 3.18: Time varying precise survival function alongside with lower and upper bounds of survival function of the network in Figure 3.17 (imprecise distribution parameters).

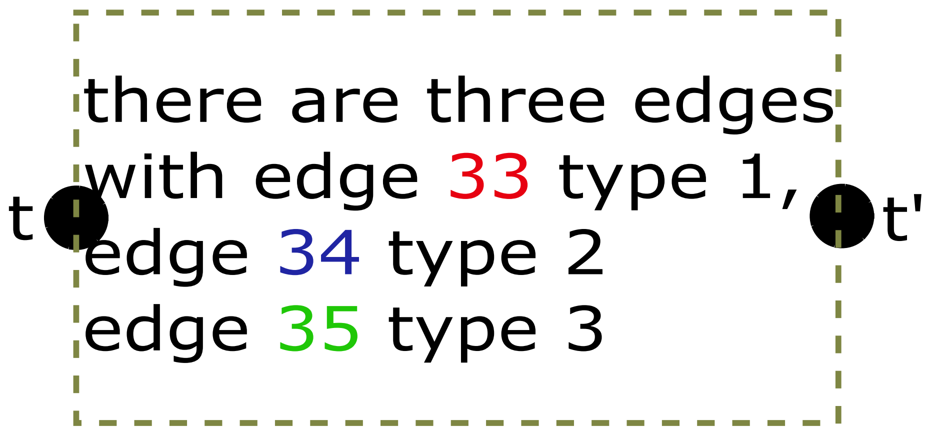

Figure 3.19: Grey box of the network in Figure 3.17.

3.6 Conclusion

In this Chapter efficient analytical and simulation approaches for analysing precise and imprecise system reliability have been presented. All the methods are based on the survival signature, which has been proven to be an effective method to estimate the survival function of systems with multiple component types.

In the proposed approach, the system model needs to be analysed only once in order to conduct a reliability analysis, which represents a significant computational advantage. Performing a survival analysis on complex systems and networks has been presented as a

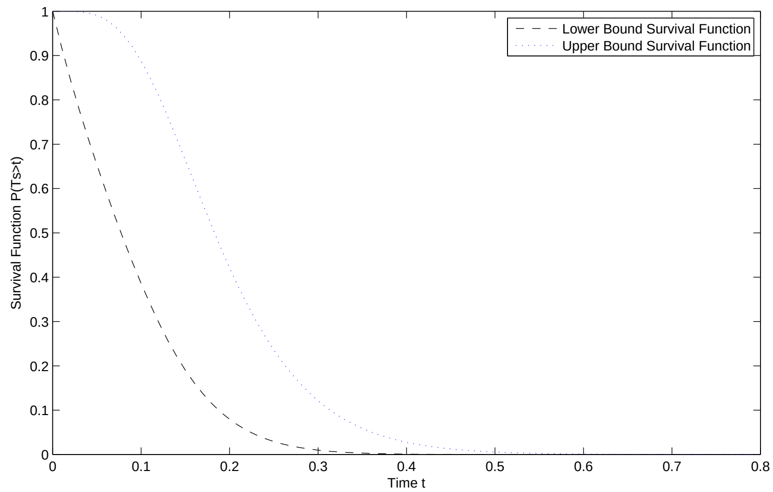

Figure 3.20: Lower and upper bounds of survival function of the network in Figure 3.17 (imprecise survival signature).

novel pathway for system reliability. In addition, the effect of imprecision, for example resulting from incomplete data or the unknown system configuration, has been taken into account in the system reliability analysis. As a consequence, bounds of survival functions of the system can be obtained. The numerical examples in this Chapter indicate that the proposed approaches can be used to evaluate the reliability and uncertainty of complex systems efficiently.

Chapter 4

Reliability Analysis of Complex Repairable Systems

4.1 Introduction

When performing reliability analysis, it is essential to distinguish between repairable and non-repairable systems and networks. The reliability approaches discussed in Chapter 3 are largely applicable to non-repairable systems.

In this Chapter, we examine the peculiar aspects of complex repairable systems as well as discuss method for analysing their reliability. An algorithm based on the survival signature is proposed to analyse the complex system with repairable components. This approach is efficient since the survival signature of the complex repairable system only needs to be calculated once while Monte Carlo simulation is used to generate component transition times.

Section 4.2 gives an introduction about the relationship between a repairable system and its components. What is more, a survival signature-based simulation method is proposed to analyse reliability of repairable systems in this Section. Section 4.3 shows the applicability and performance of the proposed approaches by analysing some numerical examples.

4.2 Repairable System Reliability Analysis Based on Survival Signature

4.2.1 Repairable System and Its Components

Non-repairable components are those that are discarded or replaced with new ones when they fail. In real engineering applications, however, there are always exist repairable components, which are not replaced following the occurrence of a failure; rather, they are repaired and put into operation again. The sketch map of the repairable component is shown in Figure 4.1.

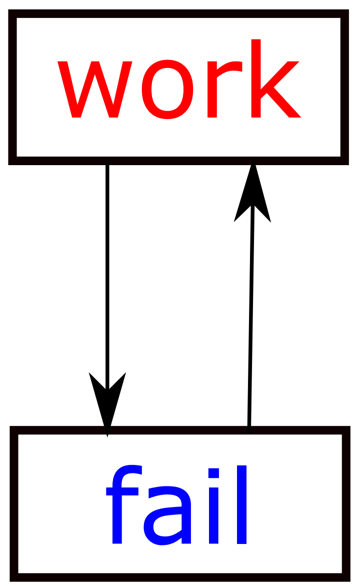

Figure 4.1: Sketch map of a repairable component.

If the system with m components is repairable, a schematic diagram of the repairable components status and the corresponding system performance is presented in Figure 4.2.

For a structure function method, it is a necessary to identify whether the system works or not at each critical time point. The critical time point is the beginning time for each component failure and the finish time for each component repair. In this Chapter, a survival signature-based simulation method is proposed to analyse the repairable system reliability.

4.2.2 Proposed Method for Repairable System Analysis

Algorithm 2 can easily be extended to analyse systems with repairable components. Assume that there are j_k possible transitions for the components of type k. The probability

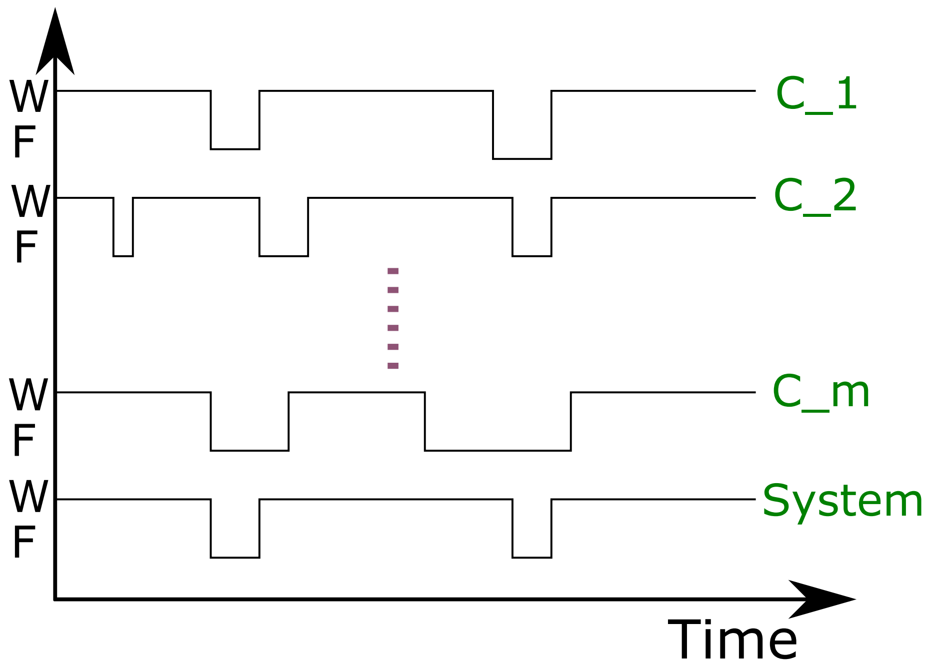

Figure 4.2: Schematic diagram of the repairable components status and the corresponding system performance.

of going from state $s = l$ to state $s' = m$ is given by $p_{klm} = P(X'_k = m \mid X_k = l)$. Let $F_{kl} = \sum_m P(X'_k = m \mid X_k = l)$ represent the CDF of the component of type k to exit from its state l, i.e. to undergo a transition leading to a state $m \neq l$.

Let us assume for the moment that there is only one possible transition to exit from the state $s = l$. For instance, a component in working status $s = 1$ can fail and enter in the state $s' = 2$; the component in the state $s = 2$ can only be repaired and return in the status $s' = 1$. Hence, $p_{k21} = P(X'_k = 2 \mid X_k = 1) = p_{k2}$ represents the probability of failure for component k, $p_{k12} = p_{k1}$ the probability of repair.

The Monte Carlo simulation is performed as follows:

Step 0. Initialise variables (i.e. $t_{old} = 0$ and counters (i.e. V_t));

Step 1. Sample the transition times t_i for $i = 1, 2, \dots, C$ for each component of the system from the corresponding CDF, F_{kl}, and stored in a vector $Vt = \{t_1, t_2, \dots, t_C\}$, set $t_{old} = 0$;

Step 2. Identify the first transition time, i.e. $min(Vt)$ and the corresponding component z. Hence, t_1 represents the first transition of the system, t_2 the second transition and so on;

Step 3. At each transition time t_i, calculate the number of components in working status (i.e. $C_{t_i} = (C_1, C_2, ..., C_K)$). The corresponding "production level" is obtained by evaluating the survival signature for the number of components in working status;

Step 4. Collect the value of the survival signature at time t_i, Φ_{t_i}, in a counter Vr representing the survival function as follows: $Vr(j) = Vr(j) + \Phi_{t_i} \quad \forall j : t_{old} \leq j \cdot dt < min(Vt)$.

Step 5. Set $t_{old} = min(Vt)$ and sample the new status of the component z from the probability mass function $P(s = m) = \frac{F_{klm}(t_i)}{F_{kl}(t_i}$;

Step 6. Update the vector of transition times Vt by sampling the next transition time t'_z of the component z of type k in status m from F_{km}, where k is the component type of the component z and m its status. Hence: $Vt(z) = t_z + t'_z$;

Step 7. If $min(Vt) < T_F$ (i.e. the final time), return to point 2.

The above steps are repeated for N samples and the survival function obtained by averaging the vector Vr over the number of samples. The flow chart of the proposed algorithm is shown in Figure 4.3 and the pseudo-code is shown in Algorithm 3.

4.3 Numerical Example

4.3.1 Circuit Bridge System

In this example the components of the bridge system shown in Figure 3.3 are considered repairable. Hence, the components can be in two different status: working ($s = 1$) and not-working ($s = 2$). Two cases are analysed considering different distributions for the repair times as shown in Table 4.1.

Analytical solutions are not available for analysing repairable systems and the system can only be analysed by adopting simulation methods such as the Algorithm 3. An example of the evolution of the system is represented in Figure 4.4.

The survival function $P(T_S > t)$ reaches a stationary level that depends on the ratio between the mean failure time and mean repair time. The estimated survival function is shown in Figures 4.5-4.8.

It is important to notice that the proposed approach (Algorithm 3) does not require the introduction of additional component types to analyse a system with repairable components. In order to verify the correctness of Algorithm 3 which is based on survival signature, the results have been compared with the solution of simulation method based on the structural function. The minimum path sets of the Bridge system shown in Figure

Algorithm 3

Require: *N*: Number of simulations; *dt*: Discretisation time; F_k: CDF failure times, $Vc = [m_1, m_2, \dots, m_k]$: Number of components per type; Nt: number of discretisation steps.

```
Set V_r(1 : Nt) = 0                                   ▷ Ititialise counter
Set C = sum(V_c)                                      ▷ Compute total number of components
Set Φ ← Survival signature                            ▷ Compute the survival signature
Set V_s = Initial component Status                    ▷ System initial conditions
for n = 1 : N do                                      ▷ loop over number of samples
    for i = 1 : C do                                  ▷ loop over number of components
        V_t(i) ~ F_kl                                 ▷ Sample transition time component z of type k in state l
    end for
    u = 1                                             ▷ Initialise counter
    while min(V_t) ≤ Nt * dt do
        [t_z, z] = min(Vt)                            ▷ Identify first system transition t_z
                                                      ▷ and corresponding omponent index z, component type k_z
        while u · dt ≤ V_j do
            Vr(u) = Vr(u) + Φ(V_k)                    ▷ Update counter
            u ← u + 1                                 ▷ Update index
        end while
        if V_s(z) is working then
            V_c(k) = V_c(k) − 1                       ▷ Update component counter
            V_s(z) NOT working                        ▷ Update component status
        else
            V_c(k) = V_c(k) + 1                       ▷ Update component counter
            Set V_s(z) working                        ▷ Update component status
        end if
        V_t(z) ~ F_kl                                 ▷ Sample new transition time component z
                                                      ▷ of type k_z in the state l = V_j(z)
    end while
end for
Vr = Vr/N                                             ▷ Normalise counter
```

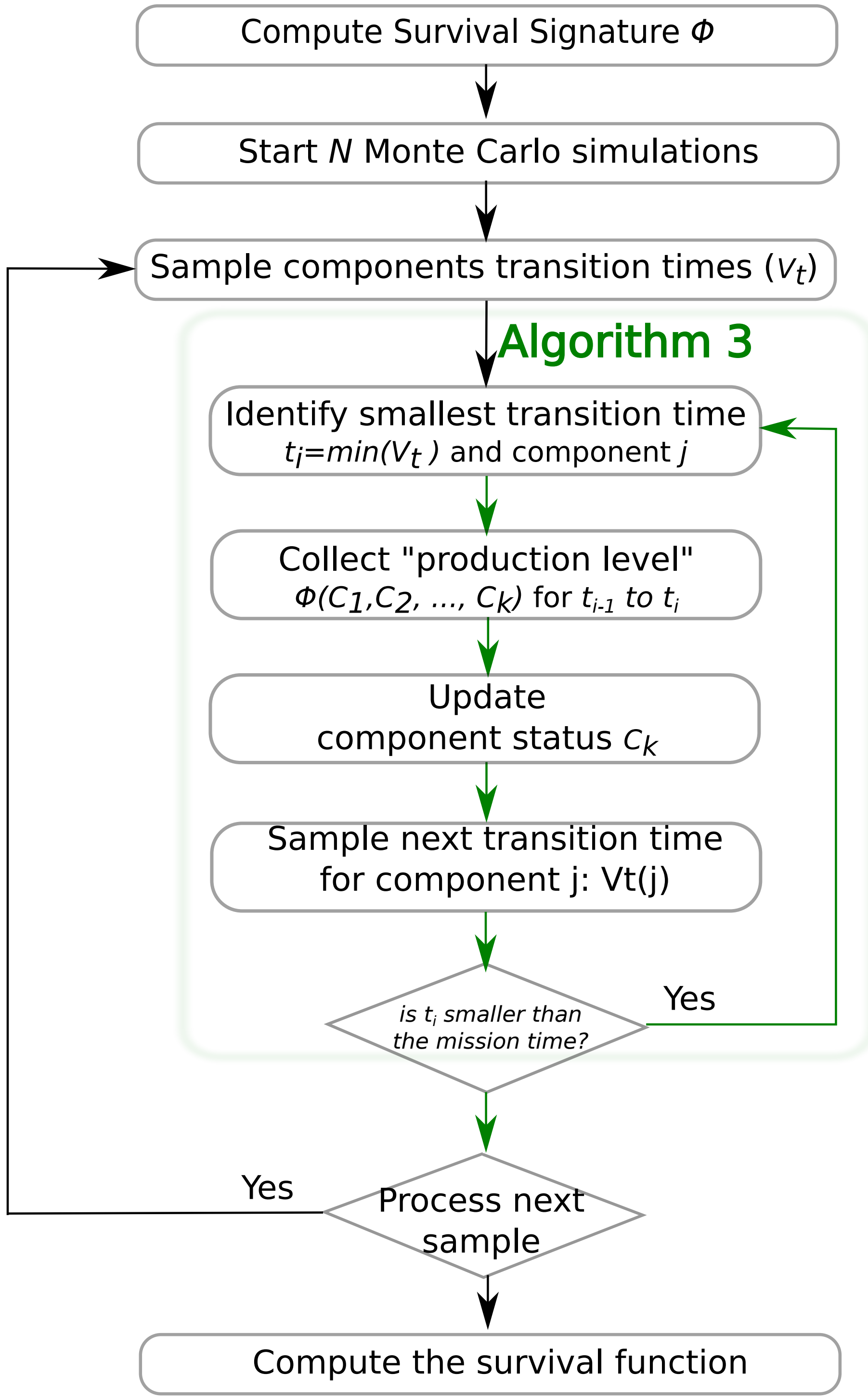

Figure 4.3: Flow chart of Algorithm 3.

3.3 are [1,2,3], [1,2,5,6], [1,3,4,5] and [1,4,6]. $N = 5000$ samples have been used to estimate the reliability of the system and the results shown in Figures 4.5 and 4.7 are in perfect agreement with the results obtained using Algorithm 3. Figures 4.6 and 4.8 compare the

Table 4.1: Parameters of repairable components in the bridge system. State 1: Working, State 2: Not-working.

Component type (k)	Transition (s)	Distribution	Parameters
		CASE A	
1	$1 \rightarrow 2$	Exponential	0.8
1	$2 \rightarrow 1$	Weibull	0.9 and 1.2
2	$1 \rightarrow 2$	Exponential	1.5
2	$2 \rightarrow 1$	Weibull	1.3 and 1.8
		CASE B	
1	$1 \rightarrow 2$	Exponential	0.8
1	$2 \rightarrow 1$	Uniform	0.2 and 0.6
2	$1 \rightarrow 2$	Exponential	1.5
2	$2 \rightarrow 1$	Uniform	0.1 and 0.2

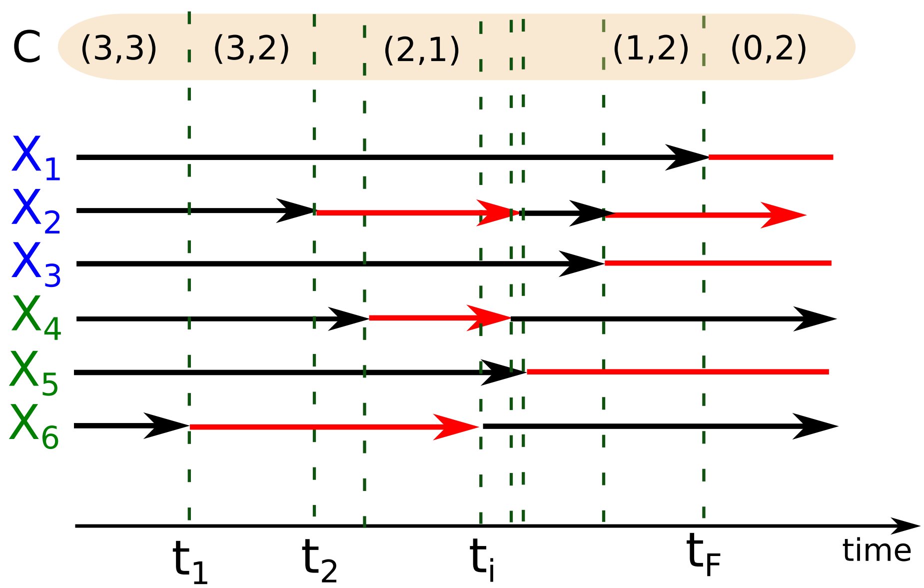

Figure 4.4: Example of a realization of the number of working components C_k as a function of time.

variance of the estimator as a function of the number of samples adopting the Algorithm 3 based on survival signature and Monte Carlo method based on structural function.

4.3.2 Complex System

In order to illustrate the efficiency and the applicability of the proposed simulation approaches, a complex system composed by 14 repairable components of six different types is analysed. The reliability block diagram of the system is shown in Figure 4.9 and the failure type and distribution parameters of the components are reported in Table 4.2. The

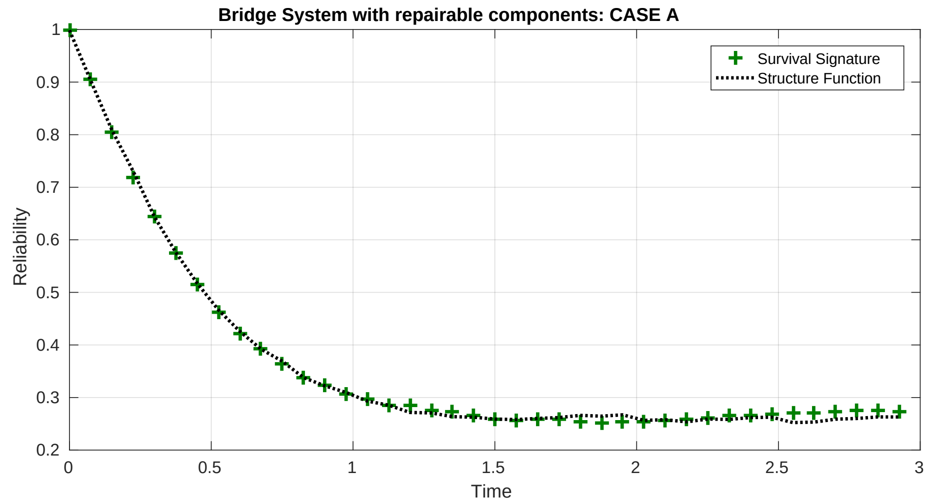

Figure 4.5: CASE A: Survival function of the circuit bridge system with repairable components calculated by means of Algorithm 3 and a simulation method based on structure function.

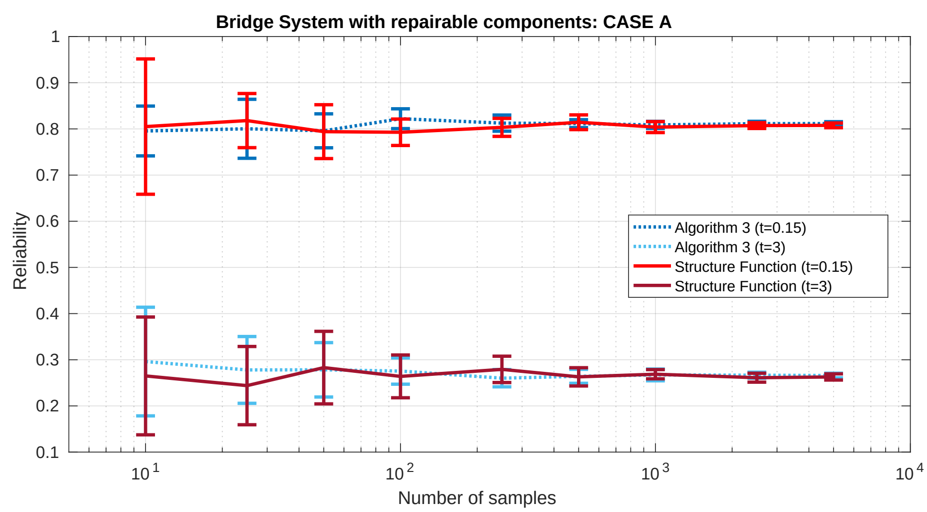

Figure 4.6: CASE A: Standard deviation of the estimator for the circuit bridge system with repairable components calculated by means of Algorithm 3 and a simulation method based on structure function.

survival signature of this system can be referred in Table 7.2 in the Appendix.

First, the system is analysed without considering the repairs (i.e. transition $2 \rightarrow 1$ is not allowed). Hence, the reliability of the system can be estimated adopting the proposed Algorithms 1 and 2 and the results are shown in Figure 4.10.

In case of repairable components or system, Algorithm 3 needs to be used. The proposed approach is generally applicable and allows to estimate the reliability of complex repairable system based on the survival signature. Figure 4.11 shows the survival function for the case of repairable components.

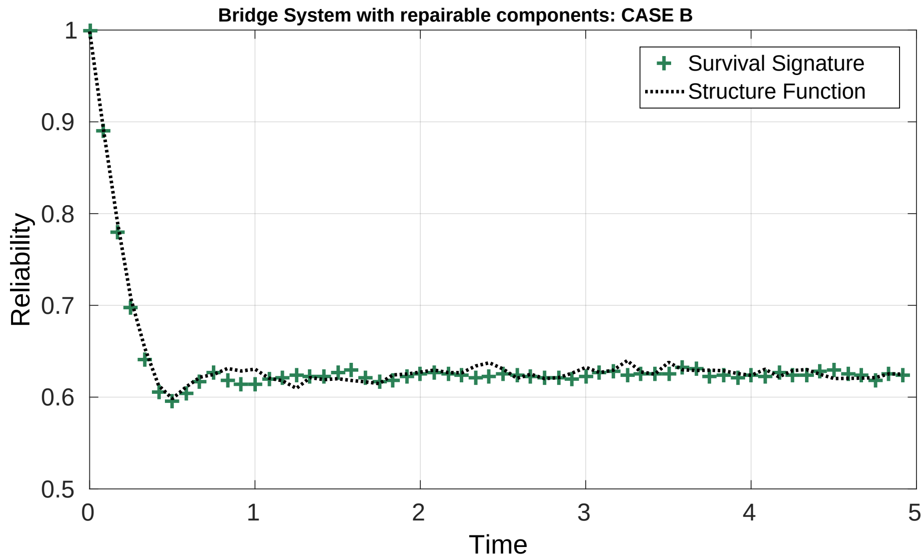

Figure 4.7: CASE B: Survival function of the circuit bridge system with repairable components calculated by means of Algorithm 3 and a simulation method based on structure function.

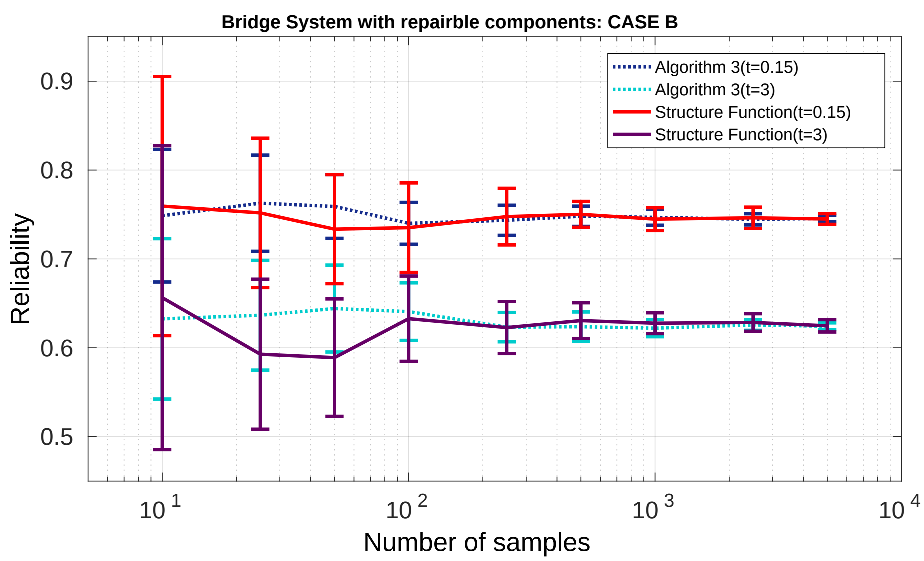

Figure 4.8: CASE B: Standard deviation of the estimator for the circuit bridge system with repairable components calculated by means of Algorithm 3 and a simulation method based on structure function.

4.4 Conclusion

The survival signature has been shown to be a practical method for performing reliability analysis of complex systems and networks with multiple component types. An algorithm which bases on the survival signature is proposed to analyse complex system with repairable components.

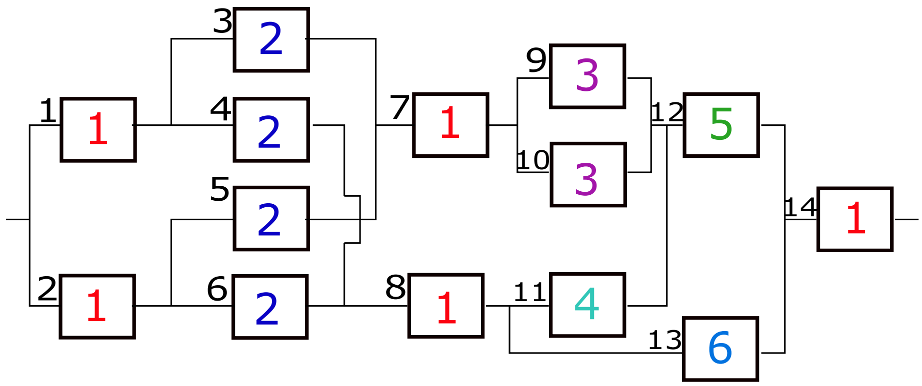

Figure 4.9: The complex repairable system with 14 components which belong to six types. The numbers inside the component boxes indicate the component type. The numbers outside the component boxes indicate the component indices.

Table 4.2: Components failure (transition $1 \rightarrow 2$) and repair (transition $2 \rightarrow 1$) data for each component type of the complex system.

Component type (k)	Transition (s)	Distribution Type	Parameters
1	$1 \rightarrow 2$	Exponential	2.3
1	$2 \rightarrow 1$	Uniform	(0.4,0.6)
2	$1 \rightarrow 2$	Exponential	1.2
2	$2 \rightarrow 1$	Uniform	(0.9,1.1)
3	$1 \rightarrow 2$	Weibull	(1.7,3.6)
3	$2 \rightarrow 1$	Uniform	(0.6,0.8)
4	$1 \rightarrow 2$	Lognormal	(1.5,2.6)
4	$2 \rightarrow 1$	Uniform	(1.0,1.2)
5	$1 \rightarrow 2$	Weibull	(3.2,2.5)
5	$2 \rightarrow 1$	Uniform	(1.2,1.4)
6	$1 \rightarrow 2$	Gamma	(3.1,1.5)
6	$2 \rightarrow 1$	Uniform	(1.1,1.3)

It is more efficient to simulate a system and network using the survival signature rather than the full structure function (and for very large systems this may not be possible). This is because it is not necessary to estimate the all cut sets of the system, exchangeable components can be grouped together and characterised by a single distribution and finally the simulation code is simple and more robust. The case studies presented in this Chapter indicate that the proposed approaches can be used to evaluate the reliability of complex repairable systems efficiently.

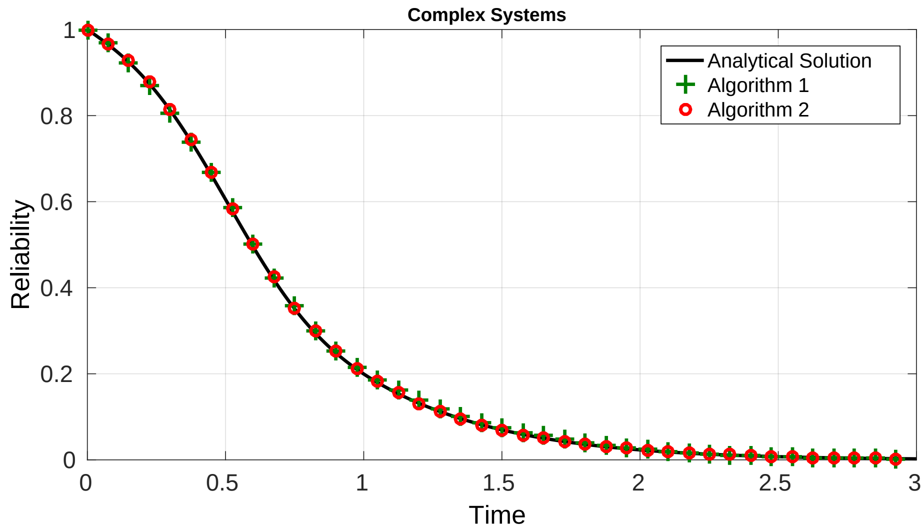

Figure 4.10: Survival function of the complex system calculated by Algorithms 1 and 2 and compared with analytical solution.

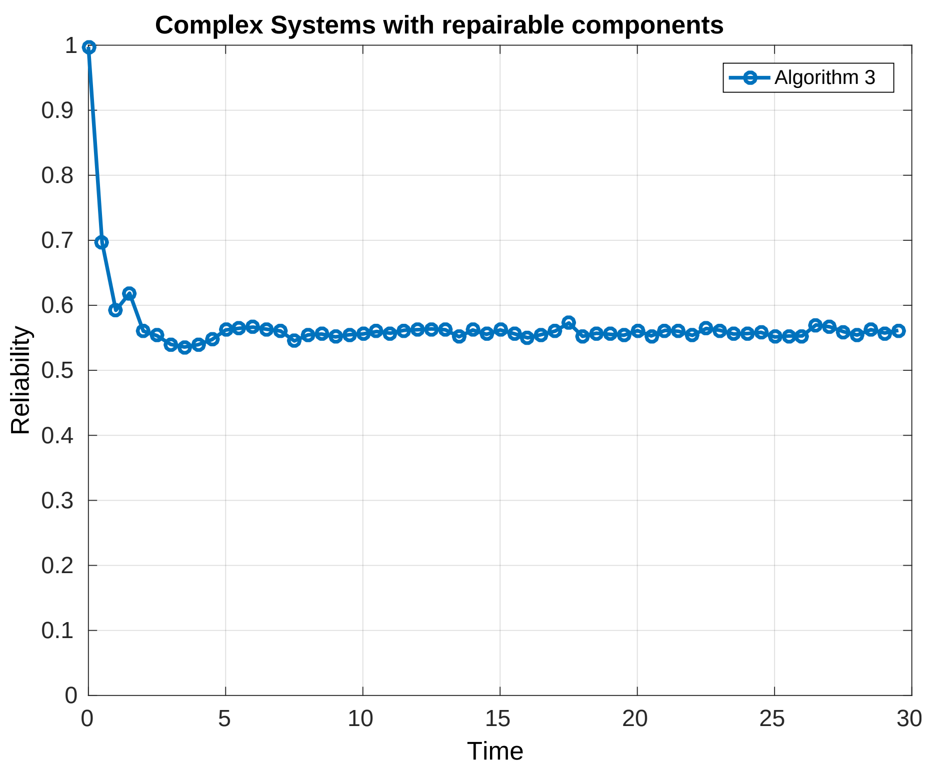

Figure 4.11: Survival function of the complex system with repairable components.

Chapter 5

Importance and Sensitivity Analysis of Complex Systems

5.1 Introduction

Component importance analysis is a sensitivity analysis which identifies the most "critical" component of the system, and in reliability theory, the most important component is the one that contributes the most to the system reliability and should be paid most attention. To be specific, component importance measures can help the engineers to rank components in order of decreasing (or increasing) importance, and to determine which components are important to the system reliability. Then the critical components should be given priority with respect to improvements or maintenance, which can improve the system performance [90].

In this Chapter, a new component importance measure is introduced as the relative importance index (RI). The survival signature-based component importance measures are applicable to both non-repairable and repairable complex systems. Through simulation methods based on survival signature, upper and lower bounds of the survival function of the system or relative importance index can be obtained efficiently. On this basis, the survival function of system and the importance degree of components can be quantified. In order to deal with the epistemic uncertainty when performing sensitivity analysis, the probability bounds analysis which is based on a pinching method is used.

Section 5.2 introduces the relative importance index as a component importance measure for non-repairable systems. Also, it considers the imprecision within the relative importance index. Then, the component importance measures for repairable systems are proposed in Section 5.3. After that, Section 5.4 gives an introduction about sensitivity analysis for systems under epistemic uncertainty with probability bounds analysis. Finally, some numerical examples are analysed in Section 5.5.

5.2 Component Importance Measures of Non-repairable Systems

5.2.1 Definition of Relative Importance Index

An important objective of a reliability and risk analysis is to identify those components or events that are most important (critical) from a reliability/safety point of view. These components should be given priority with respect to improvements or maintenance. Importance measures are important tools to evaluate and rank the impact of individual components within a system [91], which will allow one to study the relationship among components and the system. Importance measures have many applications in probabilistic risk analysis and there are many approaches based on various measures of influence and response. These importance measures provide a numerical rank to determine which components are more critical to system failure or more important to system reliability improvement.

A new importance measure is introduced herein as relative importance index indicated by RI, which is utilised to quantify the difference between the probability that the system functions if the ith component works and the probability that the system functions if the ith component is not working. The measure $RI_i(t)$ expresses the importance degree of a specific component during the survival time.

The relative importance index $RI_i(t)$ can be expressed as follows:

$$RI_i(t) = P(T_S > t \mid T_i > t) - P(T_S > t \mid T_i \leq t) \tag{5.1}$$

where $P(T_S > t \mid T_i > t)$ represents the probability that the system functions if the ith component works; $P(T_S > t \mid T_i \leq t)$ represents the probability that the system functions knowing that the ith component has failed.

The relative importance index $RI_i(t)$ is a function of time and it reveals the trend of the survival functions $P(T_S > t \mid T_i > t)$ and $P(T_S > t \mid T_i \leq t)$ of the system. This measure quantifies the degree of the influence in each component characterisation, i.e., the bigger the value of $RI_i(t)$, the bigger is the influence of the ith component on the estimation of the system reliability at a specific time t, and vice versa. At each point in time the largest RI over all components shows the most "critical" component. This helps to allocate resources for inspection, maintenance and repair in an optimal manner over the lifetime of a system.

The relative importance measures can be interpreted in a similar way as the Birnbaum's measure which quantifies the difference between the system reliability when component i is given as functioning and the system reliability when component i is given as failed. However, relative importance index $RI_i(t)$ uses the survival function instead of reliability equations to represent system reliability based on different conditions (work or fail) of the component i, which would usually be used to determine component importance for complex systems and networks with multiple types of component, because the survival functions of $P(T_S > t \mid T_i > t)$ and $P(T_S > t \mid T_i \leq t)$ can be obtained easily by using the survival signature.

5.2.2 Imprecision Within Relative Importance Index

Now taking imprecise probabilistic characterisations of the component failure probabilities into account, the set of all possible probability distribution functions can be represented as distributional p-boxes indicated with $M : P \in M$. The relative importance index can be defined as:

$$RI_i(t \mid P) = P(T_S > t \mid T_i > t) - P(T_S > t \mid T_i \leq t) \tag{5.2}$$

Therefore, the lower and upper bounds of relative importance index are:

$$\underline{RI_i}(t) =^{inf}_{P \in M} RI_i(t \mid P) \tag{5.3}$$

$$\overline{RI_i}(t) =^{sup}_{P \in M} RI_i(t \mid P) \tag{5.4}$$

Illustrative Example: Now let us calculate the relative importance index of component 4 of the system in Figure 3.3 in Chapter 3. First calculate the survival signature of the system in Figure 5.1 and Figure 5.2, which represents that the component 4 of type 2 works and fails at time t respectively.

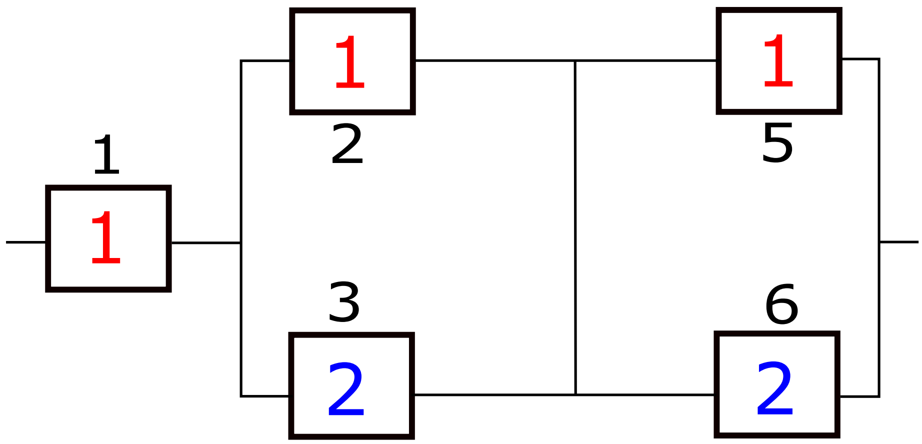

Figure 5.1: Component 4 works at time t.

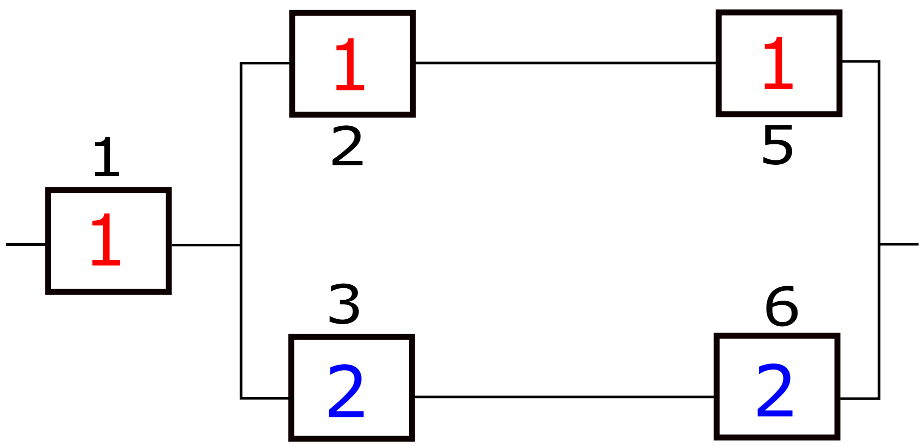

Figure 5.2: Component 4 fails at time t.

The survival signature of the two circumstances can be expressed as $\widetilde{\Phi}_1(l_1, l_2)$ and $\widetilde{\Phi}_0(l_1, l_2)$, and the results can be seen in Table 5.1 and Table 5.2 respectively.

So the relative importance index of component 4 can be expressed as:

$$RI_i(t \mid P) = P(T_S > t \mid T_i > t) - P(T_S > t \mid T_i \leq t)$$
$$= \sum_{l_1=0}^{m_1} \sum_{l_2=0}^{m_2-1} \widetilde{\Phi}_1(l_1, l_2) P(\bigcap_{k=1}^{2} \{C_k(t) = l_k\}) - \sum_{l_1=0}^{m_1} \sum_{l_2=0}^{m_2-1} \widetilde{\Phi}_0(l_1, l_2) P(\bigcap_{k=1}^{2} \{C_k(t) = l_k\})$$
$$= \sum_{l_1=0}^{m_1} \sum_{l_2=0}^{m_2-1} [\widetilde{\Phi}_1(l_1, l_2) - \widetilde{\Phi}_0(l_1, l_2)] P(\bigcap_{k=1}^{2} \{C_k(t) = l_k\}) \quad (5.5)$$

If the components' failure times have precise distribution parameters, e.g. $\lambda_1 = 0.8$ and $\lambda_2 = 1.6$, M degenerates to a probability function $P \equiv M = \{1 - e^{-\lambda t} : \lambda_1 =$

Table 5.1: Survival signature of the system in Figure 5.1

l_1	l_2	$\widetilde{\Phi}_1(l_1, l_2)$
0	0	0
0	1	0
0	2	0
1	0	0
1	1	0
1	2	1/3
2	0	0
2	1	1/3
2	2	2/3
3	0	1
3	1	1
3	2	1

Table 5.2: Survival signature of the system in Figure 5.2

l_1	l_2	$\widetilde{\Phi}_0(l_1, l_2)$
0	0	0
0	1	0
0	2	0
1	0	0
1	1	0
1	2	1/3
2	0	0
2	1	0
2	2	2/3
3	0	1
3	1	1
3	2	1

$0.8; \lambda_2 = 1.6\}$. Hence, the relative importance index of component 4 can be calculated by using an analytical method and the results can be seen in Figure 5.3.

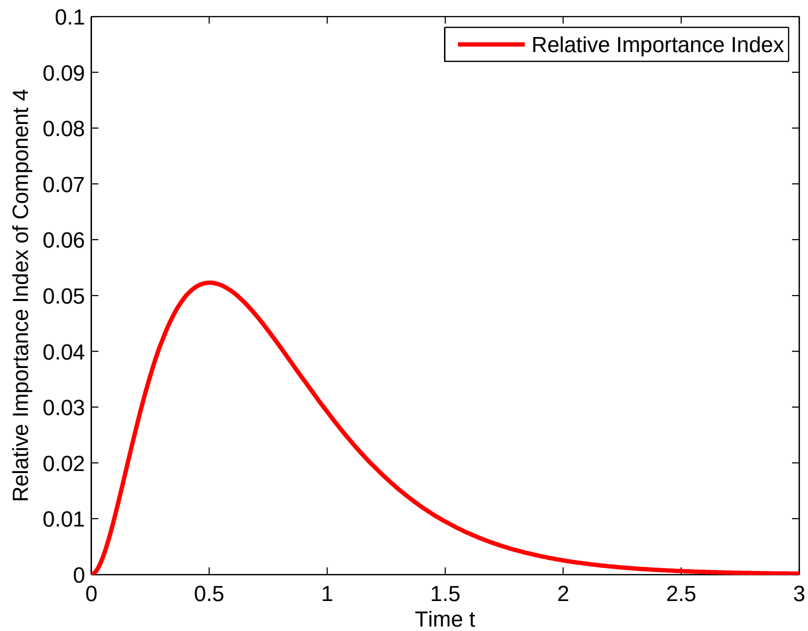

Figure 5.3: Relative importance index of Component 4 with precise distribution parameters.

Considering imprecisions within components failure times, the set of all probability distributions defines a probability p-box for each component failure time: $M = \{1 - e^{-\lambda t} : 0.4 \leq \lambda_1 \leq 1.2; 1.3 \leq \lambda_2 \leq 2.1\}$. Therefore, the lower and upper bounds of relative importance index of component 4 can be calculated through a simulation method. Figure 5.4 shows the results.

5.3 Component Importance Measures of Repairable Systems

Component importance measures are invaluable in the real engineering world to identify the weak components. The existing importance measures are mostly calculated through analytical approaches, and application of these measures to complex repairable systems may be intractable. In order to overcome intractability, two importance measures work by figuring out how much each component or component set contributes to system unavailability. What is more, another index is used to quantify the importance degree of the

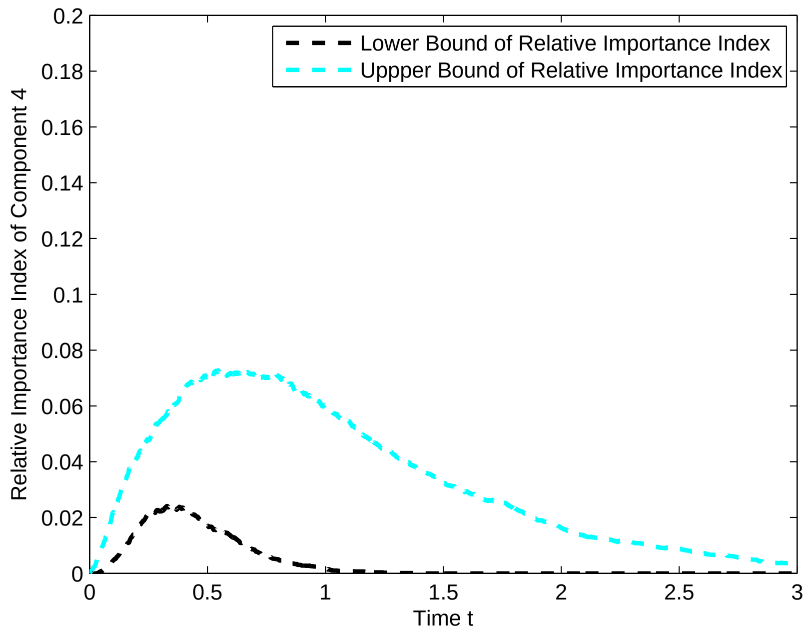

Figure 5.4: Relative importance index of Component 4 with imprecise distribution parameters.

specific component and component set. It always takes the system with m components which belong to K component types for example in this Section.

The structure function-based method and the survival signature-based method are both based on Monte Carlo simulation, which is general and useful for many problems. By generating the state evolution of each component, the structure function is computed to determine the state of the system. However, the survival signature is a summary of the structure functions, which is efficient to deal with the complex configuration systems.

5.3.1 Importance Measure of a Specific Component

The relative importance index $RI_i(t)$ of the ith component at time t that was first used in [60] can be extended to analyse the importance degree of components in the repairable system. To be specific, it is the repairable system survival function probability differences if the ith component works or not. The mathematical expression formula of the relative importance index which is based on the survival signature can be expressed as

$$RI_i^{SS}(t) = P(T_S > t \mid T_i > t) - P(T_S > t \mid T_i \leq t) \tag{5.6}$$

where $P(T_S > t \mid T_i > t)$ represents the probability that the repairable system works knowing that the ith component functions; $P(T_S > t \mid T_i \leq t)$ denotes the repairable system survival probability if the same component fails.

However, the relative importance index calculated by survival signature needs to know the survival signature of the new system. If the system is complex, it is time consuming work. Therefore, the structure function-based relative importance index is introduced to identify the specific component's importance degree, and its equation is

$$\begin{aligned} RI_i^{SF}(t) = P(T_S > t \mid x_i\ repairable \bigcap T_i > t) - \\ P(T_S > t \mid x_i\ non-repairable \bigcap T_i > t) \end{aligned} \quad (5.7)$$

where $P(T_S > t \mid x_i\ repairable \bigcap T_i > t)$ means the survival function of the repairable system if the ith component can be repaired normally; while $P(T_S > t \mid x_i\ non - repairable \bigcap T_i > t)$ indicates the probability that the system functions knowing that the same component cannot be repaired after failure.

5.3.2 Importance Measure of a Set of Components

It is sometimes important to evaluate the importance of a set of components instead of a specific one in the real engineering world. Therefore, the relative importance index for a specific component can be extended to a set of k components, which can be denoted by $RI_k(t)$. The set of components can be either in one single type or different types.

For the first situation, $RI_k(t)$ is convenient to combine with the survival signature and it is the probability difference values of the repairable system works if the components of type k are repairable or they cannot be repaired. The expression can be written as follows

$$RI_k^{SS}(t) = P(T_S > t \mid l_k\ repairable \bigcap T_i > t) - P(T_S > t \mid l_k\ non-repairable \bigcap T_i > t) \quad (5.8)$$

where $P(T_S > t \mid l_k\ repairable \bigcap T_i > t)$ represents the probability that the repairable system works if components of type k are repairable; $P(T_S > t \mid l_k\ non - repairable \bigcap T_i > t)$ denotes the probability that the repairable system functions knowing that the components of type k cannot be repaired.

For the second condition, it is more efficient to analyse the importance degree of a set of components that belong to different types. The mathematical equation of the structure function-based method is

$$RI_k^{SF}(t) = P(T_S > t \mid \underline{x}_i^k \; repairable \bigcap T_i > t) - P(T_S > t \mid \underline{x}_i^k \; non-repairable \bigcap T_i > t) \tag{5.9}$$

where $P(T_S > t \mid \underline{x}_i^k \; repairable \bigcap T_i > t)$ means the survival probability of the repairable system if the set of components of type k is repairable, here $i \in (1, 2, ..., m_k)$ and $k \in (1, 2, ..., K)$. $P(T_S > t \mid \underline{x}_i^k \; non - repairable \bigcap T_i > t)$ represents the survival function of the system given that the components of type k set is non-repairable.

It can be seen that both $RI_i(t)$ and $RI_k(t)$ are time dependent and both of them can be calculated by the survival signature-based method and the structure function-based method, respectively. What is more, they reveal the trend of the importance degree of a specific component or a set of components within the repairable system during the survival time. The greater level value of $RI_i(t)$ or $RI_k(t)$ is, the more "critical" is the ith component or the set of components on the repairable system reliability at a specific time t, and vice versa. This helps to allocate resources, which might include resources for reliability improvement, surveillance and maintenance, design modification, security, operating procedure, training, quality control requirements, and a wide variety of other resource expenditures. By using the importance of a specific component or a set of components, resources expenditure can be properly optimised to reduce the total life-cycle resource expenditures while keeping the risk as low as possible. In other words, for a given resource expenditure such as for maintenance, the importance measure of a specific component or set of components can be used to allocate resources to minimise the total system risk. This approach allows the risk manager to offer the "biggest bang for the buck" [92].

5.3.3 Quantify Importance Degree

In order to quantify importance degree of the specific component or a set of components, the quantitative importance index (QI) is introduced in this paper. The numerically obtained index for a repairable system is through a Monte Carlo simulation method which is based on survival signature and structure function. The failure times of the system can be calculated through each trial, after having simulated many histories of the system, estimates are made of the desired relative criticality index statistically. For a system with m components belonging to K types, the quantitative importance index of the specific component i is expressed as

$$QI_i = \frac{N_i^f}{max\{N_1^f, ..., N_i^f, ..., N_m^f\}} \tag{5.10}$$

where N_i^f represents average number of system failures if the ith component cannot repair while the other components can be repaired normally; and $max\{N_1^f, ..., N_i^f, ..., N_m^f\}$ denotes the biggest value of all N_i^f with $i = 1, 2, ..., m$.

Similarly, for components set k, the QI can be written as

$$QI_k = \frac{N_k^f}{max\{N_1^f, ..., N_k^f, ..., N_K^f\}} \tag{5.11}$$

where N_i^f denotes the average failure number of the system if the components set k are non-repairable but the other components sets are repairable; while $max\{N_1^f, ..., N_k^f, ..., N_K^f\}$ denotes the maximum number of all N_k^f with $k = 1, 2, ..., K$.

These two indices can quantify the importance degree of a specific component and a components set respectively, and the quantitative importance index values of all the components are compared with the biggest QI value. Therefore, the bigger the value is, the bigger influence of the ith component or the components set k on the repairable system.

5.4 Sensitivity Analysis for Systems Under Epistemic Uncertainty with Probability Bounds Analysis

5.4.1 Represent Epistemic Uncertainty by P-box

In real cases, the amount and quality of information to specify a probabilistic model can be limited to such an extent that the associated magnitude of imprecision makes the entire analysis meaningless. In such cases it is essential to identify those contributions to the imprecision, which influence the results most strongly. Once these are known, targeted measures and investments can be defined in order to reduce the imprecision to enable a meaningful survival analysis.

For the uncertain event X, $\Delta(X) = \overline{P}(X) - \underline{P}(X)$ is called the imprecision for the uncertain event X [93]. Since epistemic uncertainty reflects the unsureness in the predicted reliability, a decision maker might wants to reduce it by investing resources to estimate more accurately the value of each event parameter [94].

As for the system reliability, the survival function is time varying, which can be seen in Equations 3.1 and 3.2. The epistemic uncertainty within the components failure time distribution parameters propagates to the survival function of the system $P(t)$. The epistemic uncertainty will lead to the lower and upper bounds of the survival function, which can be expressed by $\underline{P}(t)$ and $\overline{P}(t)$ respectively.

Figure 5.5 shows an example of the p-box of the system survival function.

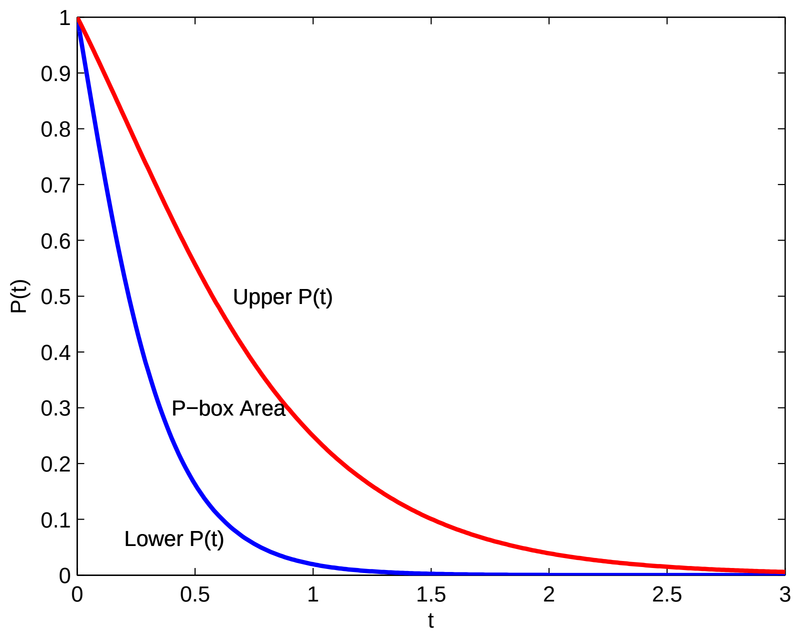

Figure 5.5: Example of p-box of the system survival function

The area of p-box, denoted by A_{PB}, reflects the degree of the epistemic uncertainty. To be specific, to calculate the survival function of a time dependent system which is described by p-box, the time interval is discretised into several subintervals, the minimum and maximum difference value at each subinterval can be found. Then summarise the product of difference value and its corresponding subintervals, which can get the value of A_{PB}. The more knowledge or information on the failure time distribution, the smaller area of the p-box. If there exists no epistemic uncertainty, which means the precise values of the components failure distribution parameters are known to us, the area of p-box A_{PB} will shrink to zero.

Since it is clear that $\underline{P}(t) \leq P(t) \leq \overline{P}(t)$, and $P(t)$ reflects the reliability of the system at different time t, the A_{PB} can be expressed by the following Equation.

$$A_{PB} = \int_0^{\infty} [\overline{P}(t) - \underline{P}(t)]dt \tag{5.12}$$

It can be seen from Equation 5.12 that A_{PB} is the difference between the estimated upper and lower survival functions of the system in presence of epistemic uncertainty.

5.4.2 Probability Bounds Analysis as Sensitivity Analysis

Ferson and Donald [95] developed probability bounds analysis (PBA) which can produce bounds around the output distribution from an assessment. These bounds enclose all the possible distributions that could actually arise given what is known and what is not known about the model (system) and its inputs (components failure distribution parameters). Therefore, PBA represents uncertainty by using a p-box.

PBA is a combination of probability theory and interval analysis, and the main advantage of PBA is that it separates aleatory uncertainty from epistemic uncertainty and propagates them differently, thus each maintains its own character [96].

System sensitivity analysis is a systematic study of how the inputs of the system influence the reliability of the system. Therefore, system sensitivity analysis has two fundamental features: one is to find out how the reliability and function of the system are influenced by the inputs, and another is to focus on improving estimates of inputs which will lead to the most improvement of the system reliability.

Because of the obvious and fundamental importance of sensitivity analysis on systems, it is essential and of interest to perform a sensitivity analysis on the systems by combining with the probability bounds analysis. This thesis uses a pinch strategy to assess the impact of epistemic uncertainty on the systems.

As we knew before, the epistemic uncertainty within the components failure time distribution will lead to lower and upper survival function bounds of the system, and the epistemic uncertainty degree can be quantified by A_{PB}. If there is extra information or data are available on an input, there will be less uncertainty degree of the whole system, which also means the A_{PB} will decrease. Therefore, it is feasible to compare the value difference before and after "pinching" an input, i.e. replacing the uncertain input with a point value or with a precise distribution function. Pinching can be applied to each input and the maximum reduction of uncertainty of the system is regarded as the most sensitive input of the system.

The estimate of the value of information for a parameter will depend on how much uncertainty is within the parameter, and how it influences the uncertainty in the system reliability. The reduction or sensitivity can be expressed by Equation 5.13.

$$100\left(1-\frac{A_{PB}^{after}}{A_{PB}^{before}}\right)\% \tag{5.13}$$

where A_{PB}^{before} is the former p-box value of A_{PB}, while A_{PB}^{after} represents the area of p-box with an input which is pinched.

The result of this Equation reflects the percentage reduction of uncertainty when the former uncertain input parameter is replaced by a more precise value. The pinching theory [97] can be applied to each system component in turn and the results of all components are got through the above Equation, then ranking the results to find out the most sensitive component. What is more, it can be extended to pinch multiple inputs simultaneously to perform the sensitivity analysis on components set of the system, and then locate which components set is more sensitive. Therefore, we can use a sensitivity index (SI) to represent Equation 5.13.

5.5 Numerical Example

5.5.1 Hydroelectric Power Plant System

Let us use the power plant system in Figure 3.11 of Chapter 3 to illustrate the relative importance index. The following part consists of two cases: Case A presents the full probability system, while Case B allows imprecision within the system. Table 3.3 shows the precise and imprecise distribution parameters of components in the hydro power plant system.

Case A: Based on the survival function it is possible to calculate the influence of each component on the system reliability for each point in time t. The basic theoretical knowledge and equations can be seen in Section 5.2, which allows to estimate the relative importance index $RI_i(t)$ of each component.

For the other component importance measures, analytical methods can be used to rank the component importance degree. The equations of Birnbaum's measure (BM), risk achievement worth (RAW) and Fussel-Vesely's measure (FV) calculate the component importance $I_i(t)$ of the ith component at time t as can be seen in Table 5.3.

Table 5.3: Component importance equations of BM, RAW and FV

Methods	Component Importance Equations
BM	$I_i^B(t) = \frac{\partial R_S(t)}{\partial R_i(t)}$
RAW	$I_i^{RAW}(t) = \frac{R_S(t)(R_i(t)=1)}{R_S(t)}$
FV	$I_i^{FV}(t) = \frac{R_S(t)-R_S(t)(R_i(t)=0)}{R_S(t)}$

In the above equations, $R_S(t)$ and $R_i(t)$ represent the reliability of the system and the ith component at time t. The Birnbaum's measure [4], $I_i^B(t)$, is defined as the rate of change in total risk of the system with respect to changes in a risk element's basic probability (or frequency). Risk achievement worth [98], $I_i^{RAW}(t)$, is the ratio of the new risk to the baseline risk of the system when the probability of the specified risk element is set to unity. And Fussel-Vesely's measure [16], $I_i^{FV}(t)$, is defined as the fractional contribution of a risk element, the total risk of the system of all scenarios containing that specified element. For the power plant in Figure 3.11, the reliability equation $R_S(t) = R_1(1-(1-R_2R_3R_4R_5)(1-R_6R_7R_8R_9))R_{10}(1-(1-R_{11})(1-R_{12}))$.

The component importance obtained at $t = 0.12$ using the proposed method for the power plant system have been compared with the results Birnbaum's measure (BM), risk achievement worth (RAW) and Fussel-Vesely's measure (FV) as shown in Table 5.4.

Table 5.4: Comparision of component importance obtained using different methods at $t = 0.12$

Components Methods	CG	$BV1$ $BV2$	$T1\,T2$	$G1$ $G2$	$CB1$ $CB2$	$CB3$	$TX1$ $TX2$
BM	0.8854	0.1181	0.1366	0.1177	0.1191	0.8846	0.2703
ranking	1	6	4	7	5	2	3
RAW	7.8947	1.9280	1.9280	1.9280	1.9280	7.8947	2.5270
ranking	1	3	3	3	3	1	2
FV	1.000	0.1346	0.1346	0.1346	0.1346	1.000	0.2215
ranking	1	3	3	3	3	1	2
RI	0.8831	0.1217	0.1401	0.1213	0.1221	0.8693	0.2656
ranking	1	6	4	7	5	2	3

According to the above table, it can be calculated that the RI method has the same component importance ranking as Birnbaum's measure. Also, the proposed RI method has the same ranking trend as RAW and FV. The RI method just needs the survival signature without calculating the reliability equation, which is useful for large systems with multiple component types.

The relative importance index values of each component over time are shown in Figure 5.6.

The relative importance index values reveal the component importance over time. The bigger the value of $RI_i(t)$ is, the more "critical" the ith component is. The above results show that $BV1$ and $BV2$ have the same relative importance index values, and the same applies to $T1$ and $T2$, $G1$ and $G2$, $CB1$ and $CB2$, $TX1$ and $TX2$. This is because the components are in a parallel configuration and they have the same failure time distribution type and parameters, which is also according to our common sense that these components have the same importance degree to the system. For component $CB3$, it has same failure time type and distribution parameters as components $CB1$ and $CB2$, but has different location in the system. Therefore, the relative importance index value of component $CB3$ is bigger than relative importance index values of components $CB1$ and $CB2$, but not as big as the relative importance index value of component CG. Components CG and $CB3$ have the same decreasing trend of relative importance index over time, while for the other

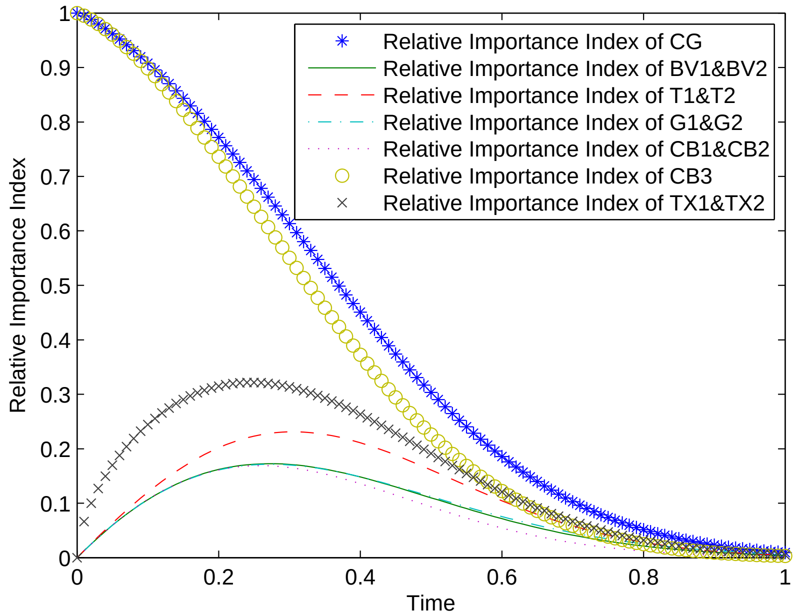

Figure 5.6: Relative importance index values of the system components.

components, the trends of relative importance index increase first, then decay with time. The relative importance index values of components $TX1$ and $TX2$ are always smaller than other components, which means they have smallest influence degree to the system reliability.

Case B: This Case considers imprecision within the distribution parameters of system components. Therefore, as a further step the imprecision can be carried forward to calculate ranges for the relative importance index. Firstly, ranges for the survival functions assuming that a given component fails or works are calculated for each component, then the associated ranges for the relative importance index for each component are determined, see Figure 5.7 and Figure 5.8.

From the above figures it can be recognised that imprecision within component failure times can lead to imprecision of relative importance index of the component.

5.5.2 Repairable Complex System

The complex repairable system in Figure 4.9 of Chapter 4 is analysed in this part; components failure and repair data for each component type of the system can be seen in Table 4.2.

Case A: Let first perform the importance measure of a specific component which is based on the structure function. The results can be seen in Figure 5.9.

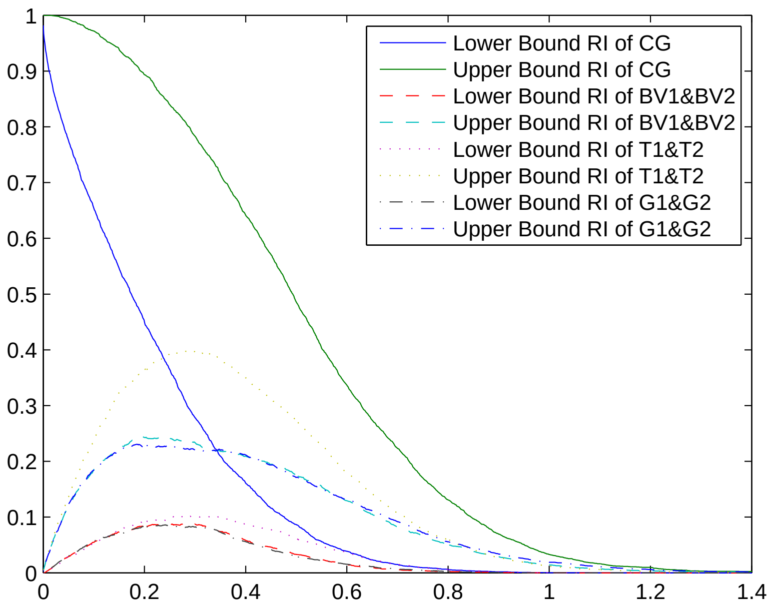

Figure 5.7: Upper and lower relative importance index of components CG, BV, T and G.

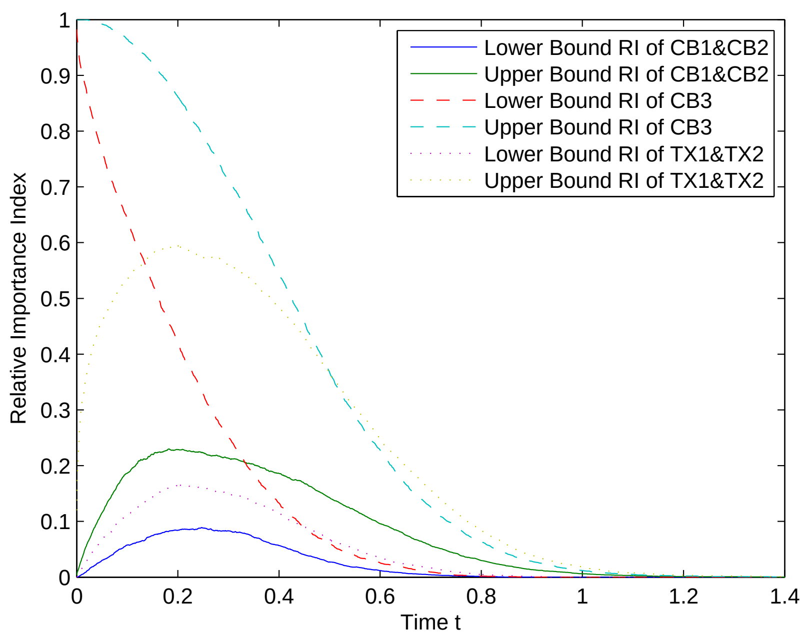

Figure 5.8: Upper and lower relative importance index of components CB and TX.

It is clear that component 14 always has a higher relative importance index than the other thirteen components, which means it is the most "critical" component in the repairable system. Then it comes to component 8. Component 13 has little relative impor-

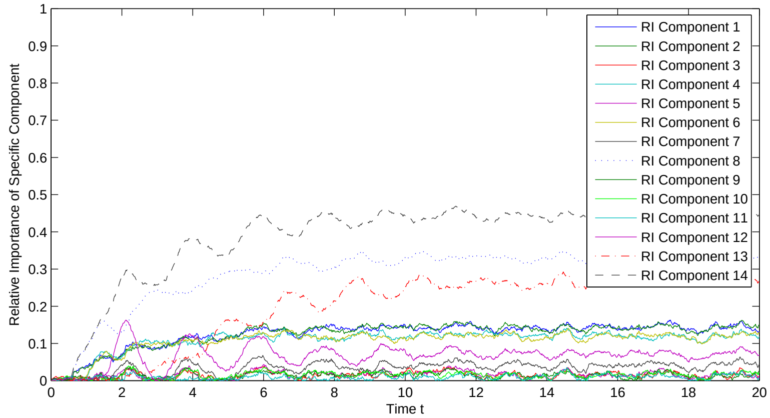

Figure 5.9: Relative importance index of the specific component in the system.

tance index values at the first time, however, its relative importance index values become bigger as time goes on, which just follows the components 14 and 8. Component 1 and component 2 have similar relative importance values, which sometimes cross over. The same circumstance occurs on components 4 and 6. The relative importance of the five components (3, 5, 7, 9, 10) is always within 0.1, which means they have less importance influence degree than other components on the repairable system.

Case B: In the real application world, sometimes people want to know the importance degree of a set of components. i.e. the relative importance index of components of set 1 to set 6 in this repairable system. Figure 5.10 shows the results of them.

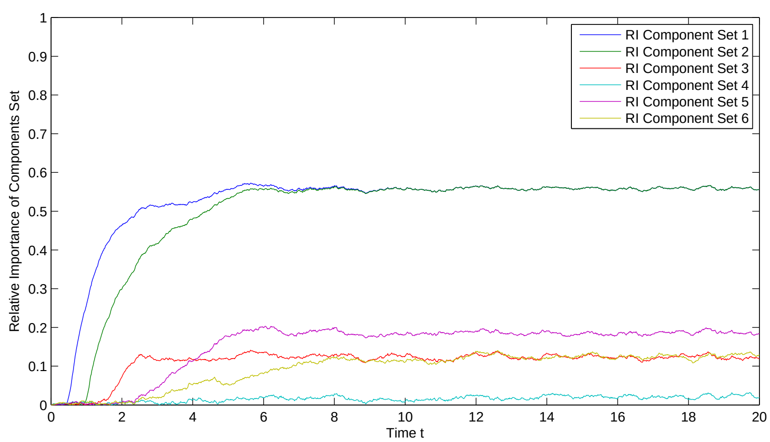

Figure 5.10: Relative importance index of the components sets with same type in system.

It can be seen that the relative importance index values of component sets 1 and 2 are bigger than other component sets. Therefore, components of types 1 and 2 are more important than components of other types in this repairable system. On the contrary, component set 4 is the least important within the system because it has the smallest values of relative importance index. The values of component set 1 are higher than 2 at the beginning time. However, their values are the same as the survival time goes on. Component set 5 has lower relative importance values than component sets 3, but the values go up and rank the third within the six component sets in the last. Component set 3 and 6 has the similar relative importance values trend, although the value of set 5 is bigger than set 6 at the beginning time.

When it comes to analyse the importance degree of a set of components which belong to different types, the efficient structure function method can be used. Suppose it is a necessary to perform sensitivity analysis on six components sets, that is set 1 with three components (5, 7, 9), set 2 with components (1, 6, 10), set 3 with components (3, 4, 13), set 4 with components (2, 8, 13), set 5 with components (12, 14) and set 6 with four components (1, 3, 7, 9). Figure 5.11 indicates the importance degree of these six components sets which belong to different component types.

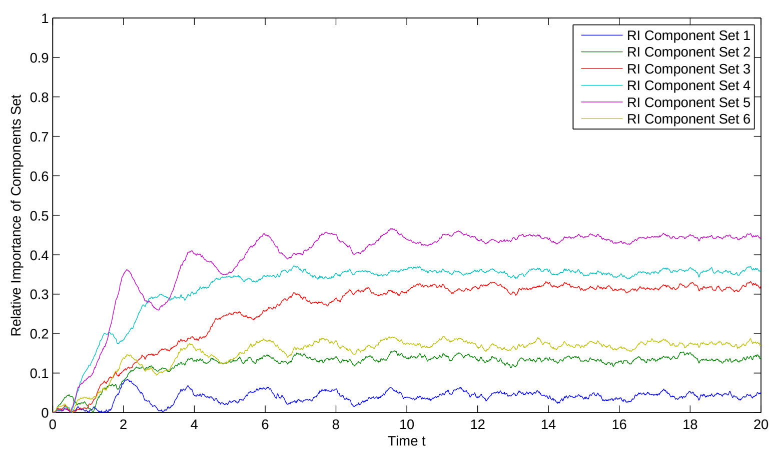

Figure 5.11: Relative importance index of the components sets with different types in system.

From the above figure, we can see that the relative importance index value of different sets at each time. For example, at time $t = 8$, the components set 5 has the biggest influence on this repairable system. Then it comes to components set 4 and 3. The components set 6 and 2 ranked the fourth and fifth respectively, while components set 1 has the least relative importance index value.

Case C: If using the quantitative importance index to quantify the importance degree during the survival time, the QI of a specific component and different types components set can be seen in Figure 5.12 and Figure 5.13, respectively.

The first figure shows that component 14 is the most "critical" one to the whole system, while the second figure indicates set 5 with components (12, 14) has the biggest influence

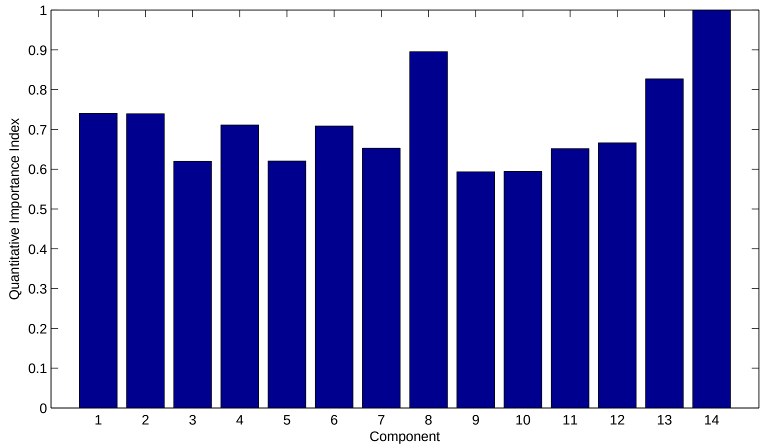

Figure 5.12: Quantitative importance index of the specific component in the system.

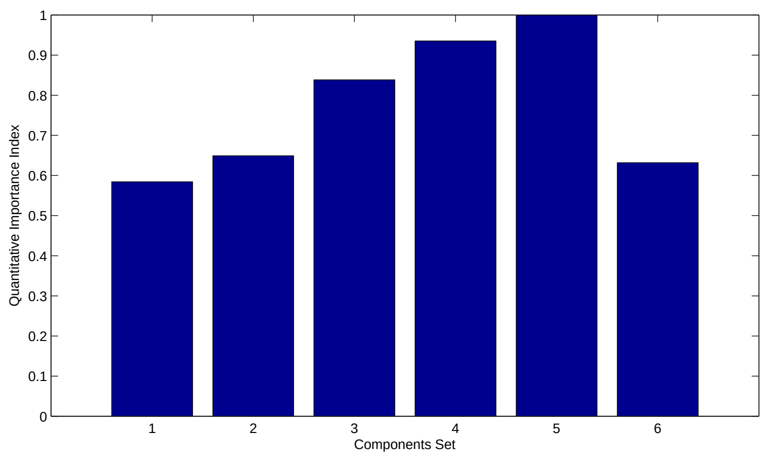

Figure 5.13: Quantitative importance index of the components sets with different types in the system.

degree on the repairable system.

5.5.3 Typical Complex System

Figure 5.14 shows a bridge system, which is a typical complex system with three component types.

The components' lieftimes all satisfy an Exponential distribution. If there exist epistemic uncertainty within the parameters, the imprecise and precise distribution parameters of all components can be seen in Table 5.5.

Accordingly, the bounds of imprecise survival function and precise survival function

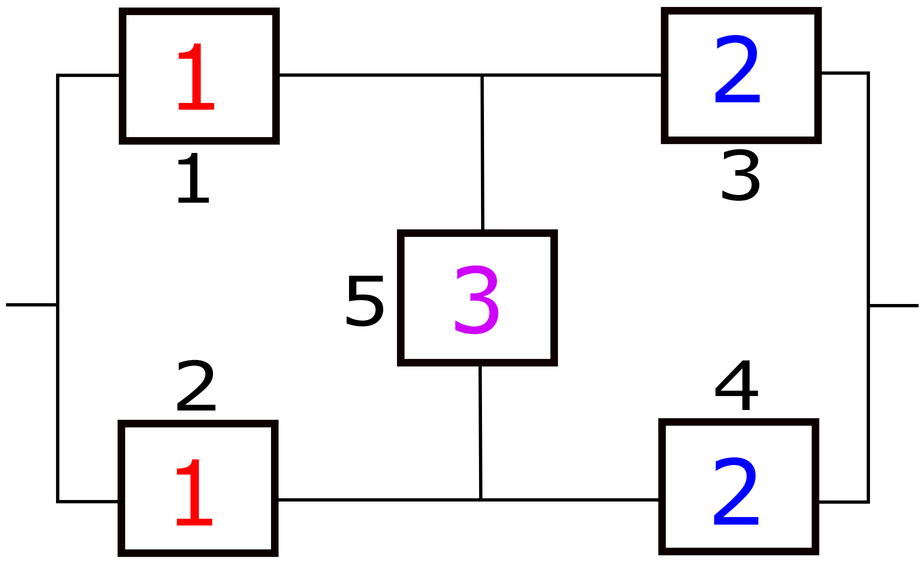

Figure 5.14: Typical complex system: the number outside the box is the component index, while the number inside the box represents the component type.

Table 5.5: Imprecise and precise distribution parameters of all components in the typical complex system

Component index	Component type	Imprecise parameter	Precise parameter
1	1	[0.24, 0.50]	0.37
2	1	[0.24, 0.50]	0.37
3	2	[0.18, 0.55]	0.365
4	2	[0.18, 0.55]	0.365
5	3	[0.21, 0.45]	0.33

of the typical system can be seen in Figure 5.15.

The area of p-box $A_{PB}^{before} = 0.2551$ by using Equation 5.12, which reflects the degree of the epistemic uncertainty. Then let us perform a sensitivity analysis on specific component and components set under epistemic uncertainty with probability bounds analysis.

Case A: Sensitivity Analysis on Specific Component

Let replace the imprecise input with a precise distribution parameter of each component, as shown in Table 5.5.

Taking component 1 as an example, we replace the imprecise distribution Exponential([0.24, 0.50]) with the precise distribution Exponential(0.37), while the other components remain the former imprecise parameters. Now the p-box of the system survival function is shown in Figure 5.16.

Therefore, it can be calculated that $A_{PB1}^{after} = 0.2112$ through Equation 5.12. The sensitivity index of component 1 can be calculated by Equation 5.13, which is 17.209%.

Similarly, the sensitivity index of components 2, 3, 4 and 5 are 17.21%, 27.95%,

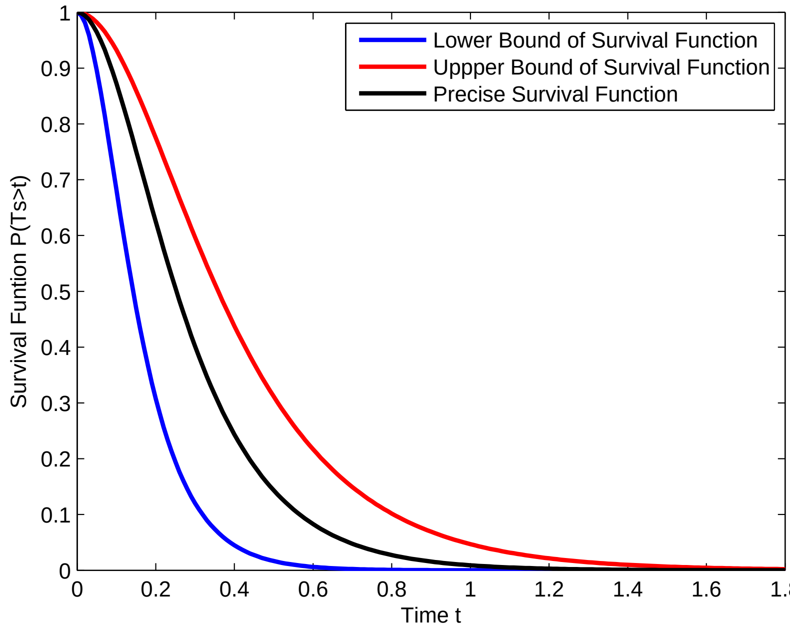

Figure 5.15: The bounds of imprecise survival function and precise survival function of the typical system

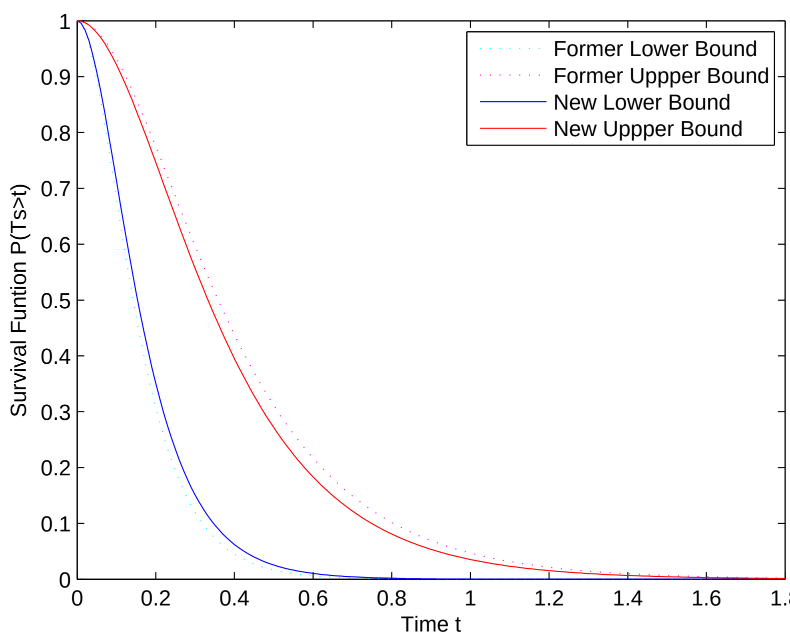

Figure 5.16: The p-box of the system survival function when component 1 is pinched by a precise distribution

27.95% and 4.98% respectively. Thus, the sequence of each component's sensitivity index is $SI_3 = SI_4 > SI_1 = SI_2 > SI_5$, which means components 3 and 4 are more sensitive than the other three components.

Case B: Sensitivity Analysis on Components Set

It is sometimes important to analyse the sensitivity degree of different components sets to the system. For instance, if people want to know the sensitivity index of these five different components sets: C[1,3], C[2,4,5], C[1,2], C[3,4] and C[5], it is necessary to replace the imprecise distribution parameters of components in each set with precise distribution parameters, which can also be seen in Table 5.5.

Considers pinching component 1 and component 3 to distributions with precise parameters, to be specific, component 1 with Exponential(0.37) while component 3 with Exponential(0.33). Then calculate the lower and upper bounds of the system survival function, which is shown in Figure 5.17.

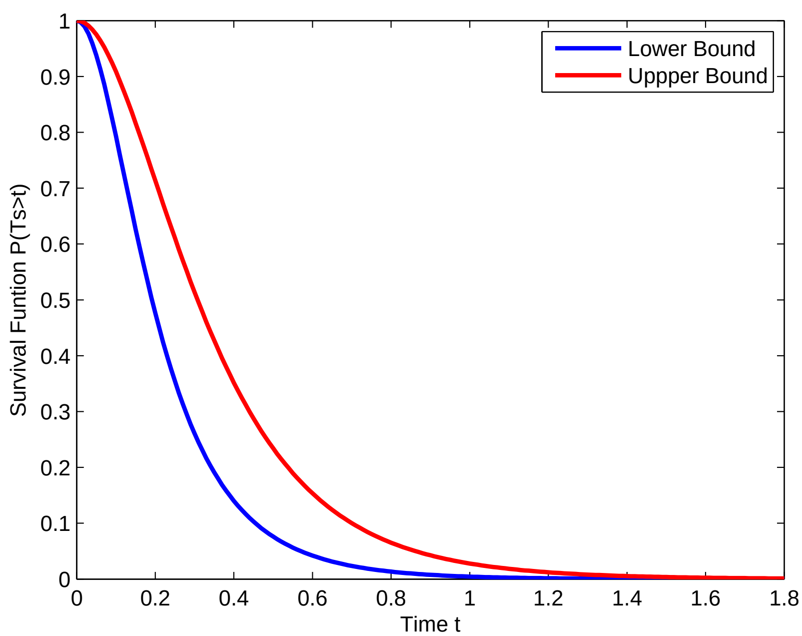

Figure 5.17: The p-box of the system survival function when components set C[1,3] is pinched by a precise distribution

Here it can be seen that the area of p-box shrinks a lot compared to the initial one, to be specific, $A_{PB13}^{after} = 0.1321$ after quantization calculation. Therefore, the percentage reduction is 48.22%.

Similarly as the above case, compare with the former epistemic uncertainty degree A_{PB}^{before}, the percentage reduction is 53.19%, 34.89%, 55.27% and 4.98% for the other four components sets, which means $SI_{C[3,4]} > SI_{C[2,4,5]} > SI_{C[1,3]} > SI_{C[1,2]} > SI_{C[5]}$.

Engineers should pay more attention on components sets C[3,4] and C[2,4,5] as they are more sensitive than the other components sets.

5.6 Conclusion

In order to quantify the influence degree of components without and with imprecision, a novel component-wise importance measure has been presented: the relative importance index. Importance measures allow us to identify the most "critical" system component at a specific time. This allows an optimal allocation of resources for repair, maintenance and inspection. This novel and efficient method is conducted in an analytical way or through simulation methods based on survival signature, which improves the computational efficiency. Using the relative importance index, the importance of the individual components is ranked to obtain a preference list for maintenance and repair.

The component importance index of complex non-repairable systems can be extended to repairable systems. In many cases, uncertainties cannot be quantified precisely since they are characterised by incomplete information, limited sampling data, ignorance, measurement errors and so on. Thus, a thorough and realistic quantitative assessment of the uncertainties is quite important. In order to find out which component or components set with epistemic uncertainty are more sensitive to the system, the probability bounds analysis which is based on pinching theory is introduced. The effectiveness and feasibility of the proposed approaches have been demonstrated with some numerical examples. The results show that the survival signature-based component importance measures are efficient to perform sensitivity analysis of non-repairable and repairable systems.

Chapter 6

Complex System Reliability Under Common Cause Failures

6.1 Introduction

A common cause failure (CCF) is an event that causes multiple components to fail simultaneously as a result of a shared (or common) cause, which exists widely in complex systems and networks. Common cause events are highly relevant to probabilistic safety assessments due to their potential adverse impact on the safety and availability of critical safety systems [99]. A number of parametric models have been developed for common cause failures over the time since the publication for the reactor safety study [100].

In this Chapter, a survival signature-based reliability analysis on complex systems and networks with common cause failures is proposed. The α-factor model distinguishes between the total failure rate of a component and the common cause failures modelled by α-factor parameters, which can be obtained through experts' judgements of the system or the past data on the system. These advantages of the α-factor model make it possible to combine with the survival signature to assess the complex system reliability after CCFs.

The standard α-factor model is proposed first in the thesis to perform system reliability analysis after CCFs. However, it has two assumptions that there must be at least one component failure of each type and the component failures of each type are independent. In order to remove these assumptions, another general α-factor model has been introduced. It takes account of all the possible combinations of the failed numbers of past events. The novel standard and general α-factor models can be expressed by the time independent equations which connect with the survival signature. Given the system components of each type that fail simultaneously after a common cause event, those of the same type are all equally likely to fail. Therefore, the survival signature remains the same as before, and it can perform the time dependent system reliability analysis after common cause failures. In many cases, however, the α-factor estimators or components failure distribution parameters cannot be quantified precisely because of limited test data, incomplete information, ignorance and so on. Thus, it is essential to take this incertitude into consideration, which will lead to the imprecise system reliability probability. The applicability of the proposed approaches are demonstrated by solving the numerical examples and, by investigating the results, the concept of design for reliability should be given more attention.

Section 6.2 proposes the time independent and time varying models for system reliability after common cause failures, while Section 6.3 shows the applicability and performance of the proposed methods by analysing a numerical example.

6.2 System Reliability after Common Cause Failures

6.2.1 Instruction of α-factor Model

The α-factor model is particularly useful in the practical engineering world as the alpha factor parameters can be got through experts' judgement of the system or past data on the system. The parameters α_r of the model are the fractions of the total probability of failure in the system that involves the failure of r components due to a common cause. For system with single component type, it can be expressed as:

$$\alpha_r = \frac{n_r}{\sum_{i=1}^{m} n_i} \tag{6.1}$$

where, n_r is the number of events with r failed components.

The α parameter estimator represents the probability that exactly r of the m components fail, given that at least one failure has occurred. It can be seen from Equation 6.1 that the sum of all the α_r will be 1.

For components which belong to multiple types, the α parameter estimator is indicated as:

$$\alpha_r^k = \frac{n_r^k}{\sum_{i=1}^{m_k} n_i^k} \tag{6.2}$$

where, n_r^k is the number of r failed components which belong to type k.

Similarly, the summation of α factor parameters of type k is 1, with $\sum_{r=1}^{m_k} \alpha_r^k = 1$.

The characters of the α-factor model make it possible to combine with the survival signature to assess the complex system reliability. In this following sections, the survival signature is introduced to analyse complex system with common cause failures. Based on the results of [47], the α-factor model can be applied to calculate system reliability in the presence of common cause failures.

6.2.2 Standard α-factor Model for System Reliability

Let assume that there is a system with m_k components belonging to type $k \in \{1, 2, ..., K\}$. When a failure event occurs, $P(f_1, f_2, ..., f_K)$ denotes the probability for how many failures occur of each component type. The survival signature $\Phi(l_1, l_2, ..., l_K)$ represents the probability that the system is functioning when l_k components of type k are working, which can be expressed as

$$\Phi(l_1, l_2, ..., l_K) = P(system\ functions \mid l_k\ components\ of\ type\ k\ work) \tag{6.3}$$

where $k \in \{1, 2, ..., K\}$.

Let $P(S_{CCF})$ denote the probability that the system still functions after a common cause failure event and can be calculated as

$$P(S_{CCF}) = \sum_{l_1=0}^{m_1} ... \sum_{l_K=0}^{m_K} \Phi(l_1, ..., l_K) P(m_1 - l_1, ..., m_K - l_K) \tag{6.4}$$

where $f_i = m_i - l_i$.

From Equation 6.4 it can be seen that unlike the $P(T_s > t)$ calculated by Equation 3.2, the survival function of the system after a CCF is independent of t. In addition, the typical merit of the survival signature can also be held. To be specific, the survival signature, which encompasses information of the system structures, and the probability distribution, which relies on the common cause failures of the components belonging to different types, are separated in the equation.

The α-factor model with estimate of the alpha factor parameters are mainly used in this Section and they are given by Equation 6.1 for a common cause group of m components.

$P(f_1, f_2, ..., f_K)$ is constructed by using the past data available on the system, combined with the α-factor model for CCFs. For this standard α-factor model, there are assumptions that $P(0, 0, ..., 0) = P(0, f_2, ..., f_K) = P(f_1, 0, ..., f_K) = ... = P(f_1, f_2, ..., 0) = 0$, as there must be at least one component failure of each type. What is more, there is another assumption that the components failures of each type are independent, which implies that $P(f_1, f_2, ..., f_K) = P(f_1) \times P(f_2) \times ... \times P(f_K)$.

Recall that $\alpha^k_{n_k}$ gives the probability that exactly n_k components fail, given that they belong to type k. For example, if there is only 1 component of type 1 failures, it follows that $P(f_1 = 1) = \alpha^1_1$. Thus, the alpha parameters provide all the information required to specify the distribution. For instance, $P(1, 1, ..., 1) = \alpha^1_1\alpha^2_1...\alpha^K_1$.

The next step is to calculate the survival signature, which will then be combined with the probability of failure considering CCF $P(f_1, f_2, ..., f_K)$ to assess the survival function of the system. It is not necessary to calculate all the survival signatures as it can identify which values are required by Equation 6.4.

6.2.3 General α-factor Model for System Reliability

The standard α-factor model for system reliability which is presented above may leads to an unsatisfactory low probability level of the system reliability, this is due to the assumption that the first failure event will affect components of all types. Therefore, it is essential to find a general α-factor model by which people can avoid assuming that the failure event has necessarily affect all common cause groups of components.

Let us assume there are K common cause component groups, where group $k \in \{1, 2, ..., K\}$ includes m_k components. The past data $n_{j_1,j_2,...,j_K}$ denote the numbers of past events with exactly j_1 failed components from group 1, exactly j_2 failure components from group 2 and the like.

The α-factor parameter $\alpha_{j_1,j_2,...,j_K}$ provides the probability that exactly j_k components of group k fail, with $k \in \{1, 2, ..., K\}$. Given that a common cause failure event has affected the overall system, however, which exact common cause groups are affected are not known. Therefore, the $\alpha_{j_1,j_2,...,j_K}$ can be estimated by Equation 6.5 as

$$\alpha_{j_1,...,j_K} = \frac{n_{j_1,...,j_K}}{(\sum_{j_1=0}^{m_1} ... \sum_{j_K=0}^{m_K} n_{j_1,...,j_K}) - n_{0,...,0}} \tag{6.5}$$

The $\sum_{j_1=0}^{m_1} ... \sum_{j_K=0}^{m_K} n_{j_1,...,j_K}$ in the denominator represents all the possible outcomes, but $n_{0,...,0}$ has to be subtracted as there is an assumption that at least one component in the system is affected by the failure event.

However, the assumption is only valid if $n_{0,...,0}$ is included in the summation. In practice, there may be no data for the number of times no components have failed. Therefore, it is necessary just arbitrarily to set $n_{0,...,0} = 0$. Since the purpose of this paper is to perform reliability analysis on complex systems in the presence of CCFs, the model without the above information is not relevant for the purpose.

The probability that the system works after a common cause failure event can also be expressed by Equation 6.4. The survival signature $\Phi(l_1, ..., l_K)$ is the same as before as it depends only on the system structures and can also be identified through Equation 6.4. However, the joint probability distribution $P(f_1, f_2, ..., f_K)$ is obtained without assuming independence among components, as it is simply given by the alpha factor parameters. To be specific, that means $P(f_1, f_2, ..., f_K) = \alpha_{j_1, j_2, ..., j_K}$.

6.2.4 Time Dependent System Reliability after CCFs

Given numbers n_k with $k \in \{1, 2, ..., K\}$ of components of each type that fail simultaneously after a common cause event, those of one type are all equally likely to be failing. Assume $C_k(t)$ expresses the number of k components function at time t after a common cause failure. Therefore,

$$P(\bigcap_{k=1}^{K} \{C_k(t) = l_k - n_k\} \mid CCF) = \prod_{k=1}^{K} \binom{m_k}{l_k - n_k} [F_k(t)]^{m_k - l_k + n_k} [1 - F_k(t)]^{l_k - n_k} \quad (6.6)$$

So the functioning of the system after the next common cause failure can be predicted by Equation 6.7.

$$P(T_s > t \mid CCF) = \sum_{l_1=0}^{m_1-n_1} ... \sum_{l_K=0}^{m_K-n_K} \Phi(l_1, ..., l_K) P(\bigcap_{k=1}^{K} \{C_k(t) = l_k - n_k\} \mid CCF) \quad (6.7)$$

It can be seen from the above Equation that the survival function of the system is dependent on time t.

6.2.5 Imprecise System Reliability after CCFs

In the application engineering world, if the system has not been in operation in the past (e.g. is a new system), or it does not usually encounter common cause failure events, there may not be enough data to estimate the values of the accurate α-factor parameters. In addition, experts can provide an estimation of the number n_k of common cause components failures, but often only an interval is predicted.

Therefore, the α-factor parameters might have imprecise values with bounds $[\underline{\alpha}^k_{m_k}, \overline{\alpha}^k_{m_k}]$. Since $P(f_1, f_2, ..., f_K)$ is estimated based on the α-factor model, the imprecision will propagate to $P(m_1 - l_1, ..., m_K - l_K)$. The survival signature remains unaffected by the imprecision since it is only influenced by uncertainty and imprecision in the system structure. Hence, the bounds of the survival probability are calculated as

$$\underline{P}(S_{CCF}) = \sum_{l_1=0}^{m_1} ... \sum_{l_K=0}^{m_K} \Phi(l_1, ..., l_K)\underline{P}(m_1 - l_1, ..., m_K - l_K) \tag{6.8}$$

$$\overline{P}(S_{CCF}) = \sum_{l_1=0}^{m_1} ... \sum_{l_K=0}^{m_K} \Phi(l_1, ..., l_K)\overline{P}(m_1 - l_1, ..., m_K - l_K) \tag{6.9}$$

For time varying system reliability after CCFs, if there is a lack of information on components failure time distribution, the cumulative distribution function $F_k(t)$ will have uncertainty. Similarly, the survival signature remains the same. Therefore, the lower survival function of the system after CCFs is

$$\underline{P}(T_S > t \mid CCF) = \sum_{l_1=0}^{m_1} ... \sum_{l_K=0}^{m_K} \Phi(l_1, ..., l_K) \times$$
$$\prod_{k=1}^{K} [\overline{P}(C_k(t) \leq l_k \mid CCF) - \overline{P}(C_k(t) \leq l_k - 1 \mid CCF)] \tag{6.10}$$

and the upper survival function is

$$\overline{P}(T_S > t \mid CCF) = \sum_{l_1=0}^{m_1} ... \sum_{l_K=0}^{m_K} \Phi(l_1, ..., l_K) \times$$
$$\prod_{k=1}^{K} [\underline{P}(C_k(t) \leq l_k \mid CCF) - \underline{P}(C_k(t) \leq l_k - 1 \mid CCF)] \tag{6.11}$$

6.3 Numerical Example

In this Section, five cases are analysed. To be specific, the first case shows the application of the proposed standard α-factor model for system reliability after common cause failures. Then, the following two cases talk about the general α-factor model. Case four illustrates the time varying system reliability after CCFs, while the last case considers imprecision within the system with common cause failures.

Figure 6.1 shows a complex system with thirteen components which belong to four $k = 4$ types ($m_1 = 3, m_2 = 4, m_3 = 2, m_4 = 4$).

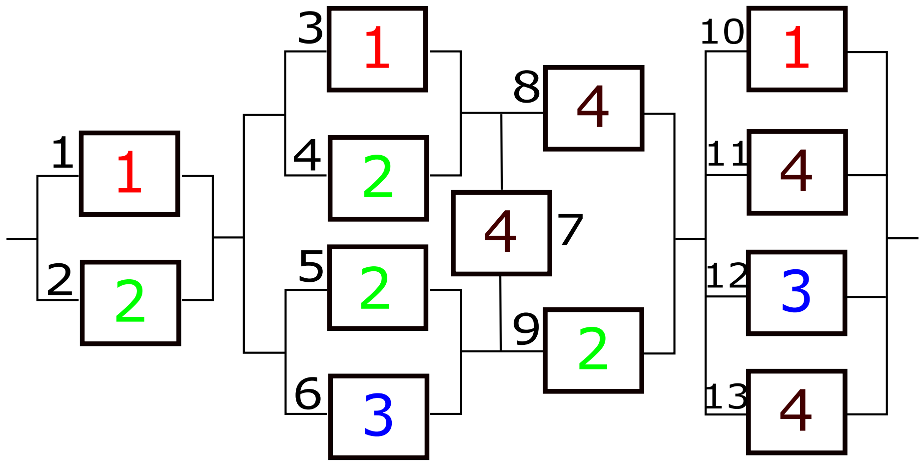

Figure 6.1: Complex system with thirteen components which belong to four types. The number inside the component box represents the type, while the number outside the box expresses the component index.

6.3.1 Case 1 (Standard α-factor Model)

Firstly, let us determine the values of the survival function by using the standard α-factor model from hypothetical data collected from the system. Suppose that the common cause failure groups are the same as component types. This is logical in the engineering world as the components of the same type have similar characteristics, so they are more likely to be influenced by the same common cause event.

we have $n_1 = 1$, $n_2 = 2$, $n_3 = 1$ for components of type 1. $n_1 = 1$ means that there has been 1 previous occurrence of failure with just one component of type 1, so the α-factor parameter estimators in this case are

$$\alpha_1^1 = \frac{n_1}{n_1 + n_2 + n_3} = \frac{1}{4} \tag{6.12}$$

$\alpha_2^1 = \frac{1}{2}$, $\alpha_3^1 = \frac{1}{4}$ and $\sum_{k=1}^{3} \alpha_k^1 = 1$.

For type 2, $n_1 = 2$, $n_2 = 1$, $n_3 = 1$ and $n_4 = 2$, which gives $\alpha_1^2 = \frac{1}{3}$, $\alpha_2^2 = \frac{1}{6}$, $\alpha_3^2 = \frac{1}{6}$ and $\alpha_4^2 = \frac{1}{3}$ respectively. There are $n_1 = 2$ and $n_2 = 1$ for type 3. Thus, $\alpha_1^3 = \frac{2}{3}$ and $\alpha_2^3 = \frac{1}{3}$.

Similarly for component type 4, the data of $n_1 = 3$, $n_2 = 3$, $n_3 = 1$ and $n_4 = 1$ lead to $\alpha_1^4 = \frac{3}{8}$, $\alpha_2^4 = \frac{3}{8}$, $\alpha_3^4 = \frac{1}{8}$ and $\alpha_4^4 = \frac{1}{8}$.

We assume that a common cause failure event occur will affect at least one component of each type. According to the α-factor model, therefore, it can be known for this system that $P(0,0,0,0) = P(0,b,c,d) = P(a,0,c,d) = P(a,b,0,d) = P(a,b,c,0) = 0$, for $a = 1,2,3$, $b = 1,2,3,4$, $c = 1,2$ and $d = 1,2,3,4$.

Based on the assumption that the failure components of different types are independent, we obtain that $P(f_1, f_2, f_3, f_4) = P(f_1)P(f_2)P(f_3)P(f_4)$.

Recall that α_1^1 represents the probability that exactly one component of the three components of type 1 fail. Hence, it follows that $P(f_1) = \alpha_1^1$. So the alpha parameters provide all the information that is required to specify the distribution. $P(1,1,1,1) = \alpha_1^1\alpha_1^2\alpha_1^3\alpha_1^4 = \frac{1}{4} \times \frac{1}{3} \times \frac{2}{3} \times \frac{3}{8} = \frac{1}{48}$.

Then, it is necessary to calculate the survival signature Φ, which is used to combine with the $P(f_1, f_2, f_3, f_4)$ to assess the survival probability of the system. For $P(1,1,1,1)$, its corresponding survival signature is $\Phi(2,3,2,3)$, which means that probability that the system works given that exactly 2 components of type one, 3 components of type two, 2 components of type three and 3 components of type four are working. There are altogether 48 possible state vectors, of which 41 combinations allow the system to function. Therefore, $\Phi(2,3,2,3) = \frac{41}{48}$.

All the values of $P(f_1, f_2, f_3, f_4)$ and their corresponding survival signature $\Phi(m_1 - f_1, m_2 - f_2, m_3 - f_3, m_4 - f_4)$ can be calculated. Based on the values of P and Φ and the Equation 6.4, the probability that the system survives after a common cause failure event $P(S_{CCF})$ is $\frac{50}{139}$.

This probability is unsatisfactorily low, which is probably due to the assumptions of the standard α-factor model. In order to relax these assumptions, it is necessary to generalise the α-factor model.

6.3.2 Case 2 (General α-factor Model One)

Let us continue to use the complex system in Figure 6.1. However, we do not assume that the common cause event will affect all the common cause failure groups. Again, the component types form the common cause failure groups as before. Therefore, it can be any number combinations of the components from type 1 to type 4. Note again that $P(0,0,0,0) = 0$ as at least one component will be affected by the CCFs.

The past data on the system can be seen in Table 7.4 in the Appendix, which has more data available to fit the requirements for implementing the general α-factor model.

It can be seen that $\sum_{j_1=0}^{3}\sum_{j_2=0}^{4}\sum_{j_3=0}^{2}\sum_{j_4=0}^{4} n_{j_1j_2j_3j_4} = 345$, and the α-factor parameters can be easily obtained by Equation 6.5. For instance, $\alpha_{2314} = \frac{1}{345}$. Since there is no assumption that independence among the components, the joint probability distribution $P(f_1, f_2, f_3, f_4)$ is simply given by the α-factor parameters, which means

$P(2,3,1,4) = \alpha_{2314} = \frac{1}{345}$.

The survival signature of the complex system in Figure 6.1 is the same as calculated before as it only depends on the structure of the system. Combining the joint probability distribution values and their corresponding survival signature, the survival probability of the system after the CCfs $P(T_s > t \mid CCF)$, which can be calculated by Equation 6.4, is $\frac{461}{934}$. This value is still low, though it is an improvement compared with the initial failure event in Case 1.

6.3.3 Case 3 (General α-factor Model Two)

It can be seen from Case 2 that this system is highly susceptible to common cause failures, as all combinations of the failed components have occurred in the past data. There is a more robust system to be analysed in this Case, with past data that can be seen in Table 6.1.

Table 6.1: Past data $n_{j_1j_2j_3j_4}$ on the system in Figure 6.1

$n_{j_1j_2j_3j_4}$	$n_{j_1j_2j_3j_4}$	$n_{j_1j_2j_3j_4}$	$n_{j_1j_2j_3j_4}$	$n_{j_1j_2j_3j_4}$	$n_{j_1j_2j_3j_4}$
$n_{0001} = 16$	$n_{0002} = 11$	$n_{0003} = 13$	$n_{0004} = 6$	$n_{0010} = 13$	$n_{0011} = 3$
$n_{0012} = 2$	$n_{0013} = 3$	$n_{0014} = 3$	$n_{0020} = 13$	$n_{0021} = 5$	$n_{0022} = 2$
$n_{0023} = 4$	$n_{0100} = 13$	$n_{0101} = 3$	$n_{0102} = 3$	$n_{0103} = 2$	$n_{0104} = 2$
$n_{0110} = 2$	$n_{0111} = 1$	$n_{0120} = 3$	$n_{0200} = 13$	$n_{0201} = 4$	$n_{0202} = 2$
$n_{0203} = 1$	$n_{0204} = 13$	$n_{0210} = 1$	$n_{0220} = 3$	$n_{0300} = 14$	$n_{0301} = 5$
$n_{0302} = 2$	$n_{0303} = 13$	$n_{0304} = 1$	$n_{0310} = 3$	$n_{0320} = 2$	$n_{0400} = 7$
$n_{0401} = 3$	$n_{0402} = 3$	$n_{0403} = 2$	$n_{0404} = 1$	$n_{0410} = 1$	$n_{0420} = 3$
$n_{1000} = 17$	$n_{1001} = 6$	$n_{1002} = 3$	$n_{1003} = 4$	$n_{1004} = 4$	$n_{1010} = 5$
$n_{1011} = 1$	$n_{1020} = 3$	$n_{1100} = 6$	$n_{1101} = 2$	$n_{1102} = 1$	$n_{1110} = 2$
$n_{1111} = 1$	$n_{1200} = 3$	$n_{1201} = 1$	$n_{1300} = 6$	$n_{1301} = 1$	$n_{1400} = 4$
$n_{1401} = 1$	$n_{1402} = 1$	$n_{2000} = 13$	$n_{2001} = 5$	$n_{2002} = 3$	$n_{2003} = 2$
$n_{2004} = 2$	$n_{2010} = 4$	$n_{2011} = 1$	$n_{2020} = 4$	$n_{2100} = 6$	$n_{2101} = 1$
$n_{2110} = 1$	$n_{2200} = 3$	$n_{2300} = 3$	$n_{2301} = 1$	$n_{3000} = 9$	$n_{3001} = 5$
$n_{3002} = 3$	$n_{3010} = 4$	$n_{3020} = 2$	$n_{3100} = 3$	$n_{3300} = 1$	$n_{3301} = 1$

It can be seen from the above table that there is one past event (n_{1111}) such that all component types are affected, and few past data where three types have been influenced ($n_{0111}, n_{1011}, n_{1101}, n_{1102}, n_{1110}, n_{1201}, n_{1401}, n_{1402}, n_{2011}, n_{2101}, n_{2110}, n_{2301}, n_{3301}$ in this case). In other words, this system has the feature that components belonging to one type are more reliable when CCFs affect the other types. Therefore, it is unlikely to have large numbers of components of more than two types fail simultaneously.

Similarly, $\sum_{j_1=0}^{3}\sum_{j_2=0}^{4}\sum_{j_3=0}^{2}\sum_{j_4=0}^{4} n_{j_1j_2j_3j_4} = 345$ and the joint probability distribution $P(f_1, f_2, f_3, f_4)$ can be found directly from α-factor parameters, with $P(f_1, f_2, f_3, f_4) =$

$\alpha_{n_1n_2n_3n_4}$. The survival signature of the system remains the same as before since its structure does not change at all.

Based on Equation 6.4, it can be found that $P(T_s > t \mid CCF) = \frac{439}{473}$, which means the system survival probability after a common cause failure is around 92.81%. This is highly improved compared with the former two cases.

The structure of the system remains the same, and the total number of the past data is identical to Case 2, but why does the vulnerability of the system to CCFs decline so drastically? It can be investigated that the higher values of $P(f_1, f_2, f_3, f_4)$, $\frac{17}{345}$, $\frac{16}{345}$, $\frac{14}{345}$, $\frac{13}{345}$ and $\frac{11}{345}$, correspond to survival signature $\Phi(l_1, l_2, l_3, l_4) = 1$. Therefore, these kinds of CCFs that are triggered will not be likely to cause system failures. While in Case 2, there are more instances in the past data where larger numbers of components from more than two types had failed, making it less likely that the system could continue to work.

6.3.4 Case 4 (Time Dependent System Reliability after CCFs)

It is assumed that all components of the same type have the same failure time distribution. Their failure types and distribution parameters are listed in Table 6.2.

Table 6.2: Failure types and distribution parameters of components of the system in Figure 6.1

Component Type	Distribution type	Parameters (α, β) or λ
1	Weibull	(1.8,2.2)
2	Exponential	1.2
3	Normal	(2.3,1.6)
4	Lognormal	(3.2,2.6)

If a common cause event occurs, the influenced components of each type will fail simultaneously with the given probability, and those from the same component type all equally likely to be failing. The survival signature remains the same as before, but the number of the working components decreases.

Let $C(0, 0, 1, 1)$ denote the common cause failure group with one component from type 3 and one from type 4, without any components failing for other two types. In order to show the results clearly, here are the other five conditions with common cause failure groups $C(1, 1, 1, 1)$, $C(0, 2, 0, 2)$, $C(2, 2, 1, 2)$, $C(2, 2, 1, 3)$ and $C(2, 3, 1, 3)$. The time dependent survival function of the system after the these conditions' common cause failures can be seen in Figure 6.2.

After the common cause failures, the survival function of the six conditions are lower than without the CCFs. It can also be seen that the more numbers and types of components are influenced by CCF, the lower of system reliability is as expected. This also agrees with

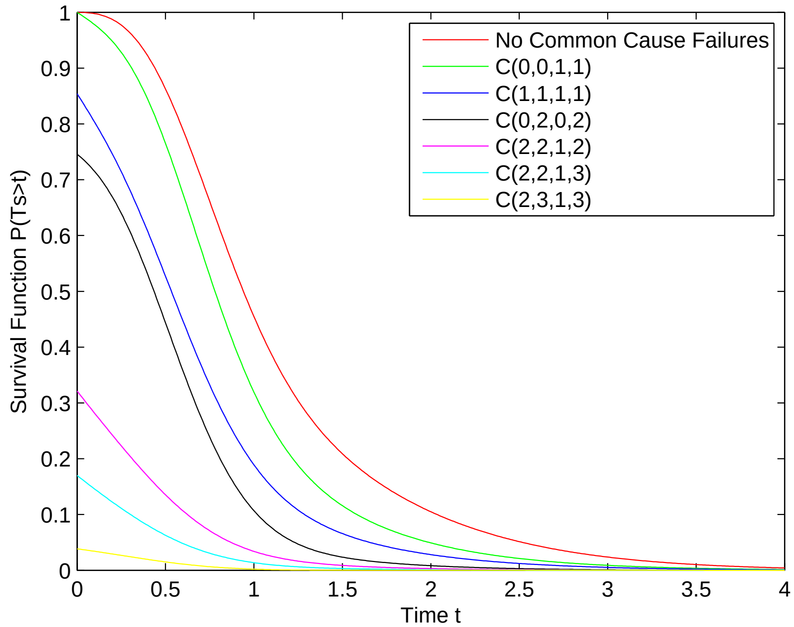

Figure 6.2: Survival functions of the system after some conditions' common cause failures.

our common sense.

6.3.5 Case 5 (Imprecise System Reliability after CCFs)

In this subsection, let us consider uncertainty in the system reliability analysis after common cause failures. In Case 1, if the system has not suffered CCF in the past, there might not be enough data to calculate the α-factor parameters, although the standard α-factor model can still be implemented by using experts' judgements.

Given that a total of 20 common cause component failures has occurred across the complex system in Figure 6.1, let two groups of experts estimate how the data would be spread. Suppose group one gives that $\alpha_1^1 = \frac{2}{5}$, $\alpha_2^1 = \frac{1}{5}$ and $\alpha_3^1 = \frac{2}{5}$ for components type 1, while for components type 2, $\alpha_1^2 = 0$, $\alpha_2^2 = \frac{2}{5}$, $\alpha_3^2 = \frac{2}{5}$ and $\alpha_4^2 = \frac{1}{5}$. $\alpha_1^3 = \frac{2}{3}$ and $\alpha_2^3 = \frac{1}{3}$ for components type 3, and for components type 4, $\alpha_1^4 = \frac{3}{7}$, $\alpha_2^4 = \frac{1}{7}$, $\alpha_3^4 = \frac{2}{7}$ and $\alpha_4^4 = \frac{1}{7}$. At this time, even the survival signature of $\Phi(2,3,2,3)$ remains the same as $\frac{41}{48}$, which is a big value within the survival signature. However, its corresponding $P(1,1,1,1)$ decreases from $\frac{1}{48}$ to 0. Let summarise the products of $P(f_1, f_2, f_3, f_4)$ and their corresponding $\Phi(m_1 - f_1, m_2 - f_2, m_3 - f_3, m_4 - f_4)$, then the survival probability $P(T_s > t \mid CCF)$ of the complex system after the next CCFs is $\frac{6}{23}$.

For group two, the estimated data are $\alpha_1^1 = \frac{1}{2}$, $\alpha_2^1 = \frac{1}{3}$, $\alpha_3^1 = \frac{1}{6}$, $\alpha_1^2 = \frac{1}{4}$, $\alpha_2^2 = \frac{1}{4}$,

$\alpha_3^2 = \frac{1}{4}$, $\alpha_4^2 = \frac{1}{4}$, $\alpha_1^3 = \frac{1}{2}$, $\alpha_2^3 = \frac{1}{2}$, $\alpha_1^4 = \frac{3}{8}$, $\alpha_2^4 = \frac{3}{8}$, $\alpha_3^4 = \frac{1}{4}$ and $\alpha_4^4 = 0$. For this circumstance, $P(1,1,1,1)$ increases to $\alpha_1^1\alpha_1^2\alpha_1^3\alpha_1^4 = \frac{1}{2} \times \frac{1}{4} \times \frac{1}{2} \times \frac{3}{8} = \frac{3}{128}$. Therefore, $P(T_s > t \mid CCF) = \frac{179}{456}$ according to Equation 6.4.

So due to the epistemic uncertainty in this example, the probability bounds that the system works after the next common cause failure event are $\underline{\overline{P}}(T_s > t \mid CCF) = [\frac{6}{23}, \frac{3}{128}]$.

As for time dependent system reliability after CCFs in this case, it is difficult to know the precise distribution parameter of components type 2 due to lack of information. Therefore, the parameter λ has imprecise values [1.0,1.3] instead of the precise value of 1.2. The imprecision within the components failure time distribution can propagate to the complex system. So now the imprecise time varying survival functions of the system after the six conditions' common cause failures can be seen in Figures 6.3 and 6.4.

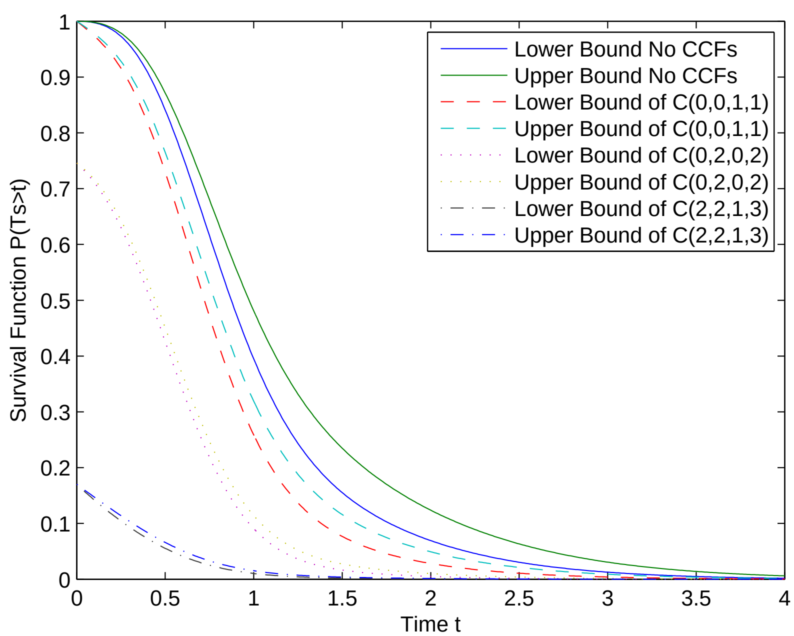

Figure 6.3: Lower and upper survival function bounds of the system after the common cause failures of C(0,0,1,1), C(0,2,0,2) and C(2,2,1,3) respectively.

In the real applications, for instance, due to confidential contracts, it is sometimes difficult to know the exact configuration of the system, which leads to imprecise survival signature. For this kind of "grey" system, the system reliability after common cause failures can also be modelled.

It can be seen that epistemic uncertainty will lead the uncertainty within the system in this Case. In order to reduce the imprecision, engineers need to improve efforts to get the precise α-factor parameters or components failure time distribution parameters.

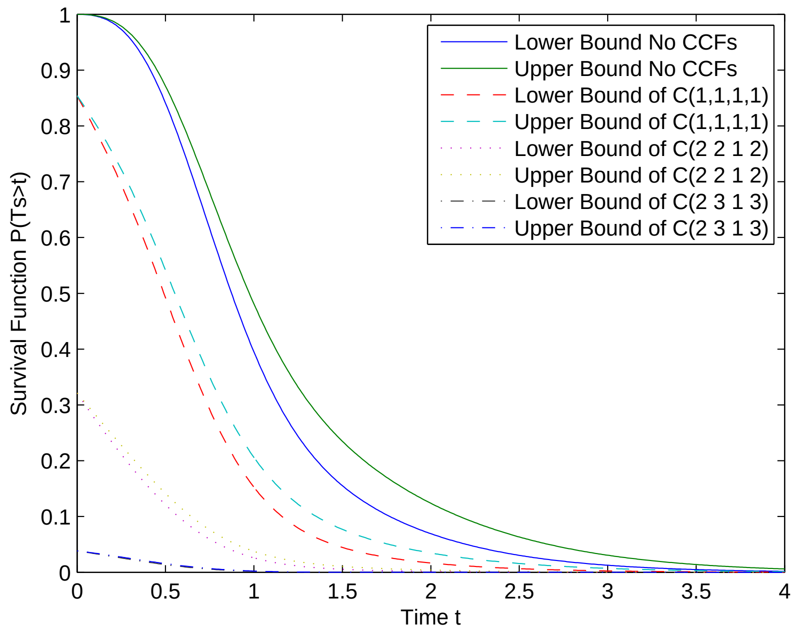

Figure 6.4: Lower and upper survival function bounds of the system after the common cause failures of C(1,1,1,1), C(2,2,1,2) and C(2,3,1,3) respectively.

6.4 Conclusion

Common cause failure events have the capability to reduce significantly the reliability and availability of systems. Therefore, it is essential to analyse and model the effects of CCF. This Chapter includes the effects of CCF into survival signature to compute the reliability analysis of complex systems.

The survival signature is a summary of system structure function, which makes it efficient to analyse complex systems, while CCFs are modelled using the standard and general α-factor models. The proposed methods are combined with the survival signature in order to perform complex system reliability analysis in the presence of common cause failure events. The effect of epistemic uncertainty, for example resulting from insufficient data, has been taken into account to perform reliability analysis on complex systems with different types after CCFs. As a result, lower and upper bounds of the system survival probability after CCFs can be obtained. The feasibility and effectiveness of the proposed measures are demonstrated by the numerical cases.

The proposed approach allows a more realistic situation of a system and can be adopted to decide whether to repair or replace the failed components immediately, or after the next common cause failure event. In other words, the administrator or designer has to consider the costs of repairing or replacing the components as soon as possible, or taking the risk of allowing the next common cause failure event to occur before performing repair or replacement. Overall, it is an example of how decision theory is incorporated in the practical engineering world.

The goal of the reliability policy is to achieve high initial reliability by focusing on reliability fundamentals during design. So design for reliability provides engineers and managers with a range of tools and techniques for incorporating reliability into the design process for complex systems [101]. For a complex system, it can examine the survival signature first, in order to find out which component combinations of failures will not allow the system to function at all. Therefore, the engineers can ensure these components are equipped to be more reliable under certain circumstances, which makes the system more reliable.

A possible drawback of the work presented in this Chapter is that there may not be enough past data on the system, which is essential to calculate the precise values of the α-factor parameters. This is either if the system does not usually experience the common cause failures, or if the system has not been run in the past. However, these α-factor models can still be implemented by using experts' judgements on the system to ascertain the $n_{j_1,j_2,\dots,j_K}$ values.

Chapter 7

Conclusion Remarks

7.1 Conclusions

This thesis investigates efficient methods for reliability and sensitivity analysis on complex systems and networks, which are the backbones of our society. In addition, the effects of imprecision and uncertainty are considered in the system reliability and component importance measures.

The simulation methods for complex system reliability analysis are based on the survival signature as proposed in Chapter 3. They are sufficient for the computation of common reliability metrics and have the crucial advantage that they can be applied to systems and networks with components whose failure times are not according to iid assumption. In addition, when it comes to large systems and networks, especially with uncertainty, it is difficult to derive the reliability metrics of interest in a purely analytical manner. However, the methodologies in this Chapter are generally applicable to any system configuration, and allow the consideration of imprecision within the system components. Therefore, the upper and lower bounds of the survival function of the system can be obtained.

In some cases, there are repairable systems which can be put into operation again after failure. Thus, the reliability approach is extended in Chapter 4 to simulate the evolution of complex repairable systems. This method is efficient, as it combines the survival signature, which needs to be calculated only once for the same system, with Monte Carlo simulation, which generates the components' transition time.

In order to identify the most important component, which contributes the largest to the system reliability, Chapter 5 presents a novel component-wise importance measure which is called relative importance index. The importance degree of the individual component at each time can be ranked through this measure, which allows an optimal allocation of resources for inspection, maintenance and repair. Furthermore, the imprecision is considered in this Chapter. As a consequence, intervals of relative importance index can be conducted in an analytical way or by a simulation approach, which is based on the survival signature. Based on the definition of relative importance index, this thesis implements a wide selection of component importance measures of specific components or components set for repairable systems and networks. As a further step, the probability bounds analysis, which is based on pinching theory, is used to identify which components or components set is most sensitive to the system.

Finally, common cause failures within the complex systems are analysed in Chapter 6. The approaches presented in this Chapter are based on the α-factor model and survival signature, which enables the reliability analysis on networks and systems with common cause failures. These different models can be used to evaluate the time-independent and time-varying system reliability, respectively. Also, the incertitude is taken into consideration here, which leads to the bounds of system reliability probability after common cause failures.

7.2 Discussion and Future Work

The feasibility and effectiveness of the presented approaches have been illustrated with numerical examples in each Chapter, and the results show that these methods are efficient to perform reliability and sensitivity analysis on complex systems and networks with imprecise probability. However, there are still several possible areas for further exploration and extension. Here are some interesting areas for possible future developments and research.

This thesis mainly focuses on classical reliability theory, which assumes that a component or a system can only be in binary states, either function or failure. However, there are multi-state systems (MSS) in real applications, and MSS reliability models allow both the systems and their components to assume more than two levels of performance [102]. Although many authors have made contributions about MSS reliability modelling and evaluation theory, no one has applied the novel survival signature theory to complex multi-state systems reliability evaluation. As this thesis stated before, the survival signature is a summary of the structure function and is efficient to analyse a complex system that has multiple components with exchangeable failure times, all of which make it possible to evaluate MSS through the survival signature. Therefore, how to calculate and use the survival signature to perform reliability analysis on such kind of systems is an interesting topic to consider. On the other hand, there are some efficient methods in the MSS area, i.e. a hybrid load flow and even driven simulation approach in [103]. It is promising to combine the survival signature with this method to perform reliability analysis on binary complex systems.

The α-factor model, which is utilised in conjunction with the survival signature to analyse complex system reliability under common cause failures is applicable. However, this model relies heavily on the past data or experts' judgements, which can be recognised as a disadvantage. Therefore, it is also possible to combine the survival signature with other CCF models if the information available on the systems and networks would better suit such models. In addition, if we were to distinguish components at risk from CCF according to e.g. location, we would need to model it more precisely, with a more detailed version of the survival signature. This is an important topic for later research.

There are special systems, such as a k-out-of-n redundancy system, in the engineering world. Calculating the reliability and importance measures for this kind of system by using the survival signature is interesting. Also, the study mainly focuses on the basic failure time models, so incorporating the time-dynamic development and survival signature into the stochastic failure models is challenging in the future work.

Resilience (ability to bounce back to a desired performance state) is a relatively new definition in network system engineering. In this view, systems should not only be reliable, i.e. having an acceptably low failure probability, but also resilient, i.e. having the ability to recover optimally from disruptions of the nominal operating conditions [104]. Therefore, it is another research challenge to consider resilience when conduct reliability analysis of complex systems with or without common cause failures, and evaluate the component importance measures as well.

When the component in the repairable system fails or is worn out, the maintenance actions like repairs and replacements need to be introduced. Given a certain cost and reward structure, an optimal repair and replacement strategy needs to be derived. Thus, it is essential to propose a general maintenance optimization approach which can be exploited in complex models.

For complex systems and networks in the real application world, in order to get information quickly on components' life distribution, accelerated life testing (ALT) is used. In practice, it means that the components are running under severe conditions (i.e. higher than usual temperature, voltage, pressure, vibration, cycling rate, load, etc.) and fail sooner than under usual conditions [105]. How to model the results from the ALT to arrive at an estimation for the lifetime during normal operating conditions is an interesting topic. Due to some conditions, however, only limited real world data can be obtained. Thus, future research may focus on using the censored data from testing or from the real world to estimate and verify the complex practical world system reliability.

Appendix

Appendix 1: Survival Signature of System in Figure 3.17

The table shows the survival signature of the complex system of Figure 3.17. The rows with survival signature values equal to either 1 or 0 have been omitted.

Table 7.1: Survival signature of a complex network in Figure 3.17; rows with $\Phi(l_1, l_2, l_3) = 0$ and $\Phi(l_1, l_2, l_3) = 1$ are omitted

l_1	l_2	l_3	$\Phi(l_1, l_2, l_3)$	l_1	l_2	l_3	$\Phi(l_1, l_2, l_3)$
2	0	2	1/400	2	0	3	7/946
2	0	4	7/473	2	0	5	2/81
2	0	6	36/973	2	0	7	15/289
2	0	8	53/767	2	0	9	4/45
2	0	10	1/9	2	1	2	3/857
2	1	3	6/577	2	1	4	13/628
2	1	5	10/289	2	1	6	15/289
2	1	7	31/427	2	1	8	3/31
2	1	9	26/209	2	1	10	75/482
2	2	2	1/204	2	2	3	2/137
2	2	4	27/931	2	2	5	6/125

Continued on next page

Table 7.1 – *Continued from previous page*

l_1	l_2	l_3	$\Phi(l_1, l_2, l_3)$	l_1	l_2	l_3	$\Phi(l_1, l_2, l_3)$
2	2	6	71/993	2	2	7	20/201
2	2	8	63/478	2	2	9	29/172
2	2	10	104/495	2	3	2	1/143
2	3	3	11/534	2	3	4	13/321
2	3	5	50/753	2	3	6	17/174
2	3	7	74/551	2	3	8	22/125
2	3	9	146/655	2	3	10	218/791
2	4	2	1/99	2	4	3	23/785
2	4	4	11/194	2	4	5	85/931
2	4	6	111/839	2	4	7	39/218
2	4	8	104/451	2	4	9	73/254
2	4	10	245/701	2	5	2	1/68
2	5	3	13/311	2	5	4	77/971
2	5	5	1/8	2	5	6	20/113
2	5	7	184/787	2	5	8	147/500
2	5	9	277/777	2	5	10	262/625
2	6	2	17/787	2	6	3	36/601
2	6	4	20/181	2	6	5	75/443
2	6	6	127/546	2	6	7	80/269
2	6	8	87/241	2	6	9	290/689
2	6	10	262/551	2	7	2	13/405
2	7	3	43/500	2	7	4	79/516
2	7	5	137/607	2	7	6	176/591
2	7	7	31/85	2	7	8	152/359

Continued on next page

Table 7.1 – *Continued from previous page*

l_1	l_2	l_3	$\Phi(l_1, l_2, l_3)$	l_1	l_2	l_3	$\Phi(l_1, l_2, l_3)$
2	7	9	89/188	2	7	10	322/625
2	8	2	3/62	2	8	3	116/937
2	8	4	106/505	2	8	5	5/17
2	8	6	249/677	2	8	7	189/442
2	8	8	214/451	2	8	9	209/409
2	8	10	486/901	2	9	2	37/500
2	9	3	161/905	2	9	4	113/400
2	9	5	145/392	2	9	6	403/929
2	9	7	463/967	2	9	8	375/734
2	9	9	405/758	2	9	10	423/767
2	10	2	38/329	2	10	3	31/122
2	10	4	88/239	2	10	5	329/743
2	10	6	59/121	2	10	7	307/596
2	10	8	388/727	2	10	9	523/957
2	10	10	5/9	2	11	2	5/27
2	11	3	192/541	2	11	4	371/817
2	11	5	74/147	2	11	6	149/283
2	11	7	300/557	2	11	8	523/957
2	11	9	218/395	2	11	10	5/9
2	12	2	183/593	2	12	3	219/473
2	12	4	100/189	2	12	5	426/773
2	12	6	5/9	2	12	7	5/9
2	12	8	5/9	2	12	9	5/9
2	12	10	5/9	3	0	2	7/946

Continued on next page

Table 7.1 – *Continued from previous page*

l_1	l_2	l_3	$\Phi(l_1, l_2, l_3)$	l_1	l_2	l_3	$\Phi(l_1, l_2, l_3)$
3	0	3	22/991	3	0	4	44/991
3	0	5	2/27	3	0	6	1/9
3	0	7	75/482	3	0	8	185/892
3	0	9	4/15	3	0	10	1/3
3	1	2	1/99	3	1	3	1/33
3	1	4	12/199	3	1	5	1/10
3	1	6	84/563	3	1	7	83/400
3	1	8	120/437	3	1	9	7/20
3	1	10	432/997	3	2	2	1/72
3	2	3	14/339	3	2	4	63/773
3	2	5	33/247	3	2	6	123/625
3	2	7	109/404	3	2	8	201/572
3	2	9	211/479	3	2	10	504/941
3	3	2	16/829	3	3	3	10/177
3	3	4	8/73	3	3	5	91/515
3	3	6	201/787	3	3	7	227/662
3	3	8	223/511	3	3	9	153/287
3	3	10	335/532	3	4	2	23/855
3	4	3	43/557	3	4	4	60/409
3	4	5	107/463	3	4	6	14/43
3	4	7	17/40	3	4	8	356/679
3	4	9	107/173	3	4	10	47/67
3	5	2	19/504	3	5	3	41/389
3	5	4	39/200	3	5	5	149/500

Continued on next page

Table 7.1 – *Continued from previous page*

l_1	l_2	l_3	$\Phi(l_1, l_2, l_3)$	l_1	l_2	l_3	$\Phi(l_1, l_2, l_3)$
3	5	6	253/623	3	5	7	157/307
3	5	8	451/743	3	5	9	575/836
3	5	10	697/926	3	6	2	6/113
3	6	3	138/961	3	6	4	10/39
3	6	5	247/655	3	6	6	393/797
3	6	7	22/37	3	6	8	463/684
3	6	9	151/204	3	6	10	397/504
3	7	2	3/40	3	7	3	173/889
3	7	4	293/882	3	7	5	353/758
3	7	6	40/69	3	7	7	303/454
3	7	8	169/231	3	7	9	486/625
3	7	10	107/132	3	8	2	77/723
3	8	3	151/577	3	8	4	382/905
3	8	5	211/378	3	8	6	589/895
3	8	7	363/500	3	8	8	649/841
3	8	9	688/857	3	8	10	361/438
3	9	2	12/79	3	9	3	87/250
3	9	4	247/473	3	9	5	555/859
3	9	6	504/697	3	9	7	661/859
3	9	8	640/801	3	9	9	9/11
3	9	10	310/373	3	10	2	167/765
3	10	3	169/372	3	10	4	98/157
3	10	5	361/500	3	10	6	547/708
3	10	7	399/499	3	10	8	71/87

Continued on next page

Table 7.1 – *Continued from previous page*

l_1	l_2	l_3	$\Phi(l_1, l_2, l_3)$	l_1	l_2	l_3	$\Phi(l_1, l_2, l_3)$
3	10	9	262/317	3	10	10	5/6
3	11	2	218/689	3	11	3	501/871
3	11	4	170/237	3	11	5	756/967
3	11	6	450/557	3	11	7	399/487
3	11	8	637/771	3	11	9	152/183
3	11	10	5/6	3	12	2	219/473
3	12	3	434/625	3	12	4	504/635
3	12	5	644/779	3	12	6	5/6
3	12	7	5/6	3	12	8	5/6
3	12	9	5/6	3	12	10	5/6
4	0	2	7/473	4	0	3	44/991
4	0	4	73/823	4	0	5	71/483
4	0	6	135/617	4	0	7	161/531
4	0	8	290/727	4	0	9	368/729
4	0	10	606/979	4	1	2	2/101
4	1	3	45/764	4	1	4	12/103
4	1	5	113/591	4	1	6	247/879
4	1	7	381/994	4	1	8	158/319
4	1	9	249/406	4	1	10	11/15
4	2	2	19/717	4	2	3	39/500
4	2	4	19/125	4	2	5	166/677
4	2	6	160/453	4	2	7	249/529
4	2	8	74/125	4	2	9	697/981
4	2	10	638/779	4	3	2	11/309

Continued on next page

Table 7.1 – *Continued from previous page*

l_1	l_2	l_3	$\Phi(l_1, l_2, l_3)$	l_1	l_2	l_3	$\Phi(l_1, l_2, l_3)$
4	3	3	79/767	4	3	4	155/788
4	3	5	31/100	4	3	6	233/536
4	3	7	281/500	4	3	8	127/186
4	3	9	332/421	4	3	10	677/777
4	4	2	6/125	4	4	3	8/59
4	4	4	86/341	4	4	5	241/625
4	4	6	426/815	4	4	7	97/149
4	4	8	149/196	4	4	9	385/456
4	4	10	852/943	4	5	2	37/571
4	5	3	111/625	4	5	4	298/933
4	5	5	294/625	4	5	6	172/281
4	5	7	641/877	4	5	8	370/451
4	5	9	353/400	4	5	10	885/958
4	6	2	7/80	4	6	3	136/589
4	6	4	249/625	4	6	5	545/972
4	6	6	287/412	4	6	7	498/625
4	6	8	319/369	4	6	9	119/131
4	6	10	325/347	4	7	2	92/779
4	7	3	11/37	4	7	4	58/119
4	7	5	97/149	4	7	6	406/527
4	7	7	713/841	4	7	8	284/317
4	7	9	549/593	4	7	10	341/361
4	8	2	117/734	4	8	3	189/500
4	8	4	183/314	4	8	5	147/200

Continued on next page

Table 7.1 – *Continued from previous page*

l_1	l_2	l_3	$\Phi(l_1, l_2, l_3)$	l_1	l_2	l_3	$\Phi(l_1, l_2, l_3)$
4	8	6	49/59	4	8	7	116/131
4	8	8	67/73	4	8	9	194/207
4	8	10	543/572	4	9	2	151/702
4	9	3	303/641	4	9	4	188/277
4	9	5	290/359	4	9	6	577/658
4	9	7	802/879	4	9	8	373/400
4	9	9	729/772	4	9	10	59/62
4	10	2	85/293	4	10	3	415/717
4	10	4	355/462	4	10	5	495/571
4	10	6	41/45	4	10	7	904/971
4	10	8	781/829	4	10	9	221/233
4	10	10	20/21	4	11	2	175/447
4	11	3	151/219	4	11	4	735/869
4	11	5	799/876	4	11	6	824/881
4	11	7	435/461	4	11	8	274/289
4	11	9	773/813	4	11	10	20/21
4	12	2	100/189	4	12	3	504/635
4	12	4	868/957	4	12	5	445/471
4	12	6	20/21	4	12	7	20/21
4	12	8	20/21	4	12	9	20/21
4	12	10	20/21	5	0	2	2/81
5	0	3	2/27	5	0	4	71/483
5	0	5	185/767	5	0	6	317/897
5	0	7	463/967	5	0	8	153/250

Continued on next page

Table 7.1 – *Continued from previous page*

l_1	l_2	l_3	$\Phi(l_1, l_2, l_3)$	l_1	l_2	l_3	$\Phi(l_1, l_2, l_3)$
5	0	9	373/500	5	0	10	55/63
5	1	2	13/405	5	1	3	56/587
5	1	4	150/803	5	1	5	19/63
5	1	6	190/439	5	1	7	365/638
5	1	8	22/31	5	1	9	622/745
5	1	10	59/63	5	2	2	13/311
5	2	3	100/817	5	2	4	4/17
5	2	5	174/469	5	2	6	259/500
5	2	7	181/273	5	2	8	88/111
5	2	9	163/182	5	2	10	77/80
5	3	2	54/989	5	3	3	33/211
5	3	4	17/58	5	3	5	328/731
5	3	6	583/963	5	3	7	373/500
5	3	8	6/7	5	3	9	400/429
5	3	10	438/449	5	4	2	22/309
5	4	3	85/428	5	4	4	115/319
5	4	5	438/823	5	4	6	69/100
5	4	7	102/125	5	4	8	167/185
5	4	9	843/883	5	4	10	393/400
5	5	2	17/183	5	5	3	1/4
5	5	4	273/625	5	5	5	359/581
5	5	6	381/497	5	5	7	544/625
5	5	8	744/797	5	5	9	123/127
5	5	10	371/376	5	6	2	59/488

Continued on next page

Table 7.1 – *Continued from previous page*

l_1	l_2	l_3	$\Phi(l_1, l_2, l_3)$	l_1	l_2	l_3	$\Phi(l_1, l_2, l_3)$
5	6	3	49/157	5	6	4	13/25
5	6	5	389/555	5	6	6	310/373
5	6	7	569/625	5	6	8	291/305
5	6	9	300/307	5	6	10	363/367
5	7	2	46/293	5	7	3	258/671
5	7	4	489/806	5	7	5	608/783
5	7	6	823/933	5	7	7	245/261
5	7	8	541/559	5	7	9	400/407
5	7	10	843/851	5	8	2	131/645
5	8	3	7/15	5	8	4	574/829
5	8	5	640/761	5	8	6	429/466
5	8	7	599/625	5	8	8	85/87
5	8	9	950/963	5	8	10	350/353
5	9	2	131/500	5	9	3	59/106
5	9	4	329/426	5	9	5	360/403
5	9	6	593/625	5	9	7	243/250
5	9	8	117/119	5	9	9	980/991
5	9	10	245/247	5	10	2	100/297
5	10	3	80/123	5	10	4	337/400
5	10	5	169/181	5	10	6	590/609
5	10	7	105/107	5	10	8	862/873
5	10	9	619/625	5	10	10	879/886
5	11	2	305/707	5	11	3	407/548
5	11	4	226/251	5	11	5	132/137

Continued on next page

Table 7.1 – *Continued from previous page*

l_1	l_2	l_3	$\Phi(l_1,l_2,l_3)$	l_1	l_2	l_3	$\Phi(l_1,l_2,l_3)$
5	11	6	284/289	5	11	7	803/813
5	11	8	101/102	5	11	9	807/814
5	11	10	879/886	5	12	2	426/773
5	12	3	644/779	5	12	4	445/471
5	12	5	436/443	5	12	6	879/886
5	12	7	879/886	5	12	8	879/886
5	12	9	879/886	5	12	10	879/886
6	0	2	36/973	6	0	3	1/9
6	0	4	135/617	6	0	5	317/897
6	0	6	368/729	6	0	7	484/733
6	0	8	699/868	6	0	9	885/958
6	1	2	28/597	6	1	3	5/36
6	1	4	97/361	6	1	5	305/719
6	1	6	281/477	6	1	7	427/573
6	1	8	181/207	6	1	9	101/105
6	2	2	31/521	6	2	3	108/625
6	2	4	204/625	6	2	5	500/999
6	2	6	567/844	6	2	7	193/236
6	2	8	509/552	6	2	9	615/628
6	3	2	25/332	6	3	3	22/103
6	3	4	345/881	6	3	5	465/802
6	3	6	351/469	6	3	7	7/8
6	3	8	159/167	6	3	9	83/84
6	4	2	73/766	6	4	3	65/248

Continued on next page

Table 7.1 – *Continued from previous page*

l_1	l_2	l_3	$\Phi(l_1, l_2, l_3)$	l_1	l_2	l_3	$\Phi(l_1, l_2, l_3)$
6	4	4	19/41	6	4	5	307/466
6	4	6	547/671	6	4	7	177/193
6	4	8	459/473	6	4	9	138/139
6	5	2	36/299	6	5	3	173/543
6	5	4	503/932	6	5	5	729/994
6	5	6	20/23	6	5	7	299/316
6	5	8	955/973	6	5	9	885/889
6	6	2	109/719	6	6	3	18/47
6	6	4	386/625	6	6	5	4/5
6	6	6	103/113	6	6	7	366/379
6	6	8	426/431	6	6	9	344/345
6	7	2	66/347	6	7	3	5/11
6	7	4	77/111	6	7	5	191/223
6	7	6	163/173	6	7	7	453/463
6	7	8	136/137	6	7	9	525/526
6	8	2	39/164	6	8	3	143/269
6	8	4	435/569	6	8	5	551/611
6	8	6	241/250	6	8	7	299/303
6	8	8	885/889	6	8	9	832/833
6	9	2	37/125	6	9	3	348/569
6	9	4	631/763	6	9	5	59/63
6	9	6	373/381	6	9	7	902/909
6	9	8	767/769	6	10	2	274/747
6	10	3	132/191	6	10	4	175/199

Continued on next page

Table 7.1 – *Continued from previous page*

l_1	l_2	l_3	$\Phi(l_1, l_2, l_3)$	l_1	l_2	l_3	$\Phi(l_1, l_2, l_3)$
6	10	5	481/500	6	10	6	363/367
6	10	7	249/250	6	10	8	713/714
6	11	2	19/42	6	11	3	603/787
6	11	4	502/545	6	11	5	49/50
6	11	6	237/238	6	11	7	624/625
6	12	2	5/9	6	12	3	5/6
6	12	4	20/21	6	12	5	879/886
7	0	2	15/289	7	0	3	75/482
7	0	4	161/531	7	0	5	463/967
7	0	6	484/733	7	0	7	800/973
7	0	8	349/371	7	1	2	8/125
7	1	3	43/228	7	1	4	216/601
7	1	5	423/767	7	1	6	568/773
7	1	7	22/25	7	1	8	950/981
7	2	2	42/533	7	2	3	161/708
7	2	4	8/19	7	2	5	521/834
7	2	6	712/889	7	2	7	914/991
7	2	8	115/117	7	3	2	55/567
7	3	3	141/518	7	3	4	58/119
7	3	5	585/841	7	3	6	435/508
7	3	7	39/41	7	3	8	417/421
7	4	2	18/151	7	4	3	290/897
7	4	4	439/789	7	4	5	360/473
7	4	6	892/991	7	4	7	97/100

Continued on next page

Table 7.1 – *Continued from previous page*

l_1	l_2	l_3	$\Phi(l_1, l_2, l_3)$	l_1	l_2	l_3	$\Phi(l_1, l_2, l_3)$
7	4	8	921/926	7	5	2	25/171
7	5	3	361/949	7	5	4	587/938
7	5	5	104/127	7	5	6	265/284
7	5	7	971/989	7	5	8	643/645
7	6	2	140/783	7	6	3	287/648
7	6	4	418/603	7	6	5	347/400
7	6	6	464/485	7	6	7	363/367
7	6	8	555/556	7	7	2	109/500
7	7	3	345/677	7	7	4	96/127
7	7	5	795/877	7	7	6	832/855
7	7	7	621/625	7	7	8	998/999
7	8	2	40/151	7	8	3	125/216
7	8	4	798/983	7	8	5	59/63
7	8	6	123/125	7	8	7	808/811
7	9	2	110/343	7	9	3	234/361
7	9	4	300/349	7	9	5	301/314
7	9	6	334/337	7	9	7	499/500
7	10	2	157/406	7	10	3	201/281
7	10	4	257/286	7	10	5	417/428
7	10	6	463/465	7	10	7	998/999
7	11	2	386/831	7	11	3	7/9
7	11	4	92/99	7	11	5	595/604
7	11	6	624/625	7	12	2	5/9
7	12	3	5/6	7	12	4	20/21

Continued on next page

Table 7.1 – *Continued from previous page*

l_1	l_2	l_3	$\Phi(l_1, l_2, l_3)$	l_1	l_2	l_3	$\Phi(l_1, l_2, l_3)$
7	12	5	879/886	8	0	2	53/767
8	0	3	185/892	8	0	4	290/727
8	0	5	153/250	8	0	6	699/868
8	0	7	349/371	8	1	2	21/253
8	1	3	29/119	8	1	4	400/877
8	1	5	665/983	8	1	6	503/587
8	1	7	298/309	8	2	2	33/332
8	2	3	80/281	8	2	4	146/283
8	2	5	216/293	8	2	6	53/59
8	2	7	373/381	8	3	2	57/479
8	3	3	75/227	8	3	4	15/26
8	3	5	381/481	8	3	6	93/100
8	3	7	803/813	8	4	2	73/513
8	4	3	51/134	8	4	4	51/80
8	4	5	341/406	8	4	6	939/985
8	4	7	839/845	8	5	2	9/53
8	5	3	10/23	8	5	4	87/125
8	5	5	345/392	8	5	6	289/298
8	5	7	243/244	8	6	2	88/435
8	6	3	162/329	8	6	4	313/417
8	6	5	73/80	8	6	6	571/582
8	6	7	434/435	8	7	2	72/299
8	7	3	132/239	8	7	4	4/5
8	7	5	196/209	8	7	6	607/614

Continued on next page

Table 7.1 – *Continued from previous page*

l_1	l_2	l_3	$\Phi(l_1, l_2, l_3)$	l_1	l_2	l_3	$\Phi(l_1, l_2, l_3)$
8	7	7	768/769	8	8	2	219/766
8	8	3	415/677	8	8	4	409/485
8	8	5	111/116	8	8	6	917/923
8	9	2	62/183	8	9	3	558/829
8	9	4	22/25	8	9	5	133/137
8	9	6	854/857	8	10	2	133/332
8	10	3	19/26	8	10	4	91/100
8	10	5	813/829	8	10	6	624/625
8	11	2	368/779	8	11	3	754/961
8	11	4	438/469	8	11	5	79/80
8	12	2	5/9	8	12	3	5/6
8	12	4	20/21	8	12	5	879/886
9	0	2	4/45	9	0	3	4/15
9	0	4	368/729	9	0	5	373/500
9	0	6	885/958	9	1	2	14/135
9	1	3	239/788	9	1	4	480/863
9	1	5	707/892	9	1	6	373/394
9	2	2	59/488	9	2	3	307/894
9	2	4	584/961	9	2	5	757/908
9	2	6	816/847	9	3	2	88/625
9	3	3	157/406	9	3	4	285/433
9	3	5	405/466	9	3	6	793/813
9	4	2	41/250	9	4	3	391/903
9	4	4	289/409	9	4	5	346/385

Continued on next page

Table 7.1 – *Continued from previous page*

l_1	l_2	l_3	$\Phi(l_1, l_2, l_3)$	l_1	l_2	l_3	$\Phi(l_1, l_2, l_3)$
9	4	6	550/559	9	5	2	17/89
9	5	3	213/442	9	5	4	513/682
9	5	5	12/13	9	5	6	97/98
9	6	2	65/292	9	6	3	428/803
9	6	4	27/34	9	6	5	589/625
9	6	6	631/635	9	7	2	69/266
9	7	3	223/381	9	7	4	509/612
9	7	5	383/400	9	7	6	808/811
9	8	2	276/913	9	8	3	67/105
9	8	4	850/983	9	8	5	345/356
9	8	6	475/476	9	9	2	291/826
9	9	3	185/268	9	9	4	660/739
9	9	5	969/991	9	9	6	908/909
9	10	2	71/173	9	10	3	346/467
9	10	4	331/361	9	10	5	557/566
9	11	2	226/473	9	11	3	725/919
9	11	4	59/63	9	11	5	618/625
9	12	2	5/9	9	12	3	5/6
9	12	4	20/21	9	12	5	879/886
10	0	2	1/9	10	0	3	1/3
10	0	4	606/979	10	0	5	55/63
10	1	2	35/278	10	1	3	359/979
10	1	4	435/662	10	1	5	262/293
10	2	2	1/7	10	2	3	35/87

Continued on next page

Table 7.1 – *Continued from previous page*

l_1	l_2	l_3	$\Phi(l_1, l_2, l_3)$	l_1	l_2	l_3	$\Phi(l_1, l_2, l_3)$
10	2	4	434/625	10	2	5	73/80
10	3	2	81/500	10	3	3	265/602
10	3	4	658/901	10	3	5	142/153
10	4	2	7/38	10	4	3	97/202
10	4	4	419/548	10	4	5	515/547
10	5	2	106/505	10	5	3	247/473
10	5	4	498/625	10	5	5	868/911
10	6	2	64/267	10	6	3	541/956
10	6	4	253/306	10	6	5	280/291
10	7	2	14/51	10	7	3	482/789
10	7	4	534/625	10	7	5	292/301
10	8	2	144/457	10	8	3	587/894
10	8	4	175/199	10	8	5	626/641
10	9	2	320/883	10	9	3	439/625
10	9	4	733/813	10	9	5	971/989
10	10	2	167/400	10	10	3	569/761
10	10	4	491/533	10	10	5	71/72
10	11	2	13/27	10	11	3	19/24
10	11	4	879/937	10	11	5	280/283
10	12	2	5/9	10	12	3	5/6
10	12	4	20/21	10	12	5	879/886

Appendix 2: Survival Signature of System in Figure 4.9

The table in this appendix shows the survival signature of the complex system of Figure 4.9. The rows with survival signature values equal to either 1 or 0 have been omitted.

Table 7.2: Survival signature of a complex system in Figure 4.9; rows with $\Phi(l_1, l_2, l_3, l_4, l_5, l_6) = 0$ and $\Phi(l_1, l_2, l_3, l_4, l_5, l_6) = 1$ are omitted

l_1	l_2	l_3	l_4	l_5	l_6	$\Phi(l_1, l_2, l_3, l_4, l_5, l_6)$
3	1	0	[0,1]	[0,1]	1	1/20
3	1	0	1	1	0	1/20
3	1	1	0	[0,1]	1	1/20
3	1	1	1	0	1	1/20
3	1	2	[0,1]	0	1	1/20
3	1	2	0	1	0	1/20
3	1	1	[0,1]	1	1	1/10
3	1	1	1	1	0	1/10
3	1	2	0	1	1	1/10
3	1	2	1	1	[0,1]	1/10
3	2	0	0	[0,1]	1	1/10
3	2	0	1	0	1	1/10
3	2	0	1	1	[0,1]	1/10
3	2	1	[0,1]	0	1	1/10
3	2	1	0	1	0	1/10
3	2	2	[0,1]	0	1	1/10
3	2	2	0	1	0	1/10
3	3	0	[0,1]	[0,1]	1	3/20

Continued on next page

Table 7.2 – *Continued from previous page*

l_1	l_2	l_3	l_4	l_5	l_6	$\Phi(l_1, l_2, l_3, l_4, l_5, l_6)$
3	3	0	1	1	0	3/20
3	3	[1,2]	[0,1]	0	1	3/20
3	3	1	0	1	0	3/20
3	3	2	0	1	0	3/20
3	4	0	[0,1]	[0,1]	1	1/5
3	4	0	1	1	0	1/5
3	4	[1,2]	[0,1]	0	1	1/5
3	4	1	0	1	0	1/5
3	4	2	0	1	0	1/5
4	1	0	[0,1]	[0,1]	1	1/5
4	1	0	1	1	0	1/5
4	1	[1,2]	[0,1]	0	1	1/5
4	1	1	0	1	0	1/5
4	1	2	0	1	0	1/5
3	2	[1,2]	[0,1]	1	1	4/15
3	2	1	1	1	0	4/15
3	2	2	1	1	0	4/15
4	2	[0,1,2]	[0,1]	0	1	11/30
4	2	0	0	1	1	11/30
4	2	0	1	1	[0,1]	11/30
4	2	1	0	1	0	11/30
4	2	2	0	1	0	11/30
3	[3,4]	1	[0,1]	1	1	2/5
3	3	1	1	1	0	2/5

Continued on next page

Table 7.2 – *Continued from previous page*

l_1	l_2	l_3	l_4	l_5	l_6	$\Phi(l_1, l_2, l_3, l_4, l_5, l_6)$
3	3	2	0	1	1	2/5
3	3	2	1	1	[0,1]	2/5
3	4	1	1	1	0	2/5
3	4	2	[0,1]	1	1	2/5
3	4	2	1	1	0	2/5
4	1	[1,2]	[0,1]	1	1	2/5
4	1	[1,2]	1	1	0	2/5
4	3	0	[0,1]	[0,1]	1	1/2
4	3	0	1	1	0	1/2
4	3	[1,2]	[0,1]	0	1	1/2
4	3	1	0	1	0	1/2
4	3	2	0	1	0	1/2
5	1	0	[0,1]	[0,1]	1	1/2
5	1	0	1	1	0	1/2
5	1	[1,2]	[0,1]	0	1	1/2
5	1	1	0	1	0	1/2
5	1	2	0	1	0	1/2
4	4	0	[0,1]	[0,1]	1	3/5
4	4	0	1	1	0	3/5
4	4	[1,2]	[0,1]	0	1	3/5
4	4	1	0	1	0	3/5
4	4	2	0	1	0	3/5
4	2	[1,2]	[0,1]	1	1	2/3
4	2	1	1	1	0	2/3

Continued on next page

Table 7.2 – *Continued from previous page*

l_1	l_2	l_3	l_4	l_5	l_6	$\Phi(l_1, l_2, l_3, l_4, l_5, l_6)$
4	2	2	1	1	0	2/3
4	3	[1,2]	[0,1]	1	1	4/5
4	3	1	1	1	0	4/5
4	3	2	1	1	0	4/5
4	4	[1,2]	[0,1]	1	1	4/5
4	4	1	1	1	0	4/5
4	4	2	1	1	0	4/5
5	2	0	[0,1]	[0,1]	1	5/6
5	2	0	1	1	0	5/6
5	2	[1,2]	[0,1]	0	1	5/6
5	2	[1,2]	0	1	0	5/6

Appendix 3: Values of $P(f_1, f_2, f_3, f_4)$ and $\Phi(l_1, l_2, l_3, l_4)$ of Case 1 in Chapter 6

Table 7.3: Values of $P(f_1, f_2, f_3, f_4)$ and their corresponding survival signature $\Phi(l_1, l_2, l_3, l_4)$, note that $f_i = m_i - l_i$

$P(f_1, f_2, f_3, f_4)$	$\Phi(l_1, l_2, l_3, l_4)$	$P(f_1, f_2, f_3, f_4)$	$\Phi(l_1, l_2, l_3, l_4)$
$P(1,1,1,1) = \frac{1}{48}$	$\Phi(2,3,2,3) = \frac{41}{48}$	$P(1,1,1,2) = \frac{1}{48}$	$\Phi(2,3,2,2) = \frac{19}{24}$
$P(1,1,1,3) = \frac{1}{144}$	$\Phi(2,3,2,1) = \frac{35}{48}$	$P(1,1,1,4) = \frac{1}{144}$	$\Phi(2,3,2,0) = \frac{2}{3}$
$P(1,1,2,1) = \frac{1}{96}$	$\Phi(2,3,1,3) = \frac{41}{48}$	$P(1,1,2,2) = \frac{1}{96}$	$\Phi(2,3,1,2) = \frac{107}{144}$

Continued on next page

Table 7.3 – *Continued from previous page*

$P(f_1, f_2, f_3, f_4)$	$\Phi(l_1, l_2, l_3, l_4)$	$P(f_1, f_2, f_3, f_4)$	$\Phi(l_1, l_2, l_3, l_4)$
$P(1,1,2,3) = \frac{1}{288}$	$\Phi(2,3,1,1) = \frac{19}{32}$	$P(1,1,2,4) = \frac{1}{288}$	$\Phi(2,3,1,0) = \frac{5}{12}$
$P(1,2,1,1) = \frac{1}{96}$	$\Phi(2,2,2,3) = \frac{17}{24}$	$P(1,2,1,2) = \frac{1}{96}$	$\Phi(2,2,2,2) = \frac{16}{27}$
$P(1,2,1,3) = \frac{1}{288}$	$\Phi(2,2,2,1) = \frac{35}{72}$	$P(1,2,1,4) = \frac{1}{288}$	$\Phi(2,2,2,0) = \frac{7}{18}$
$P(1,2,2,1) = \frac{1}{192}$	$\Phi(2,2,1,3) = \frac{49}{72}$	$P(1,2,2,2) = \frac{1}{192}$	$\Phi(2,2,1,2) = \frac{14}{27}$
$P(1,2,2,3) = \frac{1}{576}$	$\Phi(2,2,1,1) = \frac{49}{144}$	$P(1,2,2,4) = \frac{1}{576}$	$\Phi(2,2,1,0) = \frac{1}{6}$
$P(1,3,1,1) = \frac{1}{96}$	$\Phi(2,1,2,3) = \frac{9}{16}$	$P(1,3,1,2) = \frac{1}{96}$	$\Phi(2,1,2,2) = \frac{29}{72}$
$P(1,3,1,3) = \frac{1}{288}$	$\Phi(2,1,2,1) = \frac{13}{48}$	$P(1,3,1,4) = \frac{1}{288}$	$\Phi(2,1,2,0) = \frac{1}{6}$
$P(1,3,2,1) = \frac{1}{192}$	$\Phi(2,1,1,3) = \frac{1}{2}$	$P(1,3,2,2) = \frac{1}{192}$	$\Phi(2,1,1,2) = \frac{23}{72}$
$P(1,3,2,3) = \frac{1}{576}$	$\Phi(2,1,1,1) = \frac{5}{32}$	$P(1,3,2,4) = \frac{1}{576}$	$\Phi(2,1,1,0) = \frac{1}{24}$
$P(1,4,1,1) = \frac{1}{48}$	$\Phi(2,0,2,3) = \frac{5}{12}$	$P(1,4,1,2) = \frac{1}{48}$	$\Phi(2,0,2,2) = \frac{2}{9}$
$P(1,4,1,3) = \frac{1}{48}$	$\Phi(2,0,2,1) = \frac{1}{12}$	$P(1,4,1,4) = \frac{1}{48}$	$\Phi(2,0,2,0) = 0$
$P(1,4,2,1) = \frac{1}{96}$	$\Phi(2,0,1,3) = \frac{1}{3}$	$P(1,4,2,2) = \frac{1}{96}$	$\Phi(2,0,1,2) = \frac{1}{6}$
$P(1,4,2,3) = \frac{1}{96}$	$\Phi(2,0,1,1) = \frac{1}{24}$	$P(1,4,2,4) = \frac{1}{96}$	$\Phi(2,0,1,0) = 0$
$P(2,1,1,1) = \frac{1}{24}$	$\Phi(1,3,2,3) = \frac{37}{48}$	$P(2,1,1,2) = \frac{1}{24}$	$\Phi(1,3,2,2) = \frac{17}{24}$
$P(2,1,1,3) = \frac{1}{72}$	$\Phi(1,3,2,1) = \frac{31}{48}$	$P(2,1,1,4) = \frac{1}{72}$	$\Phi(1,3,2,0) = \frac{7}{12}$
$P(2,1,2,1) = \frac{1}{48}$	$\Phi(1,3,1,3) = \frac{37}{48}$	$P(2,1,2,2) = \frac{1}{48}$	$\Phi(1,3,1,2) = \frac{23}{36}$
$P(2,1,2,3) = \frac{1}{144}$	$\Phi(1,3,1,1) = \frac{11}{24}$	$P(2,1,2,4) = \frac{1}{144}$	$\Phi(1,3,1,0) = \frac{1}{4}$
$P(2,2,1,1) = \frac{1}{48}$	$\Phi(1,2,2,3) = \frac{13}{24}$	$P(2,2,1,2) = \frac{1}{48}$	$\Phi(1,2,2,2) = \frac{47}{108}$
$P(2,2,1,3) = \frac{1}{144}$	$\Phi(1,2,2,1) = \frac{25}{72}$	$P(2,2,1,4) = \frac{1}{144}$	$\Phi(1,2,2,0) = \frac{5}{18}$
$P(2,2,2,1) = \frac{1}{96}$	$\Phi(1,2,1,3) = \frac{35}{72}$	$P(2,2,2,2) = \frac{1}{96}$	$\Phi(1,2,1,2) = \frac{71}{216}$
$P(2,2,2,3) = \frac{1}{288}$	$\Phi(1,2,1,1) = \frac{13}{72}$	$P(2,2,2,4) = \frac{1}{288}$	$\Phi(1,2,1,0) = \frac{1}{18}$
$P(2,3,1,1) = \frac{1}{48}$	$\Phi(1,1,2,3) = \frac{1}{3}$	$P(2,3,1,2) = \frac{1}{48}$	$\Phi(1,1,2,2) = \frac{5}{24}$
$P(2,3,1,3) = \frac{1}{144}$	$\Phi(1,1,2,1) = \frac{1}{8}$	$P(2,3,1,4) = \frac{1}{144}$	$\Phi(1,1,2,0) = \frac{1}{12}$
$P(2,3,2,1) = \frac{1}{96}$	$\Phi(1,1,1,3) = \frac{1}{4}$	$P(2,3,2,2) = \frac{1}{96}$	$\Phi(1,1,1,2) = \frac{17}{144}$

Continued on next page

Table 7.3 – *Continued from previous page*

$P(f_1, f_2, f_3, f_4)$	$\Phi(l_1, l_2, l_3, l_4)$	$P(f_1, f_2, f_3, f_4)$	$\Phi(l_1, l_2, l_3, l_4)$
$P(2,3,2,3) = \frac{1}{288}$	$\Phi(1,1,1,1) = \frac{1}{24}$	$P(2,3,2,4) = \frac{1}{288}$	$\Phi(1,1,1,0) = 0$
$P(2,4,1,1) = \frac{1}{24}$	$\Phi(1,0,2,3) = \frac{1}{6}$	$P(2,4,1,2) = \frac{1}{24}$	$\Phi(1,0,2,2) = \frac{1}{18}$
$P(2,4,1,3) = \frac{1}{72}$	$\Phi(1,0,2,1) = 0$	$P(2,4,1,4) = \frac{1}{72}$	$\Phi(1,0,2,0) = 0$
$P(2,4,2,1) = \frac{1}{48}$	$\Phi(1,0,1,3) = \frac{1}{12}$	$P(2,4,2,2) = \frac{1}{48}$	$\Phi(1,0,1,2) = 0$
$P(2,4,2,3) = \frac{1}{144}$	$\Phi(1,0,1,1) = 0$	$P(2,4,2,4) = \frac{1}{144}$	$\Phi(1,0,1,0) = 0$
$P(3,1,1,1) = \frac{1}{48}$	$\Phi(0,3,2,3) = \frac{11}{16}$	$P(3,1,1,2) = \frac{1}{48}$	$\Phi(0,3,2,2) = \frac{5}{8}$
$P(3,1,1,3) = \frac{1}{144}$	$\Phi(0,3,2,1) = \frac{9}{16}$	$P(3,1,1,4) = \frac{1}{144}$	$\Phi(0,3,2,0) = \frac{1}{2}$
$P(3,1,2,1) = \frac{1}{96}$	$\Phi(0,3,1,3) = \frac{11}{16}$	$P(3,1,2,2) = \frac{1}{96}$	$\Phi(0,3,1,2) = \frac{13}{24}$
$P(3,1,2,3) = \frac{1}{288}$	$\Phi(0,3,1,1) = \frac{11}{32}$	$P(3,1,2,4) = \frac{1}{288}$	$\Phi(0,3,1,0) = \frac{1}{8}$
$P(3,2,1,1) = \frac{1}{96}$	$\Phi(0,2,2,3) = \frac{3}{8}$	$P(3,2,1,2) = \frac{1}{96}$	$\Phi(0,2,2,2) = \frac{5}{18}$
$P(3,2,1,3) = \frac{1}{288}$	$\Phi(0,2,2,1) = \frac{5}{24}$	$P(3,2,1,4) = \frac{1}{288}$	$\Phi(0,2,2,0) = \frac{1}{6}$
$P(3,2,2,1) = \frac{1}{192}$	$\Phi(0,2,1,3) = \frac{7}{24}$	$P(3,2,2,2) = \frac{1}{192}$	$\Phi(0,2,1,2) = \frac{11}{72}$
$P(3,2,2,3) = \frac{1}{576}$	$\Phi(0,2,1,1) = \frac{1}{16}$	$P(3,2,2,4) = \frac{1}{576}$	$\Phi(0,2,1,0) = 0$
$P(3,3,1,1) = \frac{1}{96}$	$\Phi(0,1,2,3) = \frac{1}{8}$	$P(3,3,1,2) = \frac{1}{96}$	$\Phi(0,1,2,2) = \frac{1}{24}$
$P(3,3,1,3) = \frac{1}{288}$	$\Phi(0,1,2,1) = 0$	$P(3,3,1,4) = \frac{1}{288}$	$\Phi(0,1,2,0) = 0$
$P(3,3,2,1) = \frac{1}{192}$	$\Phi(0,1,1,3) = \frac{1}{16}$	$P(3,3,2,2) = \frac{1}{192}$	$\Phi(0,1,1,2) = 0$
$P(3,3,2,3) = \frac{1}{576}$	$\Phi(0,1,1,1) = 0$	$P(3,3,2,4) = \frac{1}{576}$	$\Phi(0,1,1,0) = 0$
$P(3,4,1,1) = \frac{1}{48}$	$\Phi(0,0,2,3) = 0$	$P(3,4,1,2) = \frac{1}{48}$	$\Phi(0,0,2,2) = 0$
$P(3,4,1,3) = \frac{1}{144}$	$\Phi(0,0,2,1) = 0$	$P(3,4,1,4) = \frac{1}{144}$	$\Phi(0,0,2,0) = 0$
$P(3,4,2,1) = \frac{1}{96}$	$\Phi(0,0,1,3) = 0$	$P(3,4,2,2) = \frac{1}{96}$	$\Phi(0,0,1,2) = 0$
$P(3,4,2,3) = \frac{1}{288}$	$\Phi(0,0,1,1) = 0$	$P(3,4,2,4) = \frac{1}{288}$	$\Phi(0,0,1,0) = 0$

Appendix 4: Past Data of System in Figure 6.1

Table 7.4: Past data $n_{j_1j_2j_3j_4}$ on the system in Figure 6.1

$n_{j_1j_2j_3j_4}$	$n_{j_1j_2j_3j_4}$	$n_{j_1j_2j_3j_4}$	$n_{j_1j_2j_3j_4}$	$n_{j_1j_2j_3j_4}$	$n_{j_1j_2j_3j_4}$
$n_{0001} = 1$	$n_{0002} = 1$	$n_{0003} = 1$	$n_{0004} = 1$	$n_{0010} = 1$	$n_{0011} = 1$
$n_{0012} = 1$	$n_{0013} = 1$	$n_{0014} = 1$	$n_{0020} = 1$	$n_{0021} = 1$	$n_{0022} = 1$
$n_{0023} = 1$	$n_{0024} = 1$	$n_{0100} = 1$	$n_{0101} = 1$	$n_{0102} = 3$	$n_{0103} = 1$
$n_{0104} = 1$	$n_{0110} = 1$	$n_{0111} = 1$	$n_{0112} = 1$	$n_{0113} = 1$	$n_{0114} = 1$
$n_{0120} = 1$	$n_{0121} = 1$	$n_{0122} = 1$	$n_{0123} = 1$	$n_{0124} = 2$	$n_{0200} = 1$
$n_{0201} = 1$	$n_{0202} = 1$	$n_{0203} = 1$	$n_{0204} = 1$	$n_{0210} = 1$	$n_{0211} = 2$
$n_{0212} = 1$	$n_{0213} = 1$	$n_{0214} = 1$	$n_{0220} = 1$	$n_{0221} = 1$	$n_{0222} = 1$
$n_{0223} = 1$	$n_{0224} = 1$	$n_{0300} = 1$	$n_{0301} = 1$	$n_{0302} = 1$	$n_{0303} = 1$
$n_{0304} = 1$	$n_{0310} = 2$	$n_{0311} = 2$	$n_{0312} = 1$	$n_{0313} = 1$	$n_{0314} = 1$
$n_{0320} = 1$	$n_{0321} = 1$	$n_{0322} = 1$	$n_{0323} = 1$	$n_{0324} = 1$	$n_{0400} = 1$
$n_{0401} = 1$	$n_{0402} = 1$	$n_{0403} = 1$	$n_{0404} = 1$	$n_{0410} = 1$	$n_{0411} = 2$
$n_{0412} = 1$	$n_{0413} = 1$	$n_{0414} = 1$	$n_{0420} = 1$	$n_{0421} = 1$	$n_{0422} = 1$
$n_{0423} = 1$	$n_{0424} = 1$	$n_{1000} = 1$	$n_{1001} = 1$	$n_{1002} = 1$	$n_{1003} = 1$
$n_{1004} = 1$	$n_{1010} = 1$	$n_{1011} = 1$	$n_{1012} = 2$	$n_{1013} = 1$	$n_{1014} = 1$
$n_{1020} = 1$	$n_{1021} = 1$	$n_{1022} = 1$	$n_{1023} = 1$	$n_{1024} = 1$	$n_{1100} = 1$
$n_{1101} = 1$	$n_{1102} = 1$	$n_{1103} = 2$	$n_{1104} = 1$	$n_{1110} = 2$	$n_{1111} = 1$
$n_{1112} = 1$	$n_{1113} = 1$	$n_{1114} = 1$	$n_{1120} = 1$	$n_{1121} = 1$	$n_{1122} = 1$
$n_{1123} = 1$	$n_{1124} = 1$	$n_{1200} = 1$	$n_{1201} = 2$	$n_{1202} = 1$	$n_{1203} = 1$
$n_{1204} = 1$	$n_{1210} = 1$	$n_{1211} = 1$	$n_{1212} = 1$	$n_{1213} = 1$	$n_{1214} = 1$
$n_{1220} = 1$	$n_{1221} = 3$	$n_{1222} = 2$	$n_{1223} = 1$	$n_{1224} = 1$	$n_{1300} = 1$
$n_{1301} = 1$	$n_{1302} = 1$	$n_{1303} = 1$	$n_{1304} = 1$	$n_{1310} = 1$	$n_{1311} = 3$
$n_{1312} = 1$	$n_{1313} = 1$	$n_{1314} = 1$	$n_{1320} = 1$	$n_{1321} = 1$	$n_{1322} = 1$
$n_{1323} = 1$	$n_{1324} = 1$	$n_{1400} = 1$	$n_{1401} = 1$	$n_{1402} = 1$	$n_{1403} = 1$

Continued on next page

Table 7.4 – *Continued from previous page*

$n_{j_1j_2j_3j_4}$	$n_{j_1j_2j_3j_4}$	$n_{j_1j_2j_3j_4}$	$n_{j_1j_2j_3j_4}$	$n_{j_1j_2j_3j_4}$	$n_{j_1j_2j_3j_4}$
$n_{1404} = 2$	$n_{1410} = 4$	$n_{1411} = 1$	$n_{1412} = 1$	$n_{1413} = 1$	$n_{1414} = 1$
$n_{1420} = 1$	$n_{1421} = 1$	$n_{1422} = 1$	$n_{1423} = 1$	$n_{1424} = 1$	$n_{2000} = 1$
$n_{2001} = 1$	$n_{2002} = 1$	$n_{2003} = 1$	$n_{2004} = 1$	$n_{2010} = 3$	$n_{2011} = 1$
$n_{2012} = 1$	$n_{2013} = 1$	$n_{2014} = 1$	$n_{2020} = 1$	$n_{2021} = 1$	$n_{2022} = 1$
$n_{2023} = 2$	$n_{2024} = 1$	$n_{2100} = 1$	$n_{2101} = 1$	$n_{2102} = 1$	$n_{2103} = 2$
$n_{2104} = 1$	$n_{2110} = 1$	$n_{2111} = 1$	$n_{2112} = 1$	$n_{2113} = 1$	$n_{2114} = 1$
$n_{2120} = 1$	$n_{2121} = 1$	$n_{2122} = 1$	$n_{2123} = 1$	$n_{2124} = 1$	$n_{2200} = 1$
$n_{2201} = 2$	$n_{2202} = 1$	$n_{2203} = 1$	$n_{2204} = 1$	$n_{2210} = 1$	$n_{2211} = 1$
$n_{2212} = 1$	$n_{2213} = 1$	$n_{2214} = 1$	$n_{2220} = 1$	$n_{2221} = 2$	$n_{2222} = 1$
$n_{2223} = 1$	$n_{2224} = 3$	$n_{2300} = 1$	$n_{2301} = 1$	$n_{2302} = 1$	$n_{2303} = 1$
$n_{2304} = 1$	$n_{2310} = 1$	$n_{2311} = 1$	$n_{2312} = 1$	$n_{2313} = 1$	$n_{2314} = 1$
$n_{2320} = 1$	$n_{2321} = 1$	$n_{2322} = 1$	$n_{2323} = 1$	$n_{2324} = 1$	$n_{2400} = 1$
$n_{2401} = 1$	$n_{2402} = 2$	$n_{2403} = 1$	$n_{2404} = 1$	$n_{2410} = 1$	$n_{2411} = 1$
$n_{2412} = 1$	$n_{2413} = 1$	$n_{2414} = 1$	$n_{2420} = 1$	$n_{2421} = 1$	$n_{2422} = 1$
$n_{2423} = 1$	$n_{2424} = 1$	$n_{3000} = 2$	$n_{3001} = 1$	$n_{3002} = 2$	$n_{3003} = 1$
$n_{3004} = 1$	$n_{3010} = 1$	$n_{3011} = 1$	$n_{3012} = 1$	$n_{3013} = 1$	$n_{3014} = 1$
$n_{3020} = 1$	$n_{3021} = 1$	$n_{3022} = 1$	$n_{3023} = 3$	$n_{3024} = 1$	$n_{3100} = 1$
$n_{3101} = 1$	$n_{3102} = 1$	$n_{3103} = 1$	$n_{3104} = 1$	$n_{3110} = 1$	$n_{3111} = 1$
$n_{3112} = 2$	$n_{3113} = 1$	$n_{3114} = 1$	$n_{3120} = 1$	$n_{3121} = 1$	$n_{3122} = 1$
$n_{3123} = 1$	$n_{3124} = 1$	$n_{3200} = 1$	$n_{3201} = 1$	$n_{3202} = 1$	$n_{3203} = 1$
$n_{3204} = 1$	$n_{3210} = 1$	$n_{3211} = 3$	$n_{3212} = 1$	$n_{3213} = 1$	$n_{3214} = 1$
$n_{3220} = 1$	$n_{3221} = 1$	$n_{3222} = 2$	$n_{3223} = 1$	$n_{3224} = 1$	$n_{3300} = 1$
$n_{3301} = 1$	$n_{3302} = 1$	$n_{3303} = 1$	$n_{3304} = 2$	$n_{3310} = 1$	$n_{3311} = 1$
$n_{3312} = 1$	$n_{3313} = 1$	$n_{3314} = 3$	$n_{3320} = 1$	$n_{3321} = 1$	$n_{3322} = 1$

Continued on next page

Table 7.4 – *Continued from previous page*

$n_{j_1j_2j_3j_4}$	$n_{j_1j_2j_3j_4}$	$n_{j_1j_2j_3j_4}$	$n_{j_1j_2j_3j_4}$	$n_{j_1j_2j_3j_4}$	$n_{j_1j_2j_3j_4}$
$n_{3323} = 2$	$n_{3324} = 2$	$n_{3400} = 1$	$n_{3401} = 1$	$n_{3402} = 1$	$n_{3403} = 2$
$n_{3404} = 1$	$n_{3410} = 3$	$n_{3411} = 1$	$n_{3412} = 1$	$n_{3413} = 1$	$n_{3414} = 1$
$n_{3420} = 1$	$n_{3421} = 1$	$n_{3422} = 1$	$n_{3423} = 1$	$n_{3424} = 1$	

Bibliography

[1] J. Graunt, Natural and political observations mentioned in a following index, and made upon the bills of mortality, in: Mathematical Demography, Springer, 1977, pp. 11–20.

[2] J. H. Saleh, K. Marais, Highlights from the early (and pre-) history of reliability engineering, Reliability Engineering & System Safety 91 (2) (2006) 249–256.

[3] W. Weibull, A statistical distribution function of wide applicability, Journal of Applied Mechanics 18 (3) (1951) 293–297.

[4] Z. W. Birnbaum, On the importance of different components in a multicomponent system, Tech. rep., DTIC Document (1968).

[5] R. E. Barlow, F. Proschan, Mathematical theory of reliability, SIAM, 1996.

[6] E. Zio, Reliability engineering: Old problems and new challenges, Reliability Engineering & System Safety 94 (2) (2009) 125–141.

[7] K. Kołowrocki, B. Kwiatuszewska-Sarnecka, Reliability and risk analysis of large systems with ageing components, Reliability Engineering & System Safety 93 (12) (2008) 1821–1829.

[8] M. Modarres, What every engineer should know about reliability and risk analysis, Vol. 30, CRC Press, 1992.

[9] F. J. Samaniego, System signatures and their applications in engineering reliability, Vol. 110, Springer Science & Business Media, 2007.

[10] S. Eryilmaz, Review of recent advances in reliability of consecutive k-out-of-n and related systems, Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability 224 (3) (2010) 225–237.

[11] F. P. Coolen, T. Coolen-Maturi, Modelling uncertain aspects of system dependability with survival signatures, in: Dependability Problems of Complex Information Systems, Springer, 2015, pp. 19–34.

[12] L. Crow, Practical methods for analyzing the reliability of repairable systems, Reliability EDGE 5 (1) (2004) 1.

[13] D. F. Percy, J. R. Kearney, K. A. Kobbacy, Hybrid intensity models for repairable systems, IMA Journal of Management Mathematics 21 (4) (2008) 395–406.

[14] A. Høyland, M. Rausand, System reliability theory: models and statistical methods, Vol. 420, John Wiley & Sons, 2009.

[15] G. Levitin, A. Lisnianski, Importance and sensitivity analysis of multi-state systems using the universal generating function method, Reliability Engineering & System Safety 65 (3) (1999) 271–282.

[16] J. Fussell, How to hand-calculate system reliability and safety characteristics, Reliability, IEEE Transactions on 3 (1975) 169–174.

[17] M. J. Armstrong, Reliability-importance and dual failure-mode components, Reliability, IEEE Transactions on 46 (2) (1997) 212–221.

[18] A. Gandini, Importance and sensitivity analysis in assessing system reliability, Reliability, IEEE Transactions on 39 (1) (1990) 61–70.

[19] W. Wang, J. Loman, P. Vassiliou, Reliability importance of components in a complex system, in: Reliability and Maintainability, 2004 Annual Symposium-RAMS, IEEE, 2004, pp. 6–11.

[20] E. Zio, L. Podofillini, Monte carlo simulation analysis of the effects of different system performance levels on the importance of multi-state components, Reliability Engineering & System Safety 82 (1) (2003) 63–73.

[21] E. Zio, L. Podofillini, G. Levitin, Estimation of the importance measures of multi-state elements by monte carlo simulation, Reliability Engineering & System Safety 86 (3) (2004) 191–204.

[22] P. R. Adduri, R. C. Penmetsa, System reliability analysis for mixed uncertain variables, Structural Safety 31 (5) (2009) 375–382.

[23] F. P. Coolen, M. C. Troffaes, T. Augustin, Imprecise probability, in: International Encyclopedia of Statistical Science, Springer, 2011, pp. 645–648.

[24] S. Ferson, L. R. Ginzburg, Different methods are needed to propagate ignorance and variability, Reliability Engineering & System Safety 54 (2) (1996) 133–144.

[25] A. P. Dempster, Upper and lower probabilities induced by a multivalued mapping, The annals of mathematical statistics (1967) 325–339.

[26] G. Shafer, A mathematical theory of evidence, Vol. 1, Princeton University Press Princeton, 1976.

[27] M. C. Troffaes, G. Walter, D. Kelly, A robust bayesian approach to modeling epistemic uncertainty in common-cause failure models, Reliability Engineering & System Safety 125 (2014) 13–21.

[28] F. Tonon, Using random set theory to propagate epistemic uncertainty through a mechanical system, Reliability Engineering & System Safety 85 (1) (2004) 169–181.

[29] J. C. Helton, J. D. Johnson, W. Oberkampf, C. J. Sallaberry, Sensitivity analysis in conjunction with evidence theory representations of epistemic uncertainty, Reliability Engineering & System Safety 91 (10) (2006) 1414–1434.

[30] M. Beer, S. Ferson, V. Kreinovich, Imprecise probabilities in engineering analyses, Mechanical Systems and Signal Processing 37 (1) (2013) 4–29.

[31] B. Möller, M. Beer, Fuzzy randomness: uncertainty in civil engineering and computational mechanics, Springer Science & Business Media, 2013.

[32] E. Patelli, D. A. Alvarez, M. Broggi, M. de Angelis, An integrated and efficient numerical framework for uncertainty quantification: application to the nasa langley multidisciplinary uncertainty quantification challenge, in: 16th AIAA Non-Deterministic Approaches Conference (SciTech 2014), 2014, pp. 2014–1501.

[33] E. Patelli, Cossan: A multidisciplinary software suite for uncertainty quantification and risk management.

[34] R. C. Williamson, T. Downs, Probabilistic arithmetic. i. numerical methods for calculating convolutions and dependency bounds, International Journal of Approximate Reasoning 4 (2) (1990) 89–158.

[35] S. Ferson, V. Kreinovich, L. Ginzburg, D. S. Myers, K. Sentz, Constructing probability boxes and Dempster-Shafer structures, Vol. 835, Sandia National Laboratories, 2002.

[36] D. R. Karanki, H. S. Kushwaha, A. K. Verma, S. Ajit, Uncertainty analysis based on probability bounds (p-box) approach in probabilistic safety assessment, Risk Analysis 29 (5) (2009) 662–675.

[37] C. Simon, P. Weber, Evidential networks for reliability analysis and performance evaluation of systems with imprecise knowledge, IEEE Transactions on Reliability 58 (1) (2009) 69–87.

[38] K. Sentz, S. Ferson, Probabilistic bounding analysis in the quantification of margins and uncertainties, Reliability Engineering & System Safety 96 (9) (2011) 1126–1136.

[39] S. Ferson, W. T. Tucker, Sensitivity analysis using probability bounding, Reliability Engineering & System Safety 91 (10) (2006) 1435–1442.

[40] G. Levitin, Incorporating common-cause failures into nonrepairable multistate series-parallel system analysis, IEEE Transactions on Reliability 50 (4) (2001) 380–388.

[41] B. Dhillon, O. Anude, Common-cause failures in engineering systems: A review, International Journal of Reliability, Quality and Safety Engineering 1 (01) (1994) 103–129.

[42] D. M. Rasmuson, D. L. Kelly, Common-cause failure analysis in event assessment, Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability 222 (4) (2008) 521–532.

[43] K. Fleming, Reliability model for common mode failures in redundant safety systems, in: Modeling and Simulation. Volume 6, Part 1, 1975.

[44] K. N. Fleming, A. Mosleh, R. K. Deremer, A systematic procedure for the incorporation of common cause events into risk and reliability models, Nuclear Engineering and Design 93 (2-3) (1986) 245–273.

[45] A. Mosleh, Common cause failures: an analysis methodology and examples, Reliability Engineering & System Safety 34 (3) (1991) 249–292.

[46] D. Kelly, C. Atwood, Finding a minimally informative dirichlet prior distribution using least squares, Reliability Engineering & System Safety 96 (3) (2011) 398–402.

[47] F. P. Coolen, T. Coolen-Maturi, Predictive inference for system reliability after common-cause component failures, Reliability Engineering & System Safety 135 (2015) 27–33.

[48] F. P. Coolen, T. Coolen-Maturi, Generalizing the signature to systems with multiple types of components, in: Complex Systems and Dependability, Springer, 2012, pp. 115–130.

[49] R. G. Miller Jr, Survival analysis, Vol. 66, John Wiley & Sons, 2011.

[50] T. Augustin, F. P.A. Coolen, G. de Cooman, M. C.M. Troffaes, Introduction to imprecise probabilities, John Wiley & Sons, 2014.

[51] J. Butler, J. Jia, J. Dyer, Simulation techniques for the sensitivity analysis of multi-criteria decision models, European Journal of Operational Research 103 (3) (1997) 531–546.

[52] R. Y. Rubinstein, Optimization of computer simulation models with rare events, European Journal of Operational Research 99 (1) (1997) 89–112.

[53] S. H. Zanakis, A. Solomon, N. Wishart, S. Dublish, Multi-attribute decision making: A simulation comparison of select methods, European Journal of Operational Research 107 (3) (1998) 507–529.

[54] E. Zio, P. Baraldi, E. Patelli, Assessment of the availability of an offshore installation by monte carlo simulation, International Journal of Pressure Vessels and Piping 83 (4) (2006) 312–320.

[55] J. D. Andrews, T. R. Moss, Reliability and risk assessment, Wiley-Blackwell, 2002.

[56] W. L. Winston, J. B. Goldberg, Operations research: applications and algorithms, Vol. 3, Duxbury Press Boston, 2004.

[57] F. P. Coolen, T. Coolen-Maturi, A. H. Al-Nefaiee, Nonparametric predictive inference for system reliability using the survival signature, Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability 228 (5) (2014) 437–448.

[58] L. J. Aslett, Reliabilitytheory: Tools for structural reliability analysis. r package (2012).

[59] L. J. Aslett, F. P. Coolen, S. P. Wilson, Bayesian inference for reliability of systems and networks using the survival signature, Risk Analysis 35 (3) (2015) 1640–1651.

[60] G. Feng, E. Patelli, M. Beer, F. P. Coolen, Imprecise system reliability and component importance based on survival signature, Reliability Engineering & System Safety 150 (2016) 116–125.

[61] E. Patelli, G. Feng, Efficient simulation approaches for reliability analysis of large systems, in: International Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems, Springer, 2016, pp. 129–140.

[62] E. Patelli, G. Feng, F. P. Coolen, T. Coolen-Maturi, Simulation methods for system reliability using the survival signature, Reliability Engineering & System Safety(submitted).

[63] G. Walter, L. J. Aslett, F. Coolen, Bayesian nonparametric system reliability using sets of priors, International Journal of Approximate Reasoning 30 (2017) 67–88.

[64] F. P. Coolen, T. Coolen-Maturi, The structure function for system reliability as predictive (imprecise) probability, Reliability Engineering & System Safety 154 (2016) 180–187.

[65] L. J. Aslett, MCMC for inference on phase-type and masked system lifetime models, Ph.D. thesis, Trinity College Dublin (2012).

[66] S. Reed, An efficient algorithm for exact computation of system and survival signatures using binary decision diagrams, Reliability Engineering & System Safety 165 (2017) 257–267.

[67] R. E. Bryant, Graph-based algorithms for boolean function manipulation, Computers, IEEE Transactions on 100 (8) (1986) 677–691.

[68] C. E. Shannon, A symbolic analysis of relay and switching circuits, Electrical Engineering 57 (12) (1938) 713–723.

[69] B. Bollig, I. Wegener, Improving the variable ordering of obdds is np-complete, IEEE Transactions on Computers 45 (9) (1996) 993–1002.

[70] K. S. Brace, R. L. Rudell, R. E. Bryant, Efficient implementation of a bdd package, in: Proceedings of the 27th ACM/IEEE design automation conference, ACM, 1991, pp. 40–45.

[71] G. Hardy, C. Lucet, N. Limnios, K-terminal network reliability measures with binary decision diagrams, IEEE Transactions on Reliability 56 (3) (2007) 506–515.

[72] E. Patelli, M. Broggi, M. de Angelis, M. Beer, Opencossan: An efficient open tool for dealing with epistemic and aleatory uncertainties, in: Vulnerability, Uncertainty, and Risk Quantification, Mitigation, and Management, ASCE, 2014, pp. 2564–2573.

[73] D. A. Alvarez, Infinite random sets and applications in uncertainty analysis, Ph.D. thesis, Arbeitsbereich für Technische Mathematik am Institut fr Grundlagen der Bauingenieurwissenschaften. Leopold-Franzens-Universität Innsbruck, Innsbruck, Austria, available at `https://sites.google.com/site/diegoandresalvarezmarin/RSthesis.pdf` (2007).

[74] B. Möller, M. Beer, Fuzzy-randomness - uncertainty in civil engineering and computational mechanics, Springer-Verlag, 2004.

[75] M. Beer, S. Ferson, Imprecise probabilities-what can they add to engineering analyses?, Mechanical Systems and Signal Processing 37 (1-2) (2013) 1 – 3.

[76] R. E. Moore, R. B. Kearfott, M. J. Cloud, Introduction to interval analysis, SIAM, 2009.

[77] E. Patelli, D. A. Alvarez, M. Broggi, M. de Angelis, Uncertainty management in multidisciplinary design of critical safety systems, Journal of Aerospace Information Systems 12 (1) (2014) 140–169.

[78] M. Beer, Y. Zhang, S. T. Quek, K. K. Phoon, Reliability analysis with scarce information: Comparing alternative approaches in a geotechnical engineering context, Structural Safety 41 (2013) 1–10.

[79] M. Beer, V. Kreinovich, Interval or moments: which carry more information?, Soft Computing 17 (8) (2013) 1319–1327.

[80] B. Möller, M. Beer, Engineering computation under uncertainty–capabilities of non-traditional models, Computers & Structures 86 (10) (2008) 1024–1041.

[81] E. Patelli, H. J. Pradlwarter, G. I. Schuëller, Global sensitivity of structural variability by random sampling, Computer Physics Communications 181 (12) (2010) 2072–2081.

[82] E. Patelli, H. M. Panayirci, M. Broggi, B. Goller, P. Beaurepaire, H. J. Pradlwarter, G. I. Schuëller, General purpose software for efficient uncertainty management of large finite element models, Finite Elements in Analysis and Design 51 (2012) 31–48.

[83] S. Ferson, J. Hajagos, W. T. Tucker, Probability bounds analysis is a global sensitivity analysis, in: International Conference on Sensitivity Analysis of Model Output (SAMO), 2004.

[84] S. Ferson, C. A. Joslyn, J. C. Helton, W. L. Oberkampf, K. Sentz, Summary from the epistemic uncertainty workshop: consensus amid diversity, Reliability Engineering & System Safety 85 (1) (2004) 355–369.

[85] S. Ferson, J. G. Hajagos, Arithmetic with uncertain numbers: rigorous and (often) best possible answers, Reliability Engineering & System Safety 85 (1) (2004) 135–152.

[86] E. Wolfstetter, Stochastic dominance: theory and applications, Humboldt-Univ., Wirtschaftswiss. Fak., 1993.

[87] A. Mettas, M. Savva, System reliability analysis: the advantages of using analytical methods to analyze non-repairable systems, in: Reliability and Maintainability Symposium, 2001. Proceedings. Annual, IEEE, 2001, pp. 80–85.

[88] M. Marseguerra, E. Zio, Basics of the Monte Carlo Method with Application to System Reliability, LiLoLe Publishing, Hagen/Germany, 2002, iSBN: 3-934447-06-6.

[89] W. Liu, J. Li, An improved recursive decomposition algorithm for reliability evaluation of lifeline networks, Earthquake Engineering and Engineering Vibration 8 (3) (2009) 409–419.

[90] X. Zhu, W. Kuo, Importance measures in reliability and mathematical programming, Annals of Operations Research 212 (1) (2014) 241–267.

[91] J. E. Ramirez-Marquez, D. W. Coit, Composite importance measures for multi-state systems with multi-state components, Reliability, IEEE Transactions on 54 (3) (2005) 517–529.

[92] M. Modarres, Risk analysis in engineering: techniques, tools, and trends, CRC Press, 2006.

[93] F. Coolen, On the use of imprecise probabilities in reliability, Quality and Reliability Engineering International 20 (3) (2004) 193–202.

[94] Q. Zhang, Z. Zeng, E. Zio, R. Kang, Probability box as a tool to model and control the effect of epistemic uncertainty in multiple dependent competing failure processes, Applied Soft Computing.

[95] S. Ferson, S. Donald, Probability bounds analysis, in: International Conference on Probabilistic Safety Assessment and Management (PSAM4), 1998, pp. 1203–1208.

[96] F. O. Hoffman, J. S. Hammonds, Propagation of uncertainty in risk assessments: the need to distinguish between uncertainty due to lack of knowledge and uncertainty due to variability, Risk Analysis 14 (5) (1994) 707–712.

[97] D. Binosi, J. Papavassiliou, Pinch technique: theory and applications, Physics Reports 479 (1) (2009) 1–152.

[98] G. B. Chadwell, F. L. Leverenz, Importance measures for prioritization of mechanical integrity and risk reduction activities, American Institute of Chemical Engineers, 1999.

[99] M. Pandey, Statistical analysis of common cause failure data to support safety and reliability analysis of nuclear plant systems, Final Report (RSP-0296).

[100] A. Mosleh, K. Fleming, G. Parry, H. Paula, D. Worledge, D. M. Rasmuson, Procedures for treating common cause failures in safety and reliability studies: Volume 1, procedural framework and examples: Final report, Tech. rep., Pickard, Lowe and Garrick, Inc., Newport Beach, CA (USA) (1988).

[101] D. G. Raheja, L. J. Gullo, Design for reliability, John Wiley & Sons, 2012.

[102] A. Lisnianski, I. Frenkel, Y. Ding, Multi-state system reliability analysis and optimization for engineers and industrial managers, Springer Science & Business Media, 2010.

[103] H. George-Williams, E. Patelli, A hybrid load flow and event driven simulation approach to multi-state system reliability evaluation, Reliability Engineering & System Safety 152 (2016) 351–367.

[104] J. Park, T. P. Seager, P. S. C. Rao, M. Convertino, I. Linkov, Integrating risk and resilience approaches to catastrophe management in engineering systems, Risk Analysis 33 (3) (2013) 356–367.

[105] W. Nelson, Accelerated life testing-step-stress models and data analyses, IEEE Transactions on Reliability 29 (2) (1980) 103–108.