High Frequency: Open Problem Session

Simon Chandler-Wilde, Andrea Moiola, Euan Spence, Valery Smyshlyaev

Mathematical and Computational Aspects of Maxwell's Equations
Durham, July 12th 2016

Open Problem 1: Understanding HF Solution Behaviour

For acoustic, EM scattering problems for general bounded obstacles in 2D and 3D can we obtain, **at least for boundary traces** for BIE formulations, representations of the form

$$v(x,k) = v_0(x,k) + \sum_{j=1}^{J} v_j(x,k)\,\mathrm{e}^{\mathrm{i}k\phi_j(x)}, \qquad (1)$$

with v_0 and ϕ_j known and with the envelopes $v_j(x,k)$ smooth for large k?

And can we get rigorous k-explicit bounds on the derivatives of $v_j(x,k)$?

Open Problem 1: Understanding HF Solution Behaviour

For acoustic, EM scattering problems for general bounded obstacles in 2D and 3D can we obtain, **at least for boundary traces** for BIE formulations, representations of the form

$$v(x,k) = v_0(x,k) + \sum_{j=1}^{J} v_j(x,k)\, \mathrm{e}^{\mathrm{i}k\phi_j(x)}, \qquad (1)$$

with v_0 and ϕ_j known and with the envelopes $v_j(x,k)$ smooth for large k?

And can we get rigorous k-explicit bounds on the derivatives of $v_j(x,k)$?

Plausible that thus is possible because **high frequency asymptotic approximations - e.g. GTD - have exactly thus form, moreover with v_j known**!

Open Problem 1: Understanding HF Solution Behaviour

For acoustic, EM scattering problems for general bounded obstacles in 2D and 3D can we obtain, **at least for boundary traces** for BIE formulations, representations of the form

$$v(x,k) = v_0(x,k) + \sum_{j=1}^{J} v_j(x,k)\, \mathrm{e}^{\mathrm{i}k\phi_j(x)}, \qquad (1)$$

with v_0 and ϕ_j known and with the envelopes $v_j(x,k)$ smooth for large k?

And can we get rigorous k-explicit bounds on the derivatives of $v_j(x,k)$?

Plausible that thus is possible because **high frequency asymptotic approximations - e.g. GTD - have exactly thus form, moreover with v_j known**!

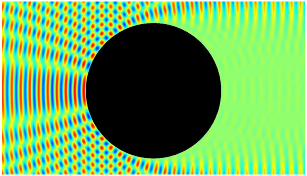
But hard in generality, **hard to write down GTD approximations uniform with respect to x, k and geometry and understand "smoothness" of v_j**. Some plausible next steps are ...

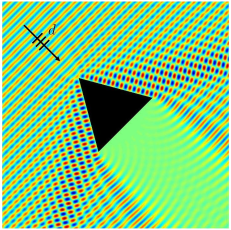
Open Problem 2: Rigorous HF bounds

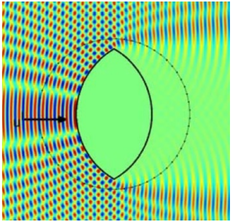
For the Dirichlet scattering problem we can show that

$$\frac{\partial u}{\partial n}(x,k) = v_0(x,k) + \sum_{j=1}^{J} v_j(x,k)\, \mathrm{e}^{\mathrm{i}k\phi_j(x)},$$

with rigorous k-explicit bounds on the unknowns $v_j(x,k)$ and their derivatives for the first two of the following problems, but not the 3rd (heuristic methods in Langdon et al. 2010).







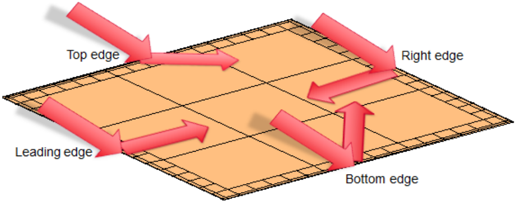
Well-known Melrose & Taylor results for C^∞ strictly convex (see Dominguez, Graham, Smyshlyaev 2007) for 1st, arguments based on Green's function for half-plane for convex polygon for 2nd, but 3rd ???

Open Problem 3: Understanding 3D HF Soln. Behaviour

For the Dirichlet scattering problem for a screen can one show that

$$\frac{\partial u}{\partial n}(x,k) \approx v_0(x,k) + \sum_{j=1}^{J} v_j(x,k)\,\mathrm{e}^{\mathrm{i}k\phi_j(x)},$$

with completely rigorous (or just heuristic) k-explicit bounds on the derivatives of the unknowns $v_j(x,k)$ and on the error in this approximation?

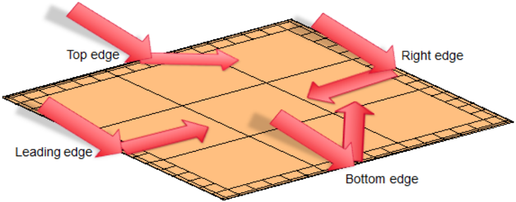


Open Problem 3: Understanding 3D HF Soln. Behaviour

For the Dirichlet scattering problem for a screen can one show that

$$\frac{\partial u}{\partial n}(x,k) \approx v_0(x,k) + \sum_{j=1}^{J} v_j(x,k)\, \mathrm{e}^{\mathrm{i}k\phi_j(x)},$$

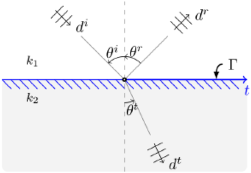
with completely rigorous (or just heuristic) k-explicit bounds on the derivatives of the unknowns $v_j(x,k)$ and on the error in this approximation?



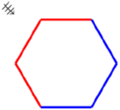
Other b.c.'s, EM scattering for PEC, convex polyhedron, ... ??

Open Problem 4: refraction at a plane interface!

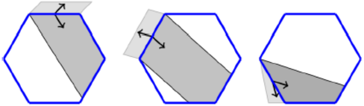
Rays refract according to Snell's law:



GO computed by beam tracing:



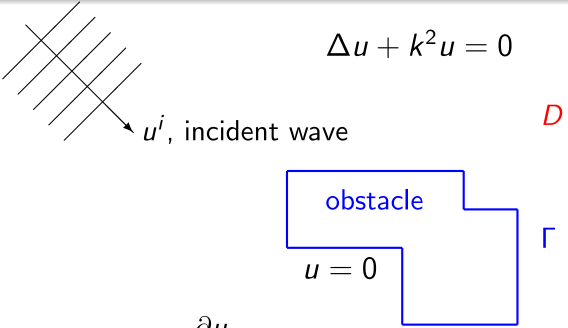
Incident wave



Primary beams from first reflection/refraction event

Plane wave reflection and refraction at a plane interface – the case $\mathrm{Im}\, k_1 > 0$ is needed to deal with beam tracing. The issue is that phase velocity and energy flow considerations can conflict.

Open Problem 5: Coercivity for the standard CFIE

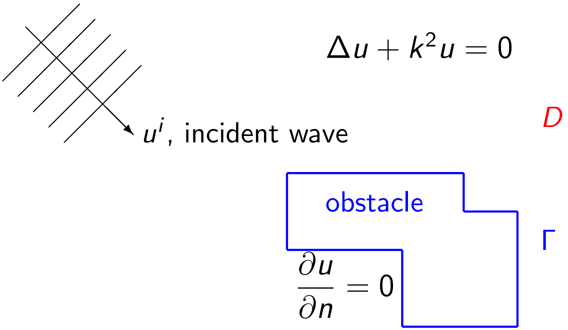


The standard CF BIE for $\dfrac{\partial u}{\partial n}$ is – Smyshlyaev talk –

$$\frac{1}{2}\frac{\partial u}{\partial \mathbf{n}}(\mathbf{x}) + \int_\Gamma \left(\frac{\partial \Phi(\mathbf{x},\mathbf{y})}{\partial \mathbf{n}(\mathbf{x})} - \mathrm{i}k\Phi(\mathbf{x},\mathbf{y}) \right) \frac{\partial u}{\partial \mathbf{n}}(\mathbf{y})\,\mathrm{d}s(\mathbf{y}) = f(\mathbf{x}), \quad \mathbf{x} \in \Gamma.$$

Spence, Kamotskii, Smyhlyaev (2015) have shown coercivity for smooth, strictly convex, but how to prove this more generally and/or without Morawetz multipliers? Numerical results (Betcke & Spence 2011) suggest that coercivity holds for all non-trapping.

Open Problem 6: Coercivity for the Neumann CFIE



Can one prove coercivity for the standard Burton & Miller CFIE, for the Neumann problem – regularised with S_0 or $S_{\mathrm{i}k}$ so as to map $L^2(\Gamma)$ to $L^2(\Gamma)$?

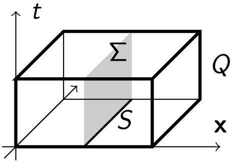
Bounbedir & Turc (2013) have proved this for a circle/sphere by eigenfunction expansions, but general strictly convex? Non-trapping? CFIE for EM scattering?

Open problem 7: trace regularity on space-time interfaces

Let $\Omega \in \mathbb{R}^n$ be Lipschitz/polytopic and bounded,
$Q = \Omega \times (0, T)$, c (piecewise) constant.
Consider inhomogeneous IBVP for 1^{st}-order wave equation:

$$\begin{cases} \nabla z + \dfrac{\partial \boldsymbol{\zeta}}{\partial t} = \boldsymbol{\Phi} & \text{in } Q, \\ \nabla \cdot \boldsymbol{\zeta} + \dfrac{1}{c^2}\dfrac{\partial z}{\partial t} = \psi & \text{in } Q, \\ z(\cdot, 0) = 0, \quad \boldsymbol{\zeta}(\cdot, 0) = \mathbf{0} & \text{on } \Omega, \\ z = 0, \quad \boldsymbol{\zeta} \cdot \mathbf{n}^x_\Omega = 0, \quad cz - \boldsymbol{\zeta} \cdot \mathbf{n}^x_\Omega = 0 & \text{one of these on } \partial\Omega \times (0, T). \end{cases}$$

Let S be a Lipschitz interface separating Ω in two components and $\Sigma = S \times (0, T)$ with unit normal $\mathbf{n}_\Sigma$. What are minimal assumptions on sources $\boldsymbol{\Phi}, \psi$ to ensure traces of v and $\boldsymbol{\zeta} \cdot \mathbf{n}_\Sigma$ are in $L^2(\Sigma)$?



Ideal: $(\psi, \boldsymbol{\Phi}) \in L^2(Q) \times \mathbf{L}^2(Q)$.
Holds for: $H^1(L^2(\Omega), 0, T) \times$
$L^2(H(\mathrm{div}; \Omega), 0, T) \cap H^{-1}_*(H_0(\mathrm{curl}; \Omega), 0, T)$
(Dirichlet case).
Similarly for Maxwell, hyperbolic systems...