Multi-particle diffusion limited aggregation

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Joint work with

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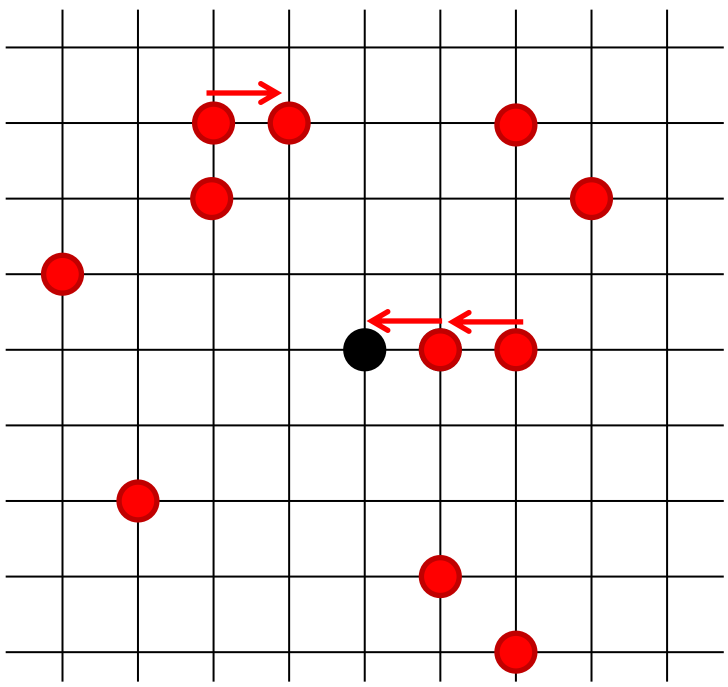
Multi-particle diffusion limited aggregation (MDLA)



Initial configuration

- Particles distributed as IID Bernoulli (μ)
- Aggregate starts at the origin of $\mathbb{Z}^d$

Dynamics of particles

Particles move as continuous-time simple random walk with exclusion rule

➢ *No two particles at same vertex*

When particle wants to jump onto aggregate:

➢ Jump is suppressed

➢ Particle added to aggregate

➢ Particle stops moving

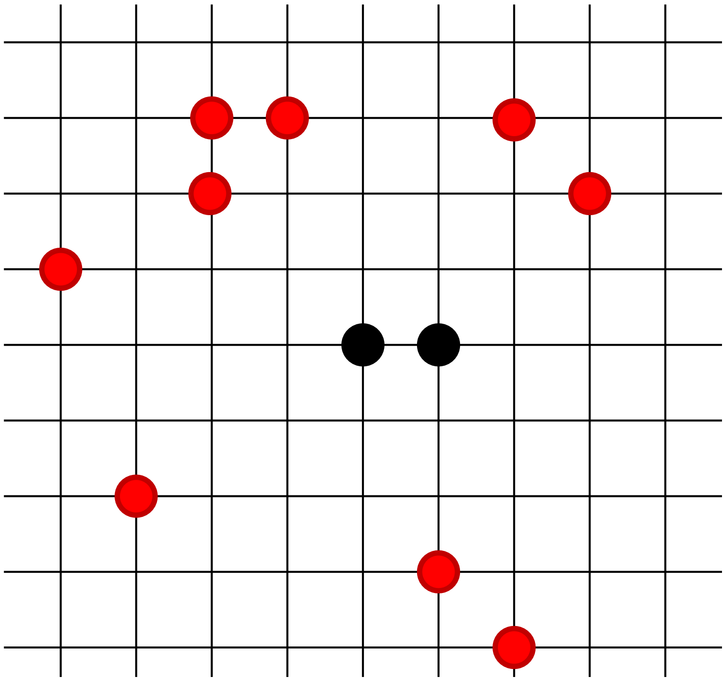
Multi-particle diffusion limited aggregation (MDLA)

History:

1980 - Introduced by Rosenstock and Marquardt to study a phenomenon in photography

1981 - Witten and Sander introduced DLA as a simplification of MDLA and studied it via simulation

1984 - Voss reintroduced MDLA as a more realistic version of DLA



Mathematics of MDLA

Question: how quickly does aggregate grow?

Does aggregate has positive speed of growth?

(that is, does it reach vertex of distance t by time t?)

Thm [Kesten, Sidoravicius]

In $d = 1$, for any $\mu \in (0,1)$, reach of aggregate is of order $\sqrt{t}$ almost surely

After our work:

Sly showed positve speed for $\mu > 1$ $(d = 1)$

Dembo and Tsai studied the case $\mu = 1$ $(d = 1)$

Main result

Thm [Sidoravicius, S.]
Unlike in dimension one, there exists a regime of positive speed of growth in dimensions $d \geq 2$

There exists $\mu_0 \in (0,1)$ such that for all $\mu > \mu_0$ we obtain

$$\mathbb{P}(\text{aggregate grows with positive speed}) > 0$$

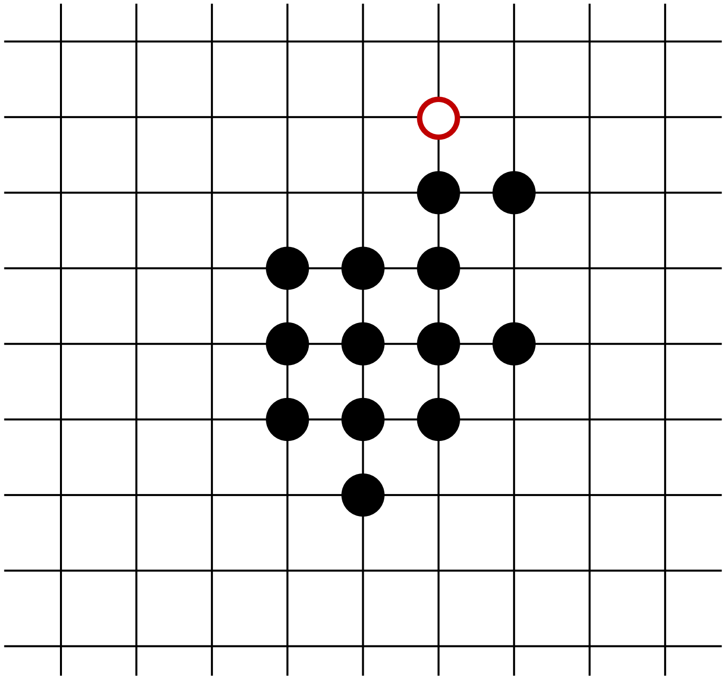
Stronger result

Aggregate grows with positive speed in all directions.

- ❖ aggregate + surrounded areas ⊃ ball of radius ct

Open problem:
What happens at low density?

Case $\mu = 1$



Equivalent to first passage percolation (FPP)

- Aggregate occupies neighbors at rate 1
- "Bulky" behavior instead of fractal-like
- Shape theorem [Richardson]

$$\frac{\text{Aggregate at time } t}{t} \to \text{a deterministic set}$$

For $\mu < 1$

- Each time aggregate tries to occupy a vertex, with probability $1 - \mu$ a hole is created
- Holes move as exclusion process

Our result for FPP in Hostile Env.

Theorem [Sidoravicius, S.]
Existence of a phase of strong survival

For any speed of **Type 2** < 1, there exists $p_0 \in (0,1)$ such that for all density of **Type 2** $< p_0$ we obtain that $\mathbb{P}(\mathbf{Type\ 1}\ \text{survives}) > 0$

Stronger result

Type-1 cluster at time t + surrounded areas ⊃ ball of radius ct

Corollary: MDLA has positive speed of growth

Coexistence

Holds for a simple variant, where passage times are deterministically set to 1 for **Type 1** and to λ for **Type 2**