Atmospheric Modelling

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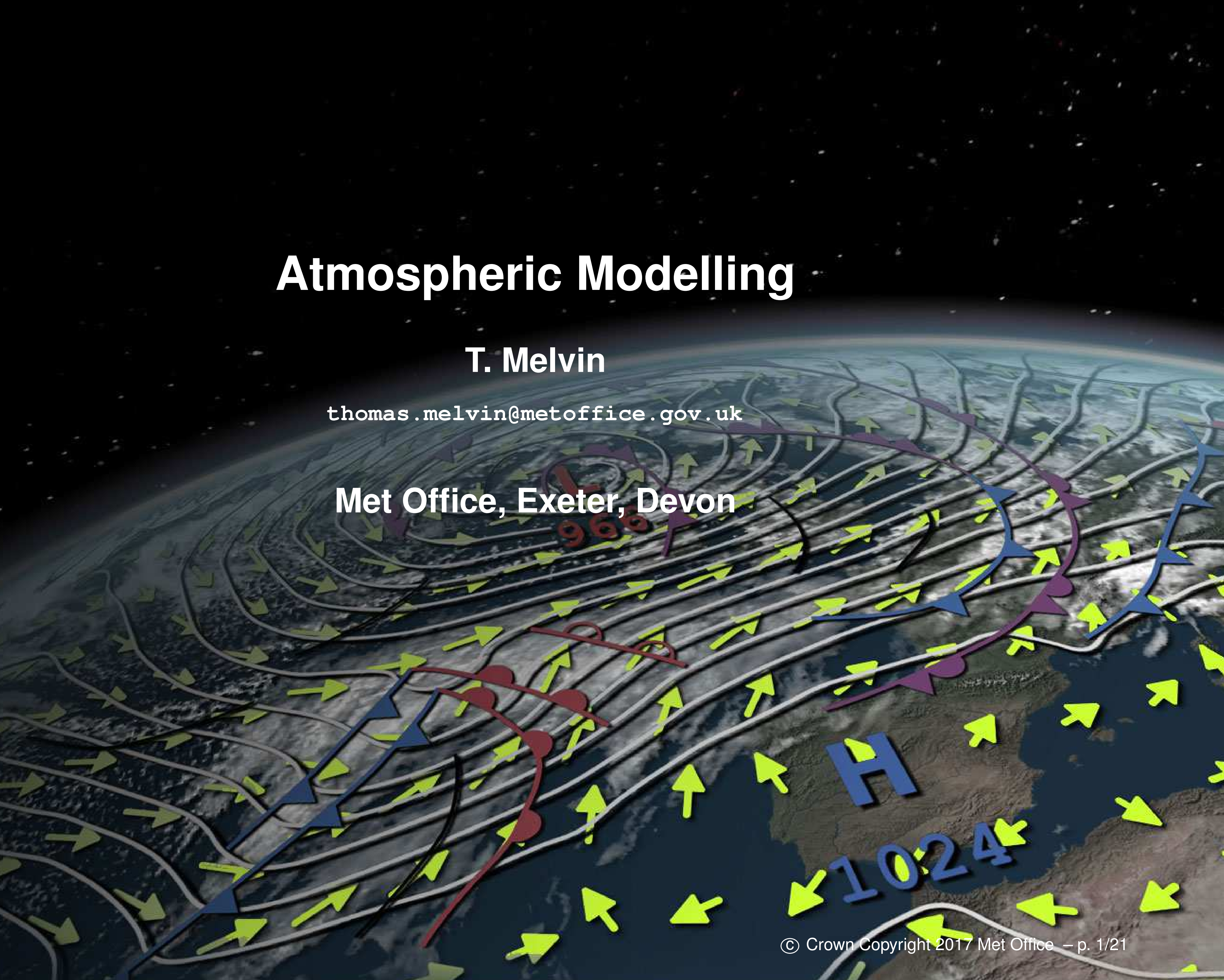
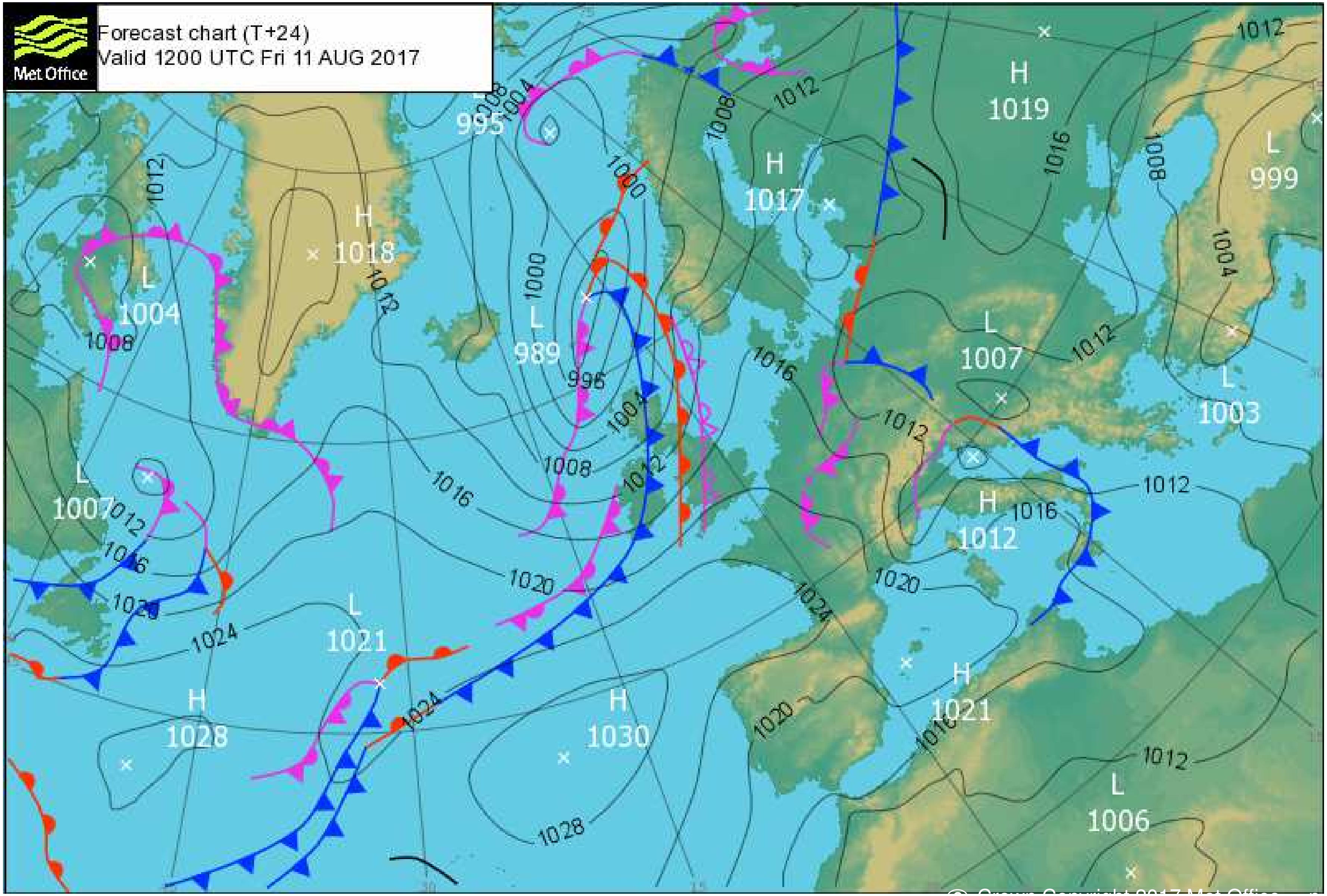
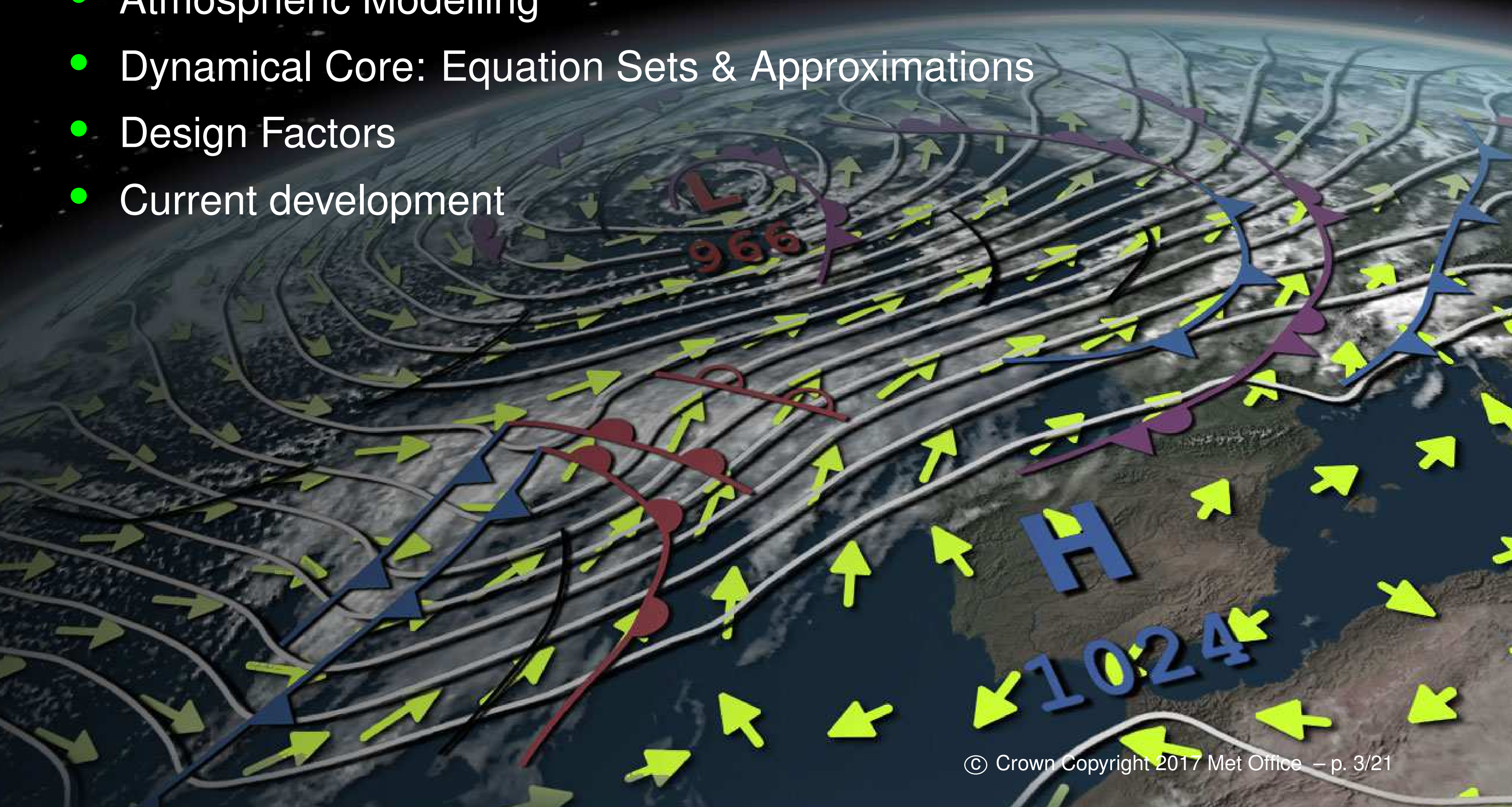


Todays Weather

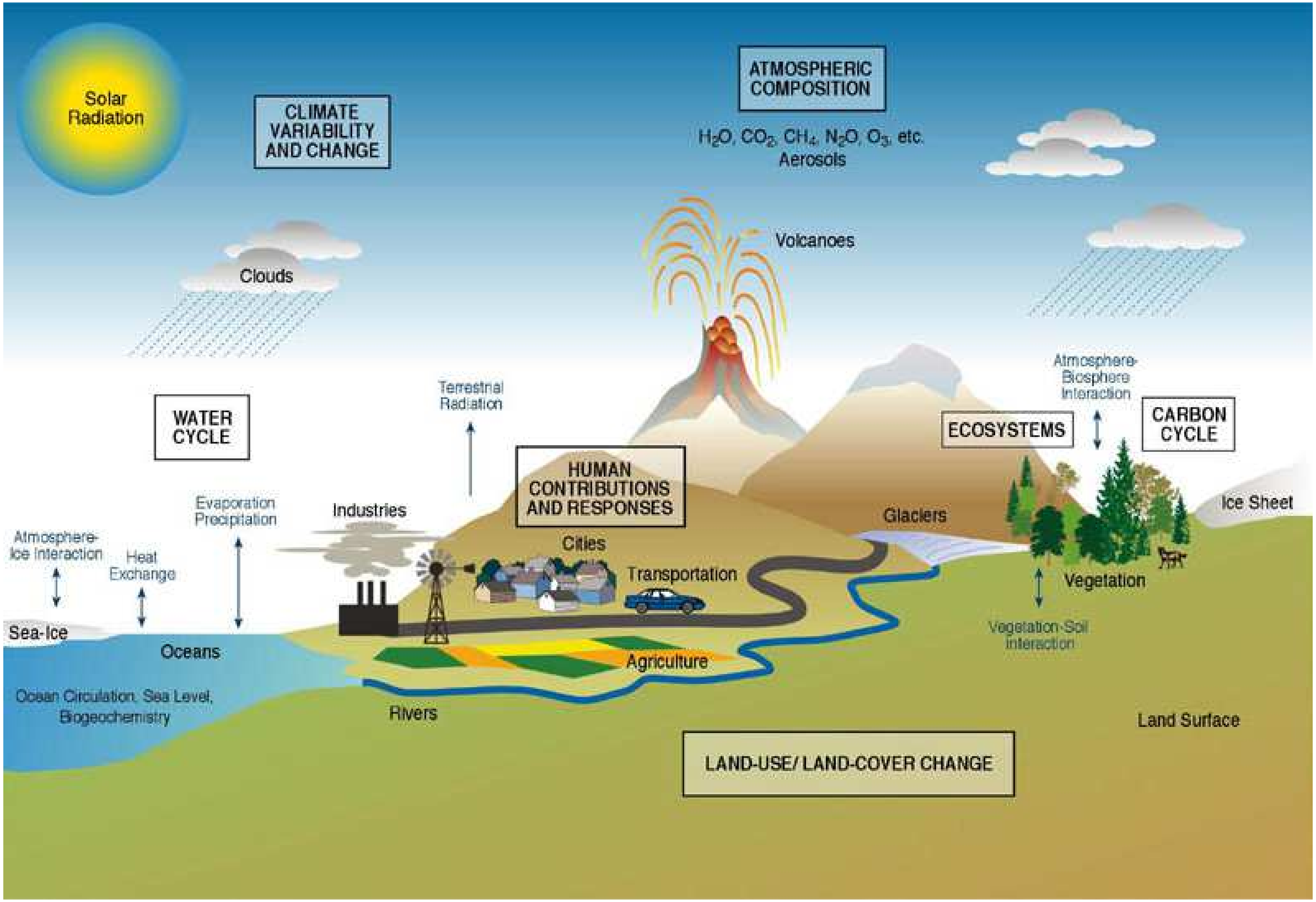


Overview

- Weather and Climate modelling: System Complexity
- Met Office approach to modelling
- Atmospheric Modelling
- Dynamical Core: Equation Sets & Approximations
- Design Factors
- Current development

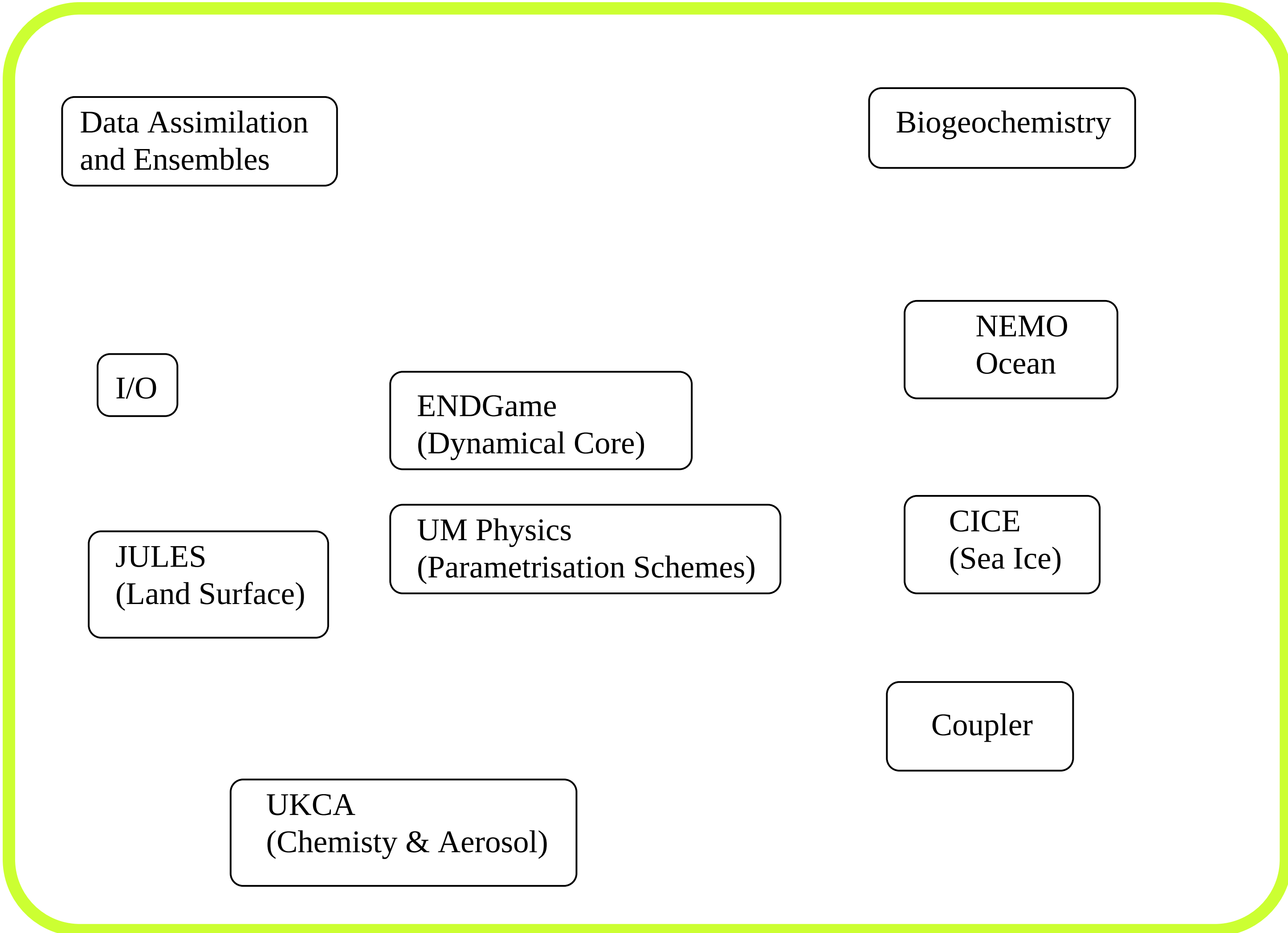


System Complexity: Physical



from: Aspen Global Change Institute.

System Complexity: Model



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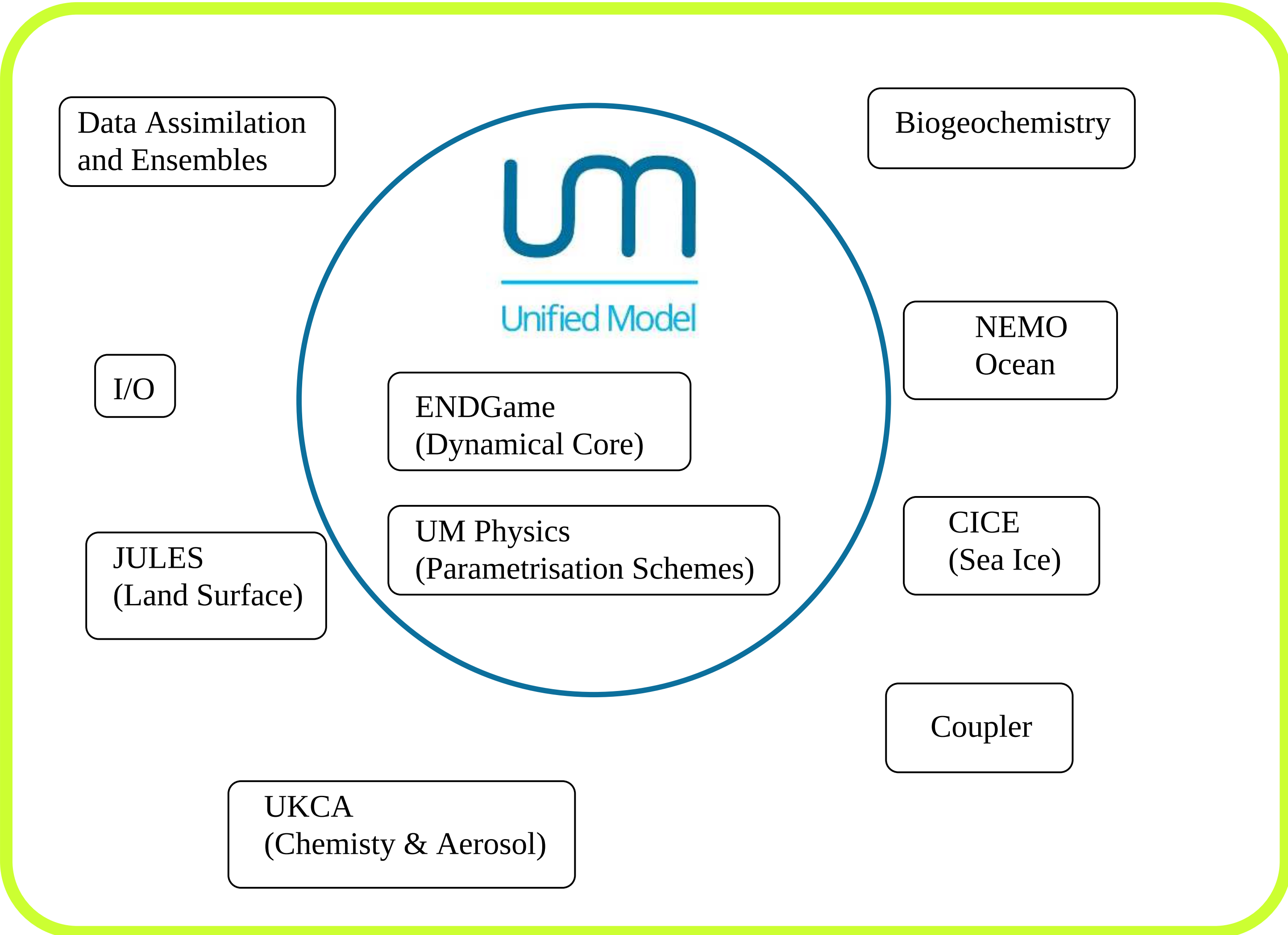


Met Office Approach: Unified Model (UM)

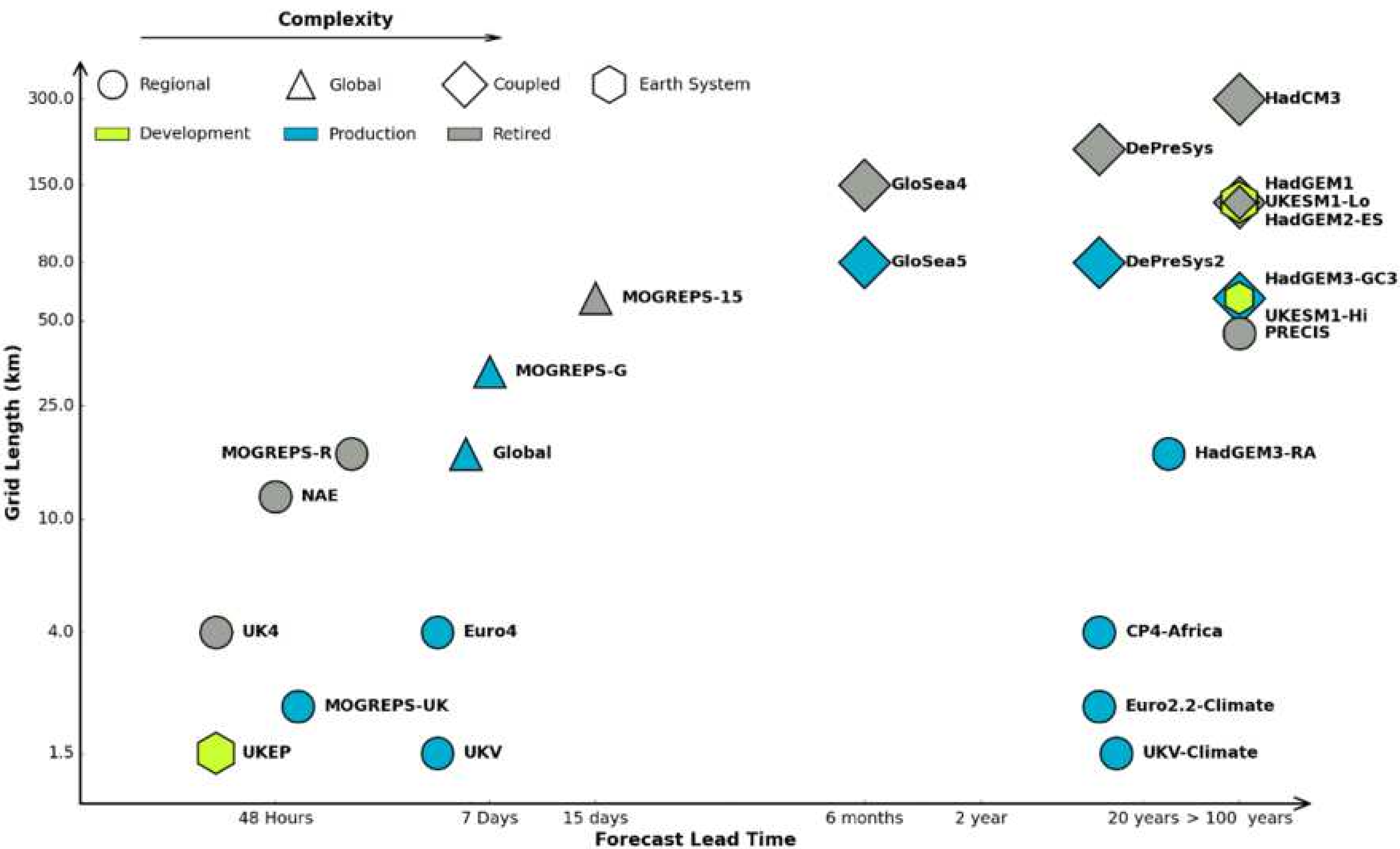
Single Model (Single numerics, Single source code) for all time and space scales

Climate Modelling: up to 100's Km for 1000's Years

Weather Forecasts: 1-10 Km for 5 days

Process Studies: 10's m for 100's Seconds

Met Office Approach: Unified Model (UM)



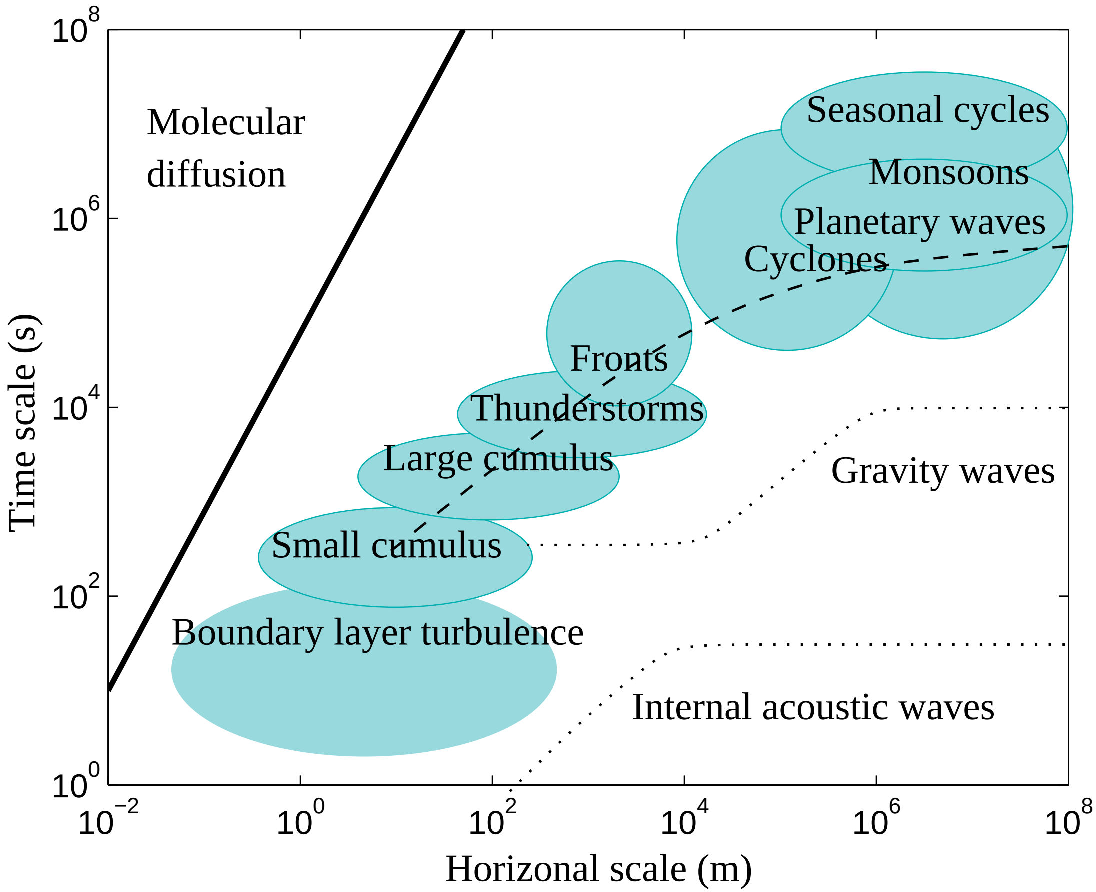
Forecast Constraints

Need to produce a forecast in a timely manner:

- Produce a forecast out to 7 days
- Global 10 km model, 70 vertical levels
- 4 Minute timestep $\Longrightarrow$ 2520 timesteps
- Resolution = 2560 x 1920 x 70 = 344 million grid points per variable
- Fixed 1 Hour time window (Including data assimilation, model run and i/o)

Algortihmic and code efficiency is critical

All Scales Problem



Phenomena occur across all time and space scales: No spectral gap

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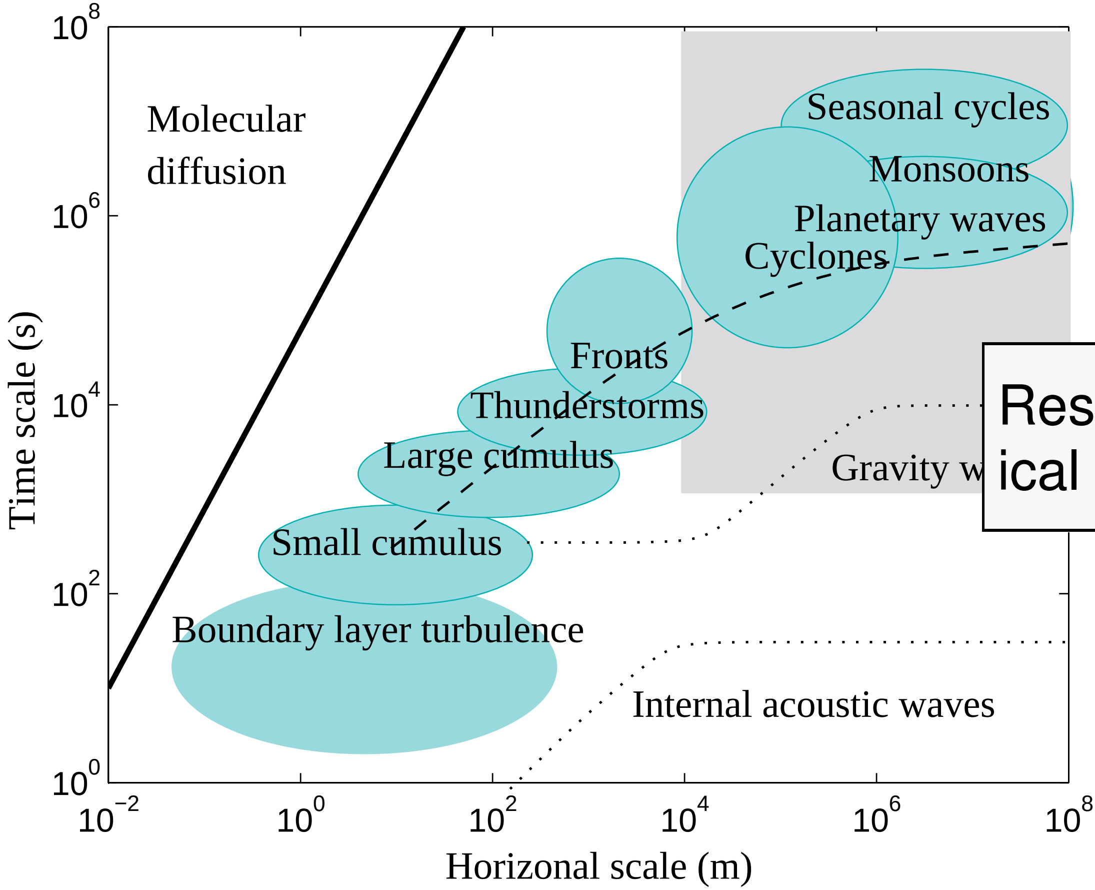
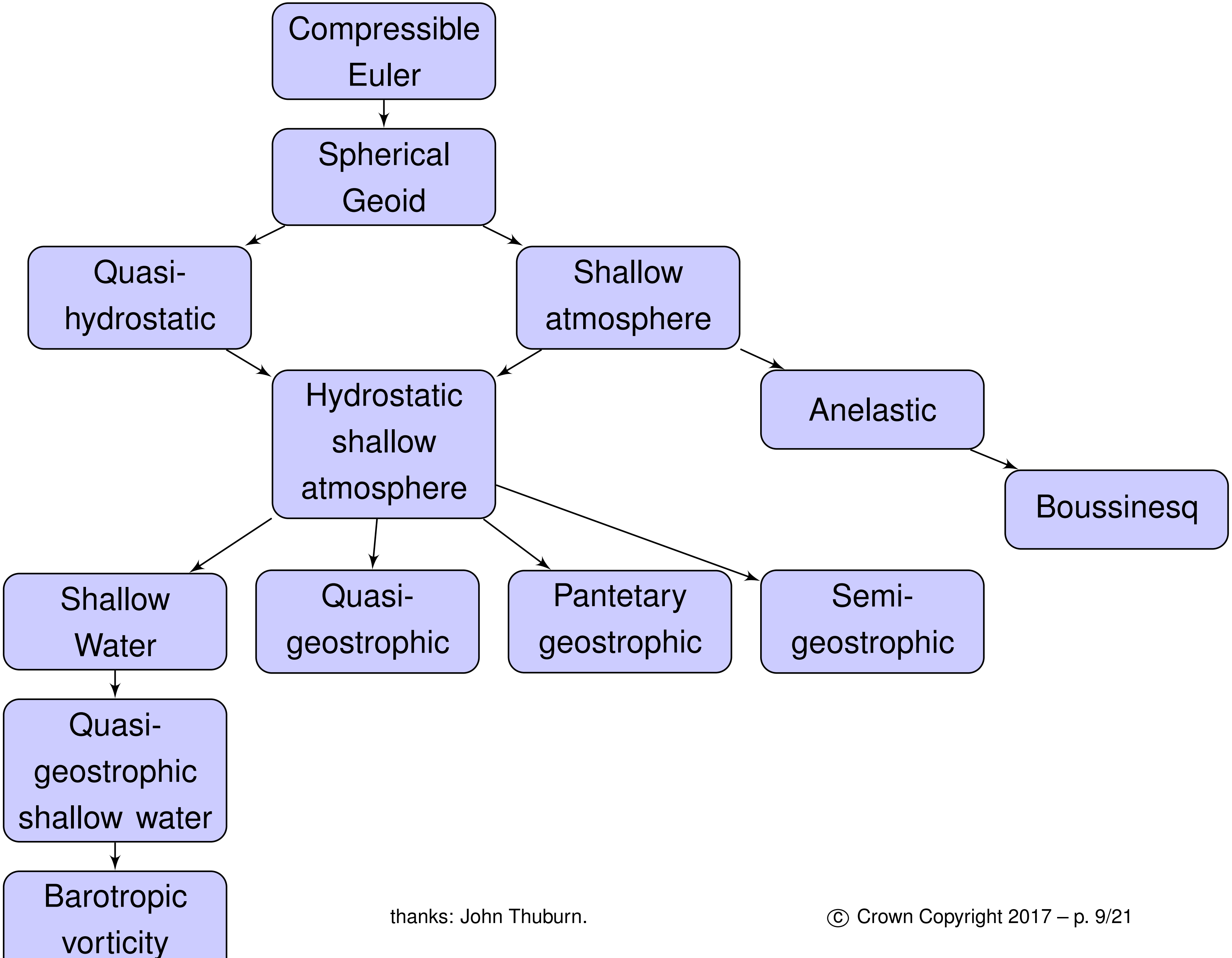


Resolution of a typical model

Phenomena occur across all time and space scales: No spectral gap

Dynamical Modelling: Equation Sets



thanks: John Thuburn.

Dynamical Core Modelling: 3D equations

Deep Atmosphere, nonhydrostatic equations: In spherical coordinates, this is the set that the Unified model uses. Only the spherical geoid approximation has been made.

$$\frac{D_r u}{Dt} - \frac{uv\tan\phi}{r} - 2\Omega\sin\phi v + \frac{c_{pd}\theta}{r\cos\phi}\frac{\partial\Pi}{\partial\lambda} = -\left(\frac{uw}{r} + 2\Omega\cos\phi w\right) + S^u$$

$$\frac{D_r v}{Dt} - \frac{u^2\tan\phi}{r} + 2\Omega\sin\phi u + \frac{c_{pd}\theta}{r\cos\phi}\frac{\partial\Pi}{\partial\phi} = -\left(\frac{vw}{r}\right) + S^v$$

$$\frac{D_r w}{Dt} + c_{pd}\theta\frac{\partial\Pi}{\partial r} + \frac{\partial\Phi}{\partial r} = \frac{(u^2+v^2)}{r} + 2\Omega\cos\phi u + S^w$$

$$\frac{D_r}{Dt}\left(\rho r^2\cos\phi\right) + \rho r^2\cos\phi\left(\frac{\partial}{\partial\lambda}\left[\frac{u}{r\cos\phi}\right] + \frac{\partial}{\partial\phi}\left[\frac{v}{r}\right] + \frac{\partial w}{\partial r}\right) = 0$$

$$\frac{D_r\theta}{Dt} = S^\theta$$

Dynamical Core Modelling: 3D equations

Shallow Atmosphere: Assume the atmosphere is a shallow shell. Replace height factors r with earths radius a and neglect certain parts of the coriolis terms. Valid when $(r-a) << a$.

$$\frac{D_a u}{Dt} - \frac{uv\tan\phi}{a} - 2\Omega\sin\phi v + \frac{c_{pd}\theta}{a\cos\phi}\frac{\partial\Pi}{\partial\lambda} = -\left(\frac{uw}{r} + 2\Omega\cos\phi w\right) + S^u$$

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Dynamical Core Modelling: 3D equations

Quasi-Hydrostatic: Neglect the vertical acceleration term Dw/Dt. This is a good approximation for horizontal scales greater than about $10km$. Filters out vertical acoustic waves

$$\frac{D_r u}{Dt} - \frac{uv\tan\phi}{r} - 2\Omega\sin\phi v + \frac{c_{pd}\theta}{r\cos\phi}\frac{\partial\Pi}{\partial\lambda} = -\left(\frac{uw}{r} + 2\Omega\cos\phi w\right) + S^u$$

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Dynamical Core Modelling: 3D equations

Hydrostatic Shallow Atmosphere: Make the shallow atmosphere and hydrostatic approximations: Hydrostatic primitive equations. Historically popular for climate modelling

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Dynamical Core Modelling: Scaling

- Vertical momentum equation scalings:

w-equation	$\frac{Dw}{Dt}$	$-\frac{u^2+v^2}{r}$	$-2\Omega u\cos\phi$	$\frac{\partial\Phi}{\partial r}$	$\frac{1}{\rho}\frac{\partial p}{\partial r}$
Scales	UW/L	U^2/a	f_0U	g	$P_0/\rho H$
Values (m/s)	10^{-7}	10^{-5}	10^{-3}	10	10

- Horizontal momentum equation scalings:

u-equation	$\frac{Du}{Dt}$	$-\frac{uv\tan\phi}{r}$	$\frac{uw}{r}$	$-2\Omega v\sin\phi$	$2\Omega w\cos\phi$	$\frac{1}{\rho r\cos\phi}\frac{\partial p}{\partial\lambda}$
v-equation	$\frac{Dv}{Dt}$	$-\frac{u^2\tan\phi}{r}$	$\frac{vw}{r}$	$-2\Omega u\sin\phi$		$\frac{1}{\rho r}\frac{\partial p}{\partial\phi}$
Scales	U^2/L	U^2/a	UW/a	f_0U	f_0W	$\delta P/\rho L$
Values (m/s)	10^{-4}	10^{-5}	10^{-8}	10^{-3}	10^{-6}	10^{-3}

Dynamical Core Modelling: Geostrophic approximations

Geostrophic: Coriolis force (f) balances the pressure gradient (∇p)

$$\mathbf{v}_G = \frac{1}{\rho f}\mathbf{k} \times \nabla_h p$$

Valid for small Rossby numbers $R_0 \equiv \frac{V}{fL} << 1$

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Quasi-geostrophic: Assume Cartesian geometry with a constant Coriolis force f_0 in geostrophic wind and include ageostrophic compontent: $\mathbf{v} = \mathbf{v}_a + \mathbf{v}_G, \quad \mathbf{v}_a << \mathbf{v}_G$. Valid for flows with $L << a$ and small perturbations around reference depth

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Plantary geostrophic: Retain the spherical geometry, valid for $L \approx a$

Semi-geostrophic: Assume Cartesian geometry. Use the full wind in the advection terms. Only drop the advection of the ageostropic component: $D\mathbf{v}_a/Dt = 0$

Dynamical Core Modelling: Shallow water approximations

Shallow Water Equations: Neglect variations with height, assume the fluid is a single layer and the wavelength λ of surface waves is much smaller than fluid depth $\lambda << d$.

$$\frac{D\mathbf{u}}{Dt} = -2\Omega \times \mathbf{u} - \nabla\left(\Phi + \Phi_0\right) = 0,$$

$$\frac{\partial \Phi}{\partial t} + \nabla.\left(\Phi \mathbf{u}\right) = 0.$$

useful for testing numerical approximations in a simplified environment.

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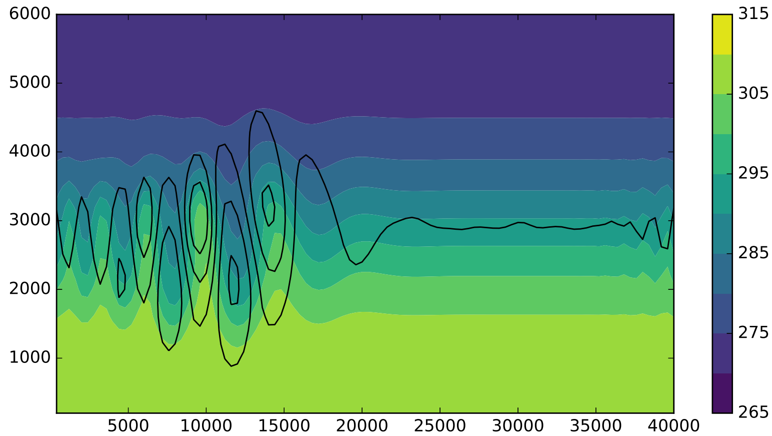
useful for testing numerical approximations in a simplified environment.
Barotropic vorticity: Describes incompressible 2D flow,

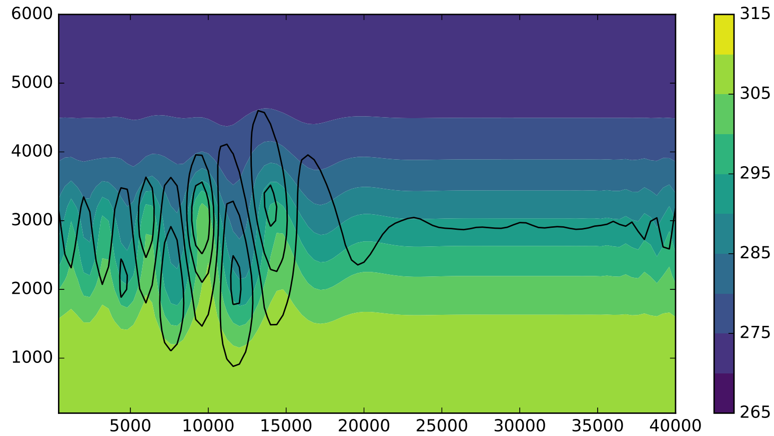
$$\frac{D\xi}{Dt} = 0,$$

$$\mathbf{u} = \nabla^{\perp}\psi, \qquad \nabla^2\psi = \xi.$$

Dynamical Core Modelling: Effects of approximations

Baroclinic wave test case. Standard test for development of mid latitude weather systems





Unapproximated model

Hydrostatic approximation

Dynamical Modelling: Design Factors

Staniforth & Thuburn (QJRMS **138**, 2012) identified ten
Essential and desirable properties of a dynamical core

1 Mass conservation

2 Accurate representation of balance and adjustment

3 Absence of, or well controlled, computational modes
Requires, at least, #velocity = 2x#pressure points

Dynamical Modelling: Design Factors

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4 Geopotential and pressure gradient should not produce unphysical vorticity

$$\nabla \times (\nabla p) = 0$$

5 Energy conserving pressure terms

$$\mathbf{u}.\nabla p + p\nabla.\mathbf{u} = \nabla.\left(\mathbf{u}p\right)$$

6 Energy conserving Coriolis terms

$$\mathbf{u}.\left(\boldsymbol{\Omega} \times \mathbf{u}\right) = 0$$

7 No spurious fast propagation of Rossby modes

8 Axial angular momentum should be conserved

These all relate to the mimetic (compatible) properties of the numerics

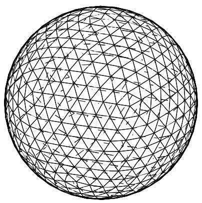
Dynamical Modelling: Design Factors

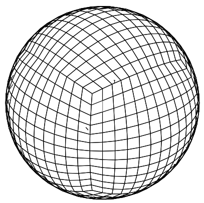
Staniforth & Thuburn (QJRMS **138**, 2012) identified ten
Essential and desirable properties of a dynamical core

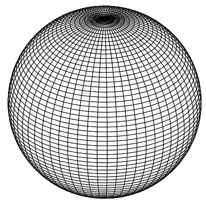
9 Accuracy at least approaching second order

10 Minimal grid imprinting

These are challenging for grids with special points
...generally require higher order schemes

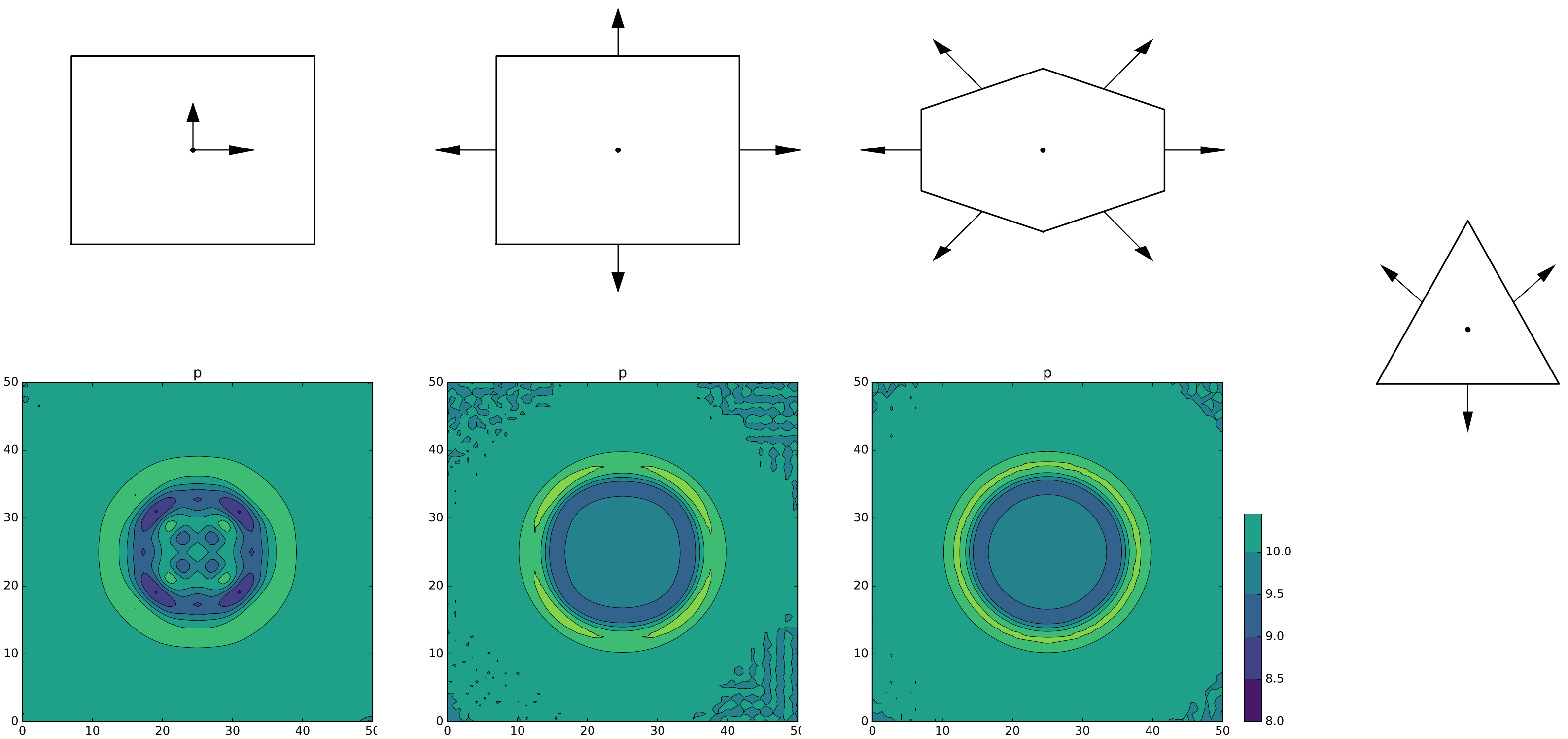






Representation of fast waves

Linear shallow water model in a Cartesian domain



Mixed finite element model

Developing a new model based suitable for future supercomputers

- Using mixed finite-element method
- Choose finite element function space to give discrete De-Rahm complex

$$
\begin{array}{ccccccc}
H_1 & & H_{curl} & & H_{div} & & L_2 \\
 & \nabla & & \nabla\times & & \nabla . & \\
\mathbb{W}_0 & \longrightarrow & \mathbb{W}_1 & \longrightarrow & \mathbb{W}_2 & \longrightarrow & \mathbb{W}_3
\end{array}
$$

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Q_{k+1} & & N_k & & RT_k & & Q_k^{DG}
\end{array}
$$

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- Accurate for arbitrary grids, (no orthogonality constraint)
- Flexibility to increase formal order of accuracy
- Builds in mimetic and conservation properties
- Generalises staggered grid finite-volume methods

Timestepping

Two main approaches used:

- Explicit
- Semi-Implicit

Timestepping

Explicit timestepping (e.g. Runge-Kutta) is simple and cheap per step but restricted by speed of fast (acoustic & inertia-gravity waves

- Explicit in the vertical: $U \approx 340m/s$, $\Delta z \approx 10m$ leads to $\Delta t < 1/4s$
- Only explicit in the horizontal: $U \approx 340m/s$, $\Delta x \approx 10Km$ leads to $\Delta t < 30s$
- Alternatively try to filter fast waves (hydrostatic, anelsatic approximations)

Timestepping

Implicit timstepping is more complex and expensive per step but much longer timestep can be taken

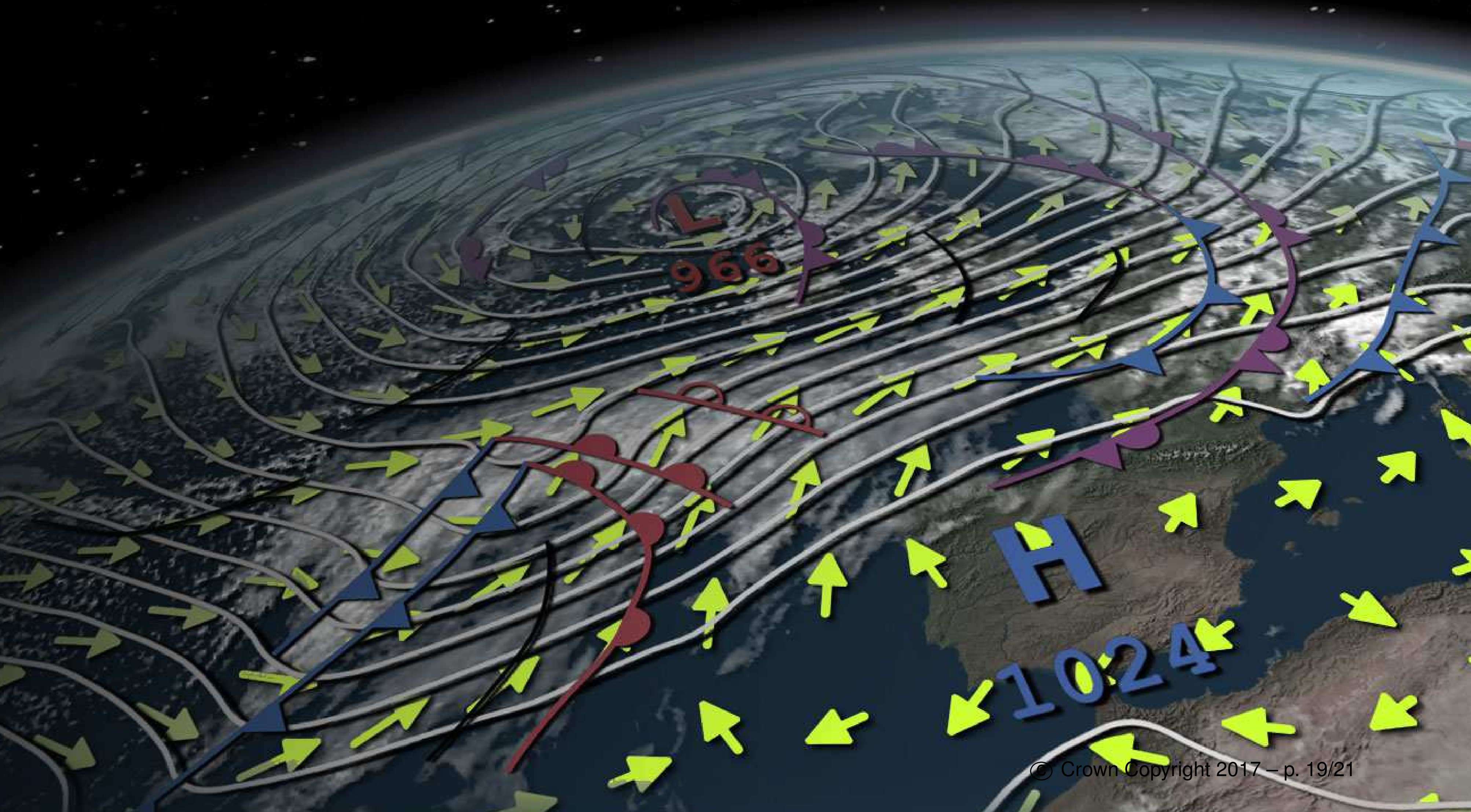
- UM uses ≈ 5 minutes for $\Delta x = 10Km$
- Forming full Jacobian for Newton method is expensive
- More common to use Quasi-Newton (semi-implicit) method

$$\begin{pmatrix} 1 & \tau\Delta t\nabla\Pi^* & 0 & \tau\Delta t\theta^*\nabla \\ \tau\Delta t\nabla\theta^* & 1 & 0 & 0 \\ \nabla.\rho^* & 0 & 1 & 0 \\ 0 & \frac{1}{\theta}^* & \frac{1}{\rho^*} & \frac{\gamma}{\Pi^*} \end{pmatrix} \begin{pmatrix} \mathbf{u}' \\ \theta' \\ \rho' \\ \Pi' \end{pmatrix} = R$$

- Only the terms for fast waves are retained
- Usually use Schur complement to reduce this to a single (Helmholtz) equation

$$H(\Pi') \equiv \alpha_1\Pi' + \alpha_2\nabla.\left(\alpha_3\nabla\left[\alpha_4\Pi'\right]\right) = RHS$$

Any Questions?



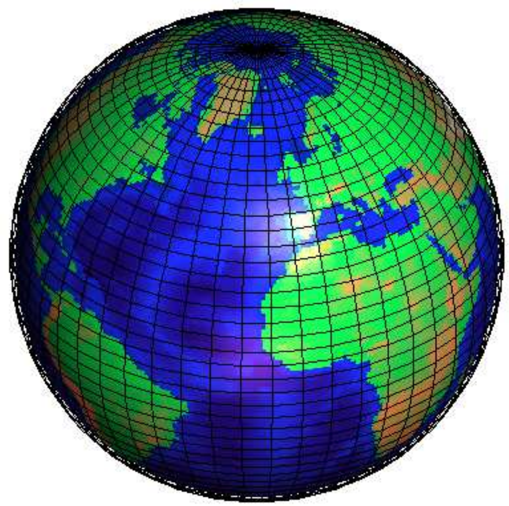
Atmospheric Model Splitting

Atmospheric model is split into two main parts

- Dynamical Core: Models all motions that are resolved on the mesh
- Physical Parameterisations: Models subgrid processes that are not resolved

Dynamical Core:

- Solves equations of motion
- Transport of fields
- Resolves large scale balances



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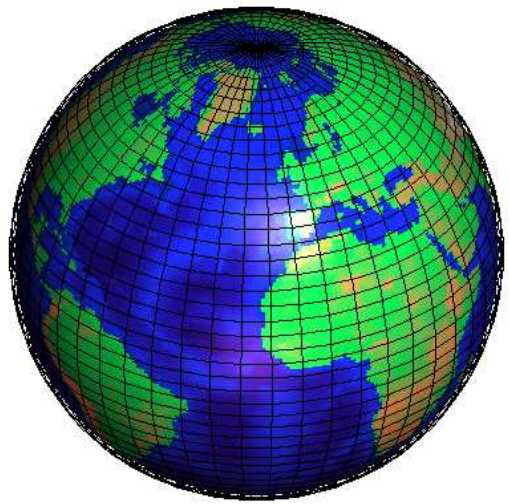
- Microphysics
- Radiation
- Boundary layer
- Gravity wave drag



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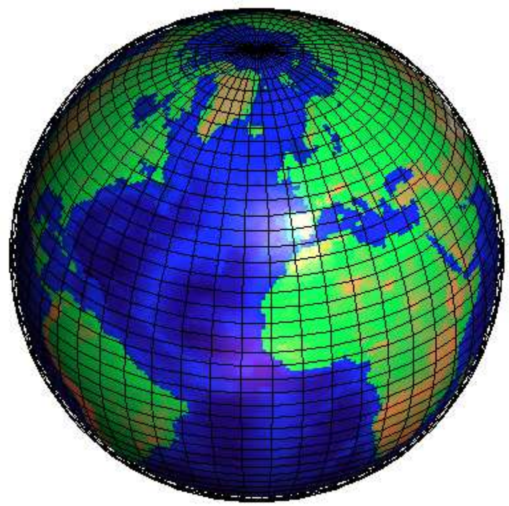

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