Exact Bayesian Inference (for Big Data)

Single- and Multi- Core Approaches

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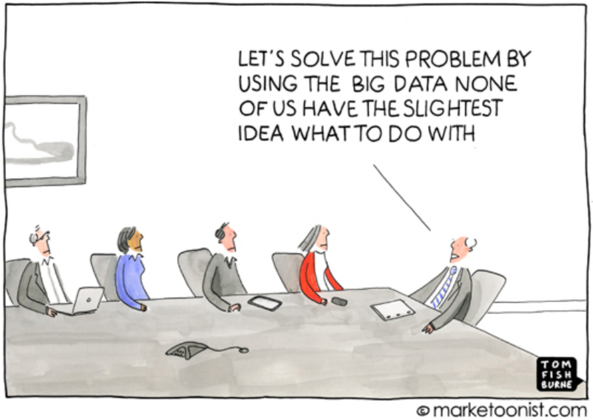
Warwick Statistics

Cartoon


LET'S SOLVE THIS PROBLEM BY USING THE BIG DATA NONE OF US HAVE THE SLIGHTEST IDEA WHAT TO DO WITH
TOM FISH BURNE
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Problem

- Big Data challenge?
  - Algorithmic 'Scalability'
- Target of interest:

$$\pi(x) \propto \prod_{i=0}^{N} f_i(x).$$

- Want to use [MCMC].
- Approaches:
  - Single-Core
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  - Problem: Metropolis move from $\theta \to \phi$ is accepted w.p.,

$$\min\left\{1, \frac{\pi(\phi)}{\pi(\theta)}\right\}$$

  - Goal: Scalability of iterative cost.
  - Lots of work!: Pseudo-Marginal; Stochastic gradient schemes. . .
- *[1] The Scalable Langevin Exact Algorithm: Bayesian Inference for Big Data* available at: https://arxiv.org/abs/1609.03436

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    1. Break data into S 'shards' (of size N/S)
    2. Separate inferences [MCMC]
    3. 'Recombine' on 'mother-core'
  - Problem: Recombining – How do you do it?
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0 - Retrospective Trust / Tricks

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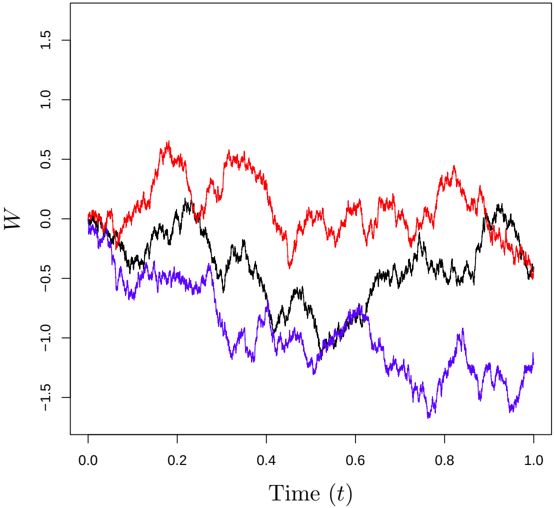

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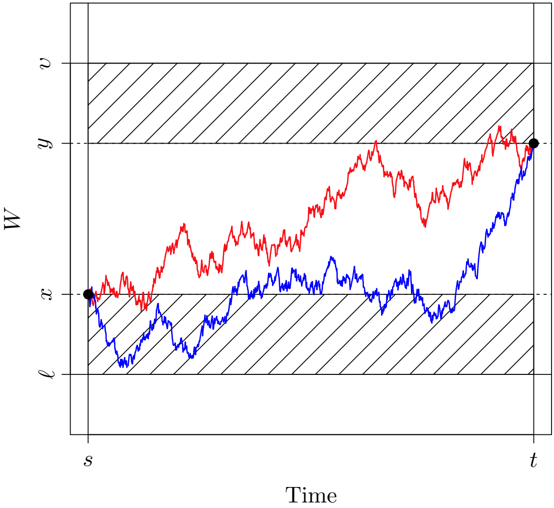

Path-space Rejection Sampling

Path-space Rejection Sampling:

- We want $X \sim \mathbb{Q}$ where:

$$\mathbb{Q} : \mathrm{d}X_t = \alpha(X_t)\,\mathrm{d}t + \Lambda^{1/2}\,\mathrm{d}B_t, \quad X_0 = x \in \mathbb{R}^d, t \in [0, T]$$

- Discretisation Free Approach!: Path-space Rejection Sampler (PRS) (see arXiv 1302.6964 for details)
  1. $X_T \sim h_T(X_0)$
  2. $X^{\mathrm{fin}} \sim \mathbb{P}|X_T$* (eg $\mathbb{W}$ or $\mathbb{O}$)
  3. (Accept / Reject)** / Assign Weight**

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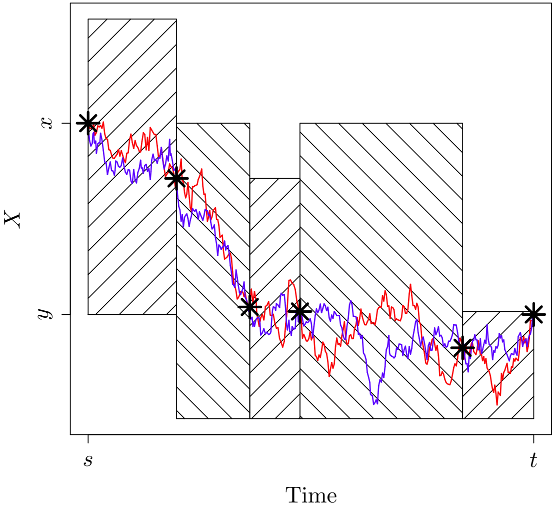

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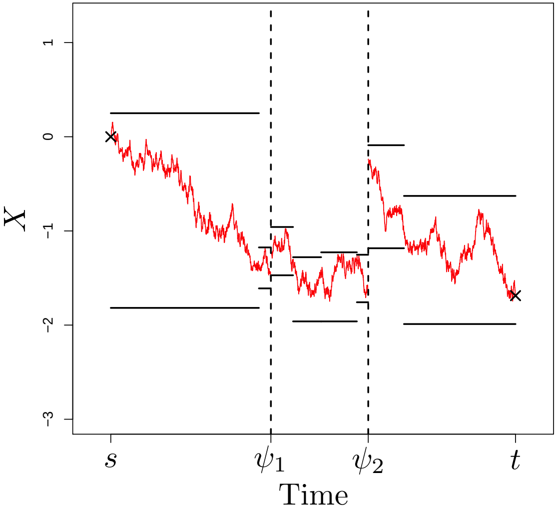

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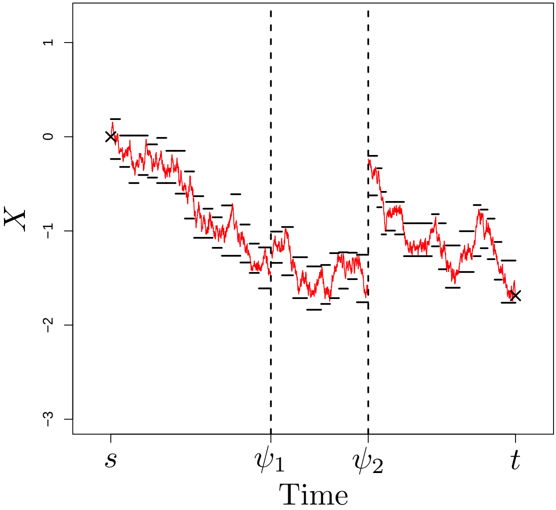

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- In $\mathbb{Q}$ set $\alpha(X_t) := \frac{1}{2}\Lambda\nabla \log \nu(X_t)$
- Invariant distribution ν
  - Direct statistical exploitation... $\nu \equiv \pi$ ($\mathbb{L}$)
  - Langevin + $\nu \equiv \pi$ + Discretisation + Correction $\Longrightarrow$ MALA
- PRS Class (however [1] $y := X_T \sim h \equiv \nu^{1/2} \ldots$)
  - $\lim_{T\to\infty} p_T(x,y) = \underbrace{w_T(x,y)\cdot\nu^{1/2}(y)}_{\propto h} \cdot \underbrace{P(X)}_{\in[0,1]} \to \nu$
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1 - Single Core: Quasi-Stationary Monte Carlo

Quasi-Stationary Monte Carlo

- Consider Brownian motion, killed at τ with intensity

$$\kappa(x) = \frac{\|\nabla \log \pi(x)\|^2 + \Delta \log \pi(x)}{2} - \ell \in \mathbb{R}_{\geq 0},$$

and the quasi-limiting distribution

$$\lim_{t \to \infty} \mathcal{L}(X_t | \tau > t).$$

- Under weak regularity conditions has quasi-stationary distribution π.
- Statistical Interpretation:
  - Big Data? → Subsampling
  - Implementation? → ScaLE

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1.2 - Subsampling

- QSMC $\equiv$ Simulating BM + inhomogeneous Poisson Process κ
- Evaluating κ is $\mathcal{O}(N)$.
- * If $\forall x$, $\kappa(x) \leq K$ (requires localisation argument) then:
  - Simulating $PP(\kappa(x)) \equiv$ Simulating $PP(K)$ and accepting w.p. $\kappa(X_t)/K$.
- We can make our algorithm worse (!) by choosing $\tilde{K} \geq K \ldots$
- Remark on coins
- Suppose $\exists A \sim \mathcal{A}$, $\tilde{\kappa_A}(\cdot) \in [0, \tilde{K}]$ such that $\mathbb{E}_{\mathcal{A}}[\tilde{\kappa_A}(x)/\tilde{K}] = \kappa(x)/\tilde{K}$ then:
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Subsampling

- Scalability $\equiv$ Finding $A \sim \mathcal{A}$ and $\tilde{\kappa}_A(\cdot)$ which are $\mathcal{O}(1)$ (trivial), such that $\tilde{K}/K \geq 1$ scales well...
- Intuition is the diffusion drift is a sum:

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1.3 - Single-Core: ScaLE

ScaLE

- Implementational Problem: Trajectory death!
- First Approach: Scalable Langevin Exact Algorithm (ScaLE)
  - Continuous time multi-level splitting / Importance sampling QSMC + SMC + Resampling

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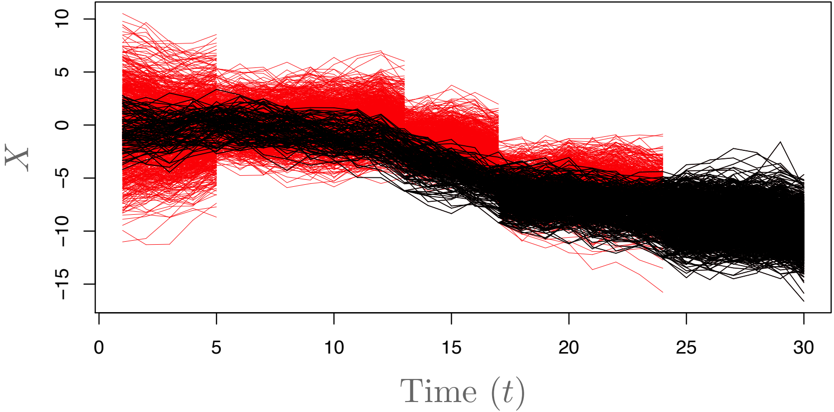
X
10
5
0
−5
−10
−15
0
5
10
15
20
25
30
Time (t)

1.4 - Summary

Summary

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  - QSMC: ‘Exact’ Bayesian Inference
  - No intrinsic cost for exactness.
  - ScaLE’s well!
- Missing Bits. . . :
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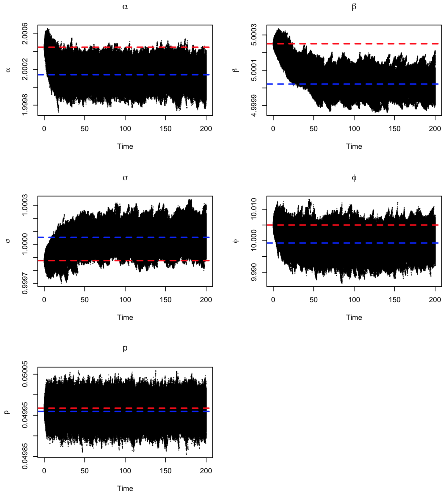
2^{27} dataset, contaminated regression model



2 - Multi-Core: Bayesian Fusion

Problem

- Recall Target:

$$\pi(x) \propto \prod_{c=1}^{C} f_c(x).$$

- C - Number of cores / experts / 'views' ...; f_c - Sub-posterior.
- Simple Approach... [Think (A)BC]
  1. Simulate $X^{(1)} \sim f_1, X^{(2)} \sim f_2, \ldots, X^{(C)} \sim f_C$.
  2. Accept if $X^{(1)} = X^{(2)} = \ldots = X^{(C)}$, else go to 1/.
  3. Return $X := X^{(1)}$ $(\sim \prod_{i=0}^{C} f_i \propto \pi)$.
- Recall Langevin: If $X_0 \sim \nu$, then $\forall t > 0, X_t \sim \nu$:
  - $\mathbb{L}_1, \ldots, \mathbb{L}_C, \mathbb{D}\mathbb{L}_1, \ldots, \mathbb{D}\mathbb{L}_C \ldots$

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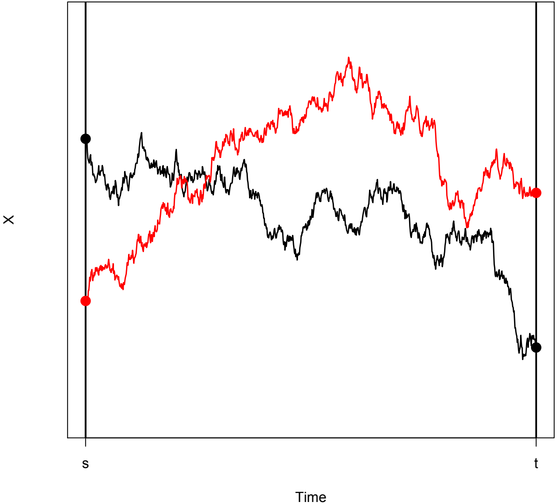
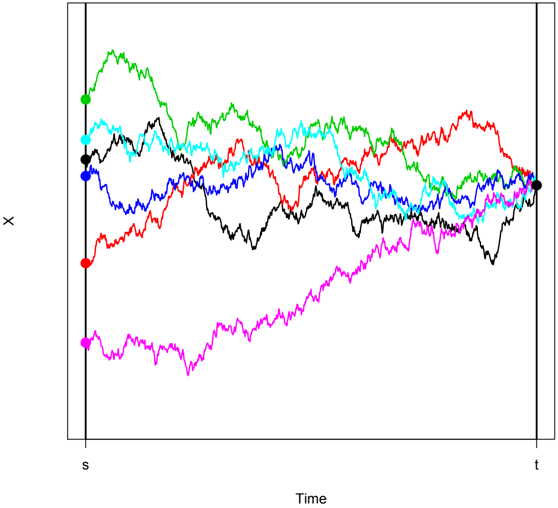
Fusion Idea



Fusion Actual



Some Details

- Fusion Measure ($\mathfrak{X} \in \mathbf{\Omega_0}$)

$$\mathrm{d}\mathbb{F}(\mathfrak{X}) \propto \mathrm{d}\left(\times_{c=1}^{C} \mathbb{DL}_c^{\boldsymbol{x}_0^{(c)}, \boldsymbol{y}_T}\right)(\mathfrak{X}) \cdot \prod_{c=1}^{C}\left[f_c^2\left(\boldsymbol{x}_0^{(c)}\right) p_{T,c}^{\mathrm{dl}}\left(\boldsymbol{y}_T \,\middle|\, \boldsymbol{x}_0^{(c)}\right) \cdot \frac{1}{f_c(\boldsymbol{y}_T)}\right],$$

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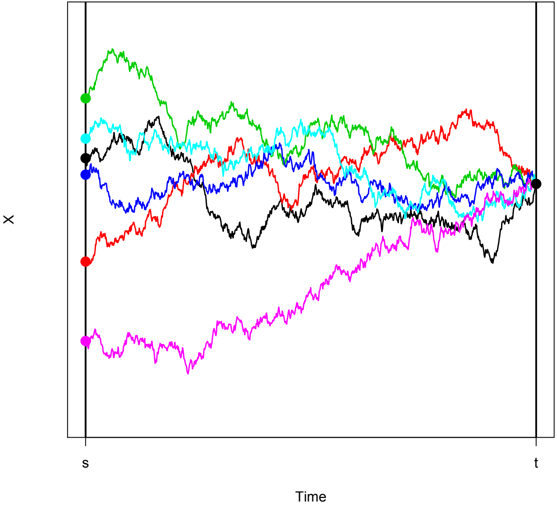

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- ‘Standard’ Multi-Core Problem ‘≡’ $\mathfrak{X} \sim \mathbb{F}$ (with practical constraints)
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  - Need RS for $h_T^{\text{bm}}(\cdot,\cdot)$ end point.
  - Accept with probability

$$P(\mathfrak{X}) := \exp\left[-\sum_{c=1}^{C}\int_0^T \kappa_c(\boldsymbol{X}_t^{(c)})\,\mathrm{d}t\right] \in [0,1]$$

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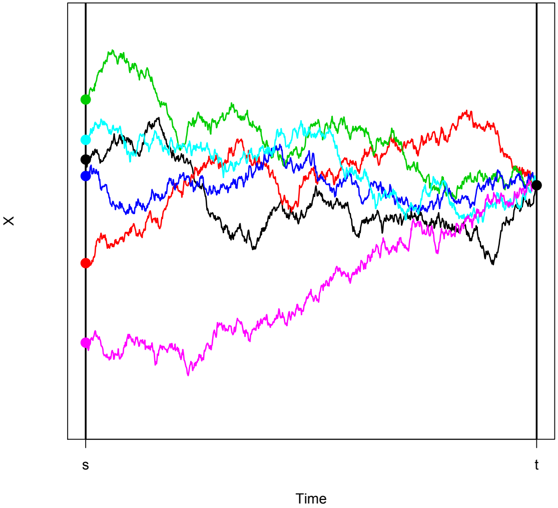
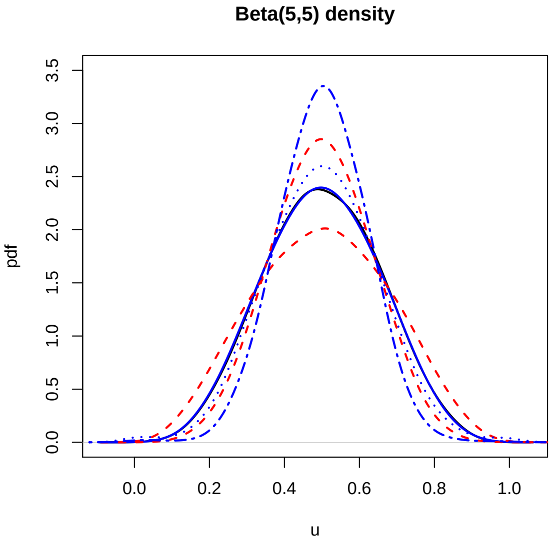


Example



Questions?