



$$\sum_{i=1}^7 x_i^2 = 1 \quad S^6$$

\Rightarrow Almost complex structure J on S^6

Non-integrable.

1954 Hirzebruch,

Sheaf Cohomology

X complex manifold $\dim_{\mathbb{C}} = n$

$\rightarrow H^q(X, \Omega^p)$ $p, q = 0, \dots, n$

X algebraic Hodge Theory $H^{p,q} = h^{p,q}$

$\sum_{p+q=k} (-1)^{p+q} h^{p,q} = \text{Euler Number}$

$\sum_{p+q=k} (-1)^{p+q} h^{p,q} = \text{Signature}(X)$

Hodge

Kähler metrics

Mannoury.



Atiyah-Singer Index Theorem ~ 1960

Hirzebruch-Riemann-Roch HRR.

$S^1 \times S^3$

S^3

$su_2 \times su_2$



S^2

$U_1 \times U_2$



4. AS methods

give results for almost complex manifolds

no need for integrability

Need new method to distinguish

Integrable from non-integrable

Non-abelian index theory ✓

Group symmetry

After hard work succeed with S^6

99% ✓