Results on anisotropic inverse boundary value problems

Suppose that $\tilde{M} \subset \mathbb{R} \times M$ and $\tilde{g} = a(t,x)(\pm dt^2 + g(x))$ where the conformal factor a is strictly positive. Consider the Cauchy data

$$\mathcal{C} := \{(u|_{\partial \tilde{M}}, \partial_\nu u|_{\partial \tilde{M}});\ \Delta_{\tilde{g}} u = 0 \text{ in } \tilde{M}\}.$$

1. If $a = 1$ and $\tilde{M} = \mathbb{R} \times M$, then $\mathcal{C}$ determines (M,g) uniquely.
   - Hyperbolic case [Belishev-Kurylev'92].
   - Elliptic case [Dos Santos Ferreira-Kurylev-Lassas-Salo'16] via a reduction to the hyperbolic case.
   - Time-domain **Maxwell** [Kurylev-Lassas'06].
2. If (M,g) is simple, then $\mathcal{C}$ and (M,g) determine a uniquely.
   - Elliptic case [Dos Santos Ferreira-Kenig-Salo-Uhlmann'09].
   - Hyperbolic case [Kian-Oksanen'16].

Outside the above framework: hyperbolic case with analytic t dependence [Eskin'07,'16].