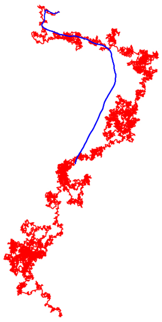
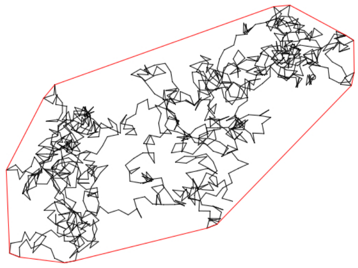
Durham University, 31 March–3 April 2014

# Aspects of random walks





# Iain MacPhee memorial day

Iain MacKenzie MacPhee
(7/11/1957–13/1/2012)

## Aspects of random walks & Iain MacPhee memorial day

# Programme

### Monday 31 March, IAS

Workshop starts with lunch at the IAS. The Monday afternoon session is sponsored by the London Mathematical Society celebration of recent appointments scheme.

12.00–13.00 Lunch.

13.00 Conference opening.

13.10–14.10 Ben Hambly: *Branching random walks and fractals.*

14.10–15.10 David Croydon: *Subsequential scaling limits of simple random walk on the two-dimensional uniform spanning tree.*

15.10–15.30 Tea/Coffee break.

15.30–16.30 Andrew Wade: *Convex hulls of random walks.*

### Tuesday 1 April, IAS

09.30–10.30 Dimitri Petritis: *Type transition for random walks on randomly directed lattices.*

10.30–11.00 Tea/Coffee break.

11.00–12.00 Denis Denisov: *Tail behaviour of stationary distribution for Markov chains with asymptotically zero drift.*

12.15–13.15 Lunch.

13.15–14.15 Stephen Connor: *Mixing time for a random walk on a ring.*

14.15–14.50 Stephen Muirhead: *Localisation in the parabolic Anderson and Bouchaud trap models.*

14.50–15.15 Dominic Yeo: *Exploring the acyclic random graph.*

15.15–15.45 Tea/Coffee break.

15.45–16.45 Nicholas Georgiou: *Non-homogeneous random walks on a semi-infinite strip.*

### Wednesday 2 April, IAS

09.30–10.30 Codina Cotar: *Density functional theory and optimal transportation with Coulomb cost.*

10.30–11.00 Tea/Coffee break.

11.00–12.00 Malwina Luczak: *Extinction times in the stochastic logistic epidemic.*

12.00–13.00 Ostap Hryniv: *Non-homogeneous random walks.*

13.00–14.00 Lunch.

14.00 Excursion.

18.30 Conference dinner.

### Thursday, 3 April: Iain MacPhee day, Maths dept.

09.30 Opening remarks (Anne Taormina).

09.40–10.10 Frank Coolen: *Nonparametric predictive reliability of series of voting systems.*

10.10–10.40 Jonty Rougier: *Server advantage in tennis matches.*

10.40–11.10 Tea/Coffee break.

11.10–12.00 Mathew Penrose: *Random bipartite geometric graphs.*

12.00–12.50 Stas Volkov: *A polling system with 3 queues and 1 server in the overload regime.*

12.50–14.00 Lunch.

14.00–15.00 Mikhail Menshikov: *Colloquium for Iain MacPhee.*

15.00 Informal reception in CM211.

16.00 Close of meeting.

# Iain MacPhee: Brief mathematical biography

Iain's undergraduate education was from the University of Melbourne, from where he also obtained a M.Sc. with dissertation *On Galton–Watson processes in varying environments* (1981). Iain studied for a Ph.D. at the University of Cambridge from 1982–1987, under the supervision of Frank Kelly; this was awarded for his thesis *On optimal strategies for stochastic decision processes* (1989).

Iain joined the staff in the Department of Mathematical Sciences at the University of Durham as a Temporary Lecturer in 1987, and his position became permanent in 1990. He was promoted to Reader in 2010.

Iain made significant and lasting contributions to teaching and administration in the department. He was involved, often as the driving force, in setting up numerous popular undergraduate courses, including material on probability, operations research, optimization, and mathematical finance, and service modules for engineering and science students. He was part of the team that introduced projects to the curriculum. He was involved in administration for teaching and learning, timetabling, and was head of the department's admissions team from 2002/3 to 2007/8, having been involved in admissions since 1993.

Iain supervised four Ph.D. students, Nikolaos H. Antoniu (1990–94), Ben P. Jordan (1993–95), Lisa J. Müller (2003–06), and Ahmad Aboalkhair (2008–12, jointly with Frank Coolen). All successfully completed their degrees.

# Publications of Iain MacPhee


1. I. M. MacPhee & H.-J. Schuh. A Galton-Watson branching process in varying environments with essentially constant offspring means and two rates of growth, *Austral. J. Statist.* **25** (1983) 329–338. MR0725212.
2. I. M. MacPhee. Contribution to discussion of 'Stochastic models of computer communication systems' (pp. 379–395) by F. P. Kelly, *J. Royal Statist. Soc. Ser. B* **47** (1985) 421–422.
3. F. P. Kelly & I. M. MacPhee. The number of packets transmitted by collision detect random access schemes, *Ann. Probab.* **15** (1987) 1557–1568. MR0905348.
4. I. M. MacPhee. *On optimal strategies in stochastic decision processes*, Ph.D. thesis, University of Cambridge, 1989.
5. I. M. MacPhee & B. P. Jordan. Optimal search for a moving target, *Probab. Engrg. Inform. Sci.* **9** (1995) 159–182. MR1357845.
6. I. M. MacPhee & I. Ziedins. Admission controls for loss networks with diverse routing, pp. 205–214 in *Stochastic Networks: Theory and Applications*, F. P. Kelly, S. Zachary, I. Ziedins (eds.), Clarendon, Oxford, 1996.
7. I. M. MacPhee & M. V. Menshikov. Critical random walks on two-dimensional complexes with applications to polling systems, *Ann. Appl. Probab.* **13** (2003) 1399–1422. MR2023881.
8. I. M. MacPhee, J. Rougier, & G. H. Pollard. Server advantage in tennis matches, *J. Appl. Probab.* **41** (2004) 1182–1186. MR2122811.
9. I. M. MacPhee & L. J. Müller. Stability criteria for controlled queueing systems, *Queueing Syst.* **52** (2006) 215–229. MR2220975.

10. I. M. MacPhee, M. V. Menshikov, S. Popov, & S. Volkov. Periodicity in the transient regime of exhaustive polling systems, *Ann. Appl. Probab.* **16** (2006) 1816–1850. MR2288706.

11. I. M. MacPhee, M. V. Menshikov, D. Petritis, & S. Popov. A Markov chain model of a polling system with parameter regeneration, *Ann. Appl. Probab.* **17** (2007) 1447–1473. MR2358630.

12. I. M. MacPhee & L. J. Müller. Stability criteria for multi-class queueing networks with re-entrant lines, *Methodol. Comput. Appl. Probab.* **9** (2007) 377–388. MR2396341.

13. I. M. MacPhee, M. V. Menshikov, D. Petritis, & S. Popov. Polling systems with parameter regeneration, the general case, *Ann. Appl. Probab.* **18** (2008) 2131–2155. MR2473652.

14. P. Coolen-Schrijner, F. P. A. Coolen, & I. M. MacPhee. Nonparametric predictive inference for system reliability with redundancy allocation, *J. Risk and Reliability* **222** (2008) 463–476.

15. I. M. MacPhee, F. P. A. Coolen, & A. M. Aboalkhair. Nonparametric predictive system reliability with redundancy allocation following component testing, *J. Risk and Reliability* **223** (2009) 181–188.

16. A. M. Aboalkhair, F. P. A. Coolen, & I. M. MacPhee. Nonparametric predictive system reliability with all subsystems consisting of one type of component, pp. 85–98 in *Risk and Decision Analysis in Maintenance Optimization and Flood Management*, M. J. Kallen & S. P. Kuniewski, IOS Press, 2009.

17. I. M. MacPhee, M. V. Menshikov, & A. R. Wade. Angular asymptotics for multi-dimensional non-homogeneous random walks with asymptotically zero drift, *Markov Process. Related Fields* **16** (2010) 351–388. MR2666858.

18. I. M. MacPhee, M. V. Menshikov, S. Volkov, & A. R. Wade. Passage-time moments and hybrid zones for the exclusion-voter model, *Bernoulli* **16** (2010) 1312–1342. MR2759181.

19. O. Hryniv, I. M. MacPhee, M. V. Menshikov, & A. R. Wade. Non-homogeneous random walks with non-integrable increments and heavy-tailed random walks on strips, *Electron. J. Probab.* **17** (2012), no. 59, 28 pp. MR2959065.

20. I. M. MacPhee, M. V. Menshikov, & M. Vachkovskaia. Dynamics of the non-homogeneous supermarket model, *Stoch. Models* **28** (2012) 533–556. MR2995522.

21. A. M. Aboalkhair, F. P. A. Coolen, & I. M. MacPhee. Nonparametric predictive reliability of series of voting systems, *European J. Oper. Res.* **226** (2013) 77–84. MR3008205.

22. I. M. MacPhee, M. V. Menshikov, & A. R. Wade. Moments of exit times from wedges for non-homogeneous random walks with asymptotically zero drifts, *J. Theoret. Probab.* **26** (2013) 1–30. MR3023832.

# Locations

- **Monday to Wednesday** the workshop will take place at the Institute for Advanced Study (IAS), which is located off Palace Green (near the Cathedral): building 21 on the University map.
- **Thursday**'s events will take place in the Department of Mathematical Sciences, on the Durham science site: building 15 on the map.
- **Accommodation** is at Collingwood college: location 6 on the map.
- The **excursion** will probably be an option to visit the Cathedral, or the botanic gardens, location 7 on the map.

# Titles & Abstracts

### Stephen Connor (University of York, UK)

*Mixing time for a random walk on a ring.*

We consider a variant of a process used in random number generation, and previously studied by Chung, Diaconis and Graham. This a random walk on the integers mod $n$ ($n$ odd), which at each step either increments by 1 or doubles its value, but where the probability of doubling is a decreasing function of $n$. We use a mixture of representation theory and probability to show that the total variation distance for this process exhibits a cutoff phenomenon.

This is joint work with Michael Bate (York).

### Frank Coolen (Durham University, UK)

*Nonparametric predictive reliability of series of voting systems.*

Since about 2007, I collaborated with Iain MacPhee on a project considering Nonparametric Predictive Inference (NPI) for system reliability, when one has failure data on components which are exchangeable with those in the system and wishes to predict the system's functioning. Initially this work was jointly with Pauline Coolen-Schrijner [1]; since 2008 it was the topic of the Ph.D. research of Ahmad Aboalkhair [2]. I will provide a brief overview of the different stages in this research, the main results of which were published in the European Journal of Operational Research [3]. Examples will be used to illustrate the results, which show important aspects of redundancy and diversity for system reliability. Finally, I will comment briefly on related work we have done since 2012.


[1] P. Coolen-Schrijner, F.P.A. Coolen, I.M. MacPhee, *Journal of Risk and Reliability* **222** (2008) 463–476.

[2] A.M. Aboalkhair, Ph.D. Thesis, Durham University, 2012.

[3] A.M. Aboalkhair, F.P.A. Coolen, I.M. MacPhee, *European Journal of Operational Research* **226** (2013) 77–84.

**Codina Cotar (University College London, UK)**

*Density functional theory and optimal transportation with Coulomb cost.*

Density functional theory (DFT) is a computationally feasible electronic structure model which simplifies full quantum mechanics and for which Walter Kohn received a Nobel prize in 1998. In the semiclassical limit, DFT reduces to a multi-marginal optimal transport (OT) problem [1]. Considerable insight into the limit problem had been built up, prior to our work, by physicists (Seidl, Perdew, Levy, Gori-Giorgi, Savin), who essentially developed a considerable amount of optimal transport theory without knowing they were doing optimal transport.

The goal of my talk is three-fold:

(i) To explain the connection between DFT and optimal transport.

(ii) To discuss what is known rigorously about the limit problem, including

- justification of the formal semiclassical limit [1];
- qualitative theory of OT problems with Coulomb cost, including the question whether 'Kantorovich minimizers' must be 'Monge minimizers' (*yes* for 2 particles, open for $N$ particles, *no* for infinitely many particles) [1, 2];
- exactly soluble cases ($N = 2$ with radial density; $N = \infty$) [1, 2].

(iii) To present a natural hierarchy of further approximations of the limit functional related to representability constraints on the pair density which survive in the classical limit, and discuss the important (open) problem of characterizing $N$-representable pair densities.

[1] C. Cotar, G. Friesecke, C. Klueppelberg, *CPAM* **66** (2013) 548–599.

[2] C. Cotar , G. Friesecke, C. Klueppelberg, Ch. Mendl, B. Pass, *Journal of Chemical Physics* **139** (2013) 164109.

[3] C. Cotar, G. Friesecke, B. Pass, arXiv 1307.6540, 2013.

**David Croydon (University of Warwick, UK)**

*Subsequential scaling limits of simple random walk on the two-dimensional uniform spanning tree.*

The work that I will describe establishes that the law of the simple random walk on the two-dimensional uniform spanning tree is tight under a particular rescaling of time and space. Whilst such a result immediately implies the existence of subsequential scaling limits for the random walks in question, our techniques further allow us to describe these limits as diffusions on random real trees embedded into Euclidean space, and derive various transition density estimates for them. This is a joint project with Martin Barlow (UBC) and Takashi Kumagai (Kyoto University).

**Denis Denisov (University of Manchester, UK)**

*Tail behaviour of stationary distribution for Markov chains with asymptotically zero drift.*

We consider a one-dimensional Markov chain with asymptotically zero drift and finite second moments of jumps which is positive recurrent. A power-like asymptotic behaviour of the invariant tail distribution is proven; such a heavy-tailed invariant measure happens even if the jumps of the chain are bounded. Our analysis is based on test functions technique and on construction of a harmonic function.

This is joint work with Korshunov and Wachtel: `http://arxiv.org/abs/1208.3066`.

**Nicholas Georgiou (Durham University, UK)**

*Non-homogeneous random walks on a semi-infinite strip.*

Let $(X_n, \eta_n)$ be a Markov chain on state space $\mathbb{Z}_+ \times S$, where $\mathbb{Z}_+$ is the non-negative integers and $S$ is a finite set. We assume a moments bound on the jumps of $X_n$ and that, roughly speaking, $\eta_n$ is close to being Markov when $X_n$ is large. Under these assumptions, we show it is possible to give a recurrence classification for $X_n$, in terms of the increment moment parameters of $X_n$ and the stationary distribution for the large-$X$ limit of $\eta_n$.

In this talk, I will describe the case where these moment parameters satisfy Lamperti-type drift conditions on each line $\mathbb{Z}_+ \times \{i\}$, for $i \in S$, and show that the classification can be seen as a generalisation of Lamperti's results for non-homogeneous random walks on $\mathbb{Z}_+$; in our setting the asymptotic behaviour of $X_n$ is determined by the "average" increment moments of $X_n$, where the average is taken over $S$ using the stationary distribution for the large-$X$ limit of $\eta_n$. We apply Lamperti's results to an embedded process that records the $X$-coordinate each time the chain returns to a given reference line and calculate moments via the Doob decomposition for $X_n$ over an excursion from this line, which explains these averages as an accumulation of the moment parameters over such an excursion.

This is joint work with Andrew Wade.

**Ben Hambly (University of Oxford, UK)**

*Branching random walks and fractals.*

Branching random walks and general branching processes can be used to encode information about many fractal sets. We will show how information about the geometric and analytic properties of random recursive fractals can be captured. By establishing a central limit theorem for general branching processes, we will be able to deduce some novel results about fractal sets. In particular we will discuss the asymptotics of the heat content for some domains with fractal boundary and the asymptotics of the eigenvalue counting function for the Brownian continuum random tree.

**Ostap Hryniv (Durham University, UK)**

*Non-homogeneous random walks.*

We study long term behaviour of certain spatially non-homogeneous random walks, as well as asymptotics of various functionals of their trajectories (such as running maxima, hitting times of particular sets, etc.) at large finite times.

Based on joint work with Iain MacPhee, Mikhail Menshikov, and Andrew Wade.

**Malwina Luczak (Queen Mary, University of London, UK)**

*Extinction times in the stochastic logistic epidemic.*

The stochastic logistic process is a well-known birth-and-death process, often used to model the spread of an epidemic within a population of size $N$. We survey some of the known results about the time to extinction for this model. Our focus is on new results for the "subcritical" regime, where the recovery rate exceeds the infection rate by more than $N^{-1/2}$, and the epidemic dies out rapidly with high probability. Previously, only a first-order estimate for the mean extinction time of the epidemic was known, even in the case where the recovery rate and infection rate are fixed constants: we derive precise asymptotics for the distribution of the extinction time throughout the subcritical regime. In proving our results, we illustrate a technique for approximating certain types of Markov chain by differential equations over long time periods.

This is joint work with Graham Brightwell.

**Stephen Muirhead (University College London, UK)**

*Localisation in the parabolic Anderson and Bouchaud trap models.*

Random walks in random environments often exhibit intermittency, meaning that their probability mass functions cannot, in general, be described with simple averaging principles. An extreme form of intermittency is localisation, where the probability mass function is eventually concentrated on a small number of sites with high probability. In this talk I present recent progress in proving localisation results in two such random walk models, the parabolic Anderson model and the Bouchaud trap model.

**Mathew Penrose (University of Bath, UK)**

*Random bipartite geometric graphs.*

Consider a bipartite random geometric graph (RGG) on the union of two independent homogeneous Poisson point processes in the plane, with distance parameter $r$ and intensities $\lambda$, $\mu$. If $\lambda$ is supercritical for the one-type RGG with distance parameter $2r$ then there exists $\mu$ such that $(\lambda, \mu)$ is supercritical; this also holds in higher dimensions.

Consider also the restriction of this graph to points in the unit square. We descibe a strong law of a large numbers as $\lambda \to \infty$, with $\mu/\lambda$ fixed, for the connectivity threshold of this graph. These results add to earlier work of Iyer and Yogeshwaran (*Adv. Appl. Probab.* 2012).

**Dimitri Petritis (Université de Rennes, France)**

*Type transition for random walks on randomly directed lattices.*

We shall examine random walks on partially directed lattices obtained from $\mathbb{Z}^2$ by keeping vertical edges bi-directional but rendering the horizontal lines uni-directional. Depending on the choice of the directions, the ordinary random walk on such lattices can be made recurrent or transient. For directions chosen as a small random perturbation of a periodic pattern, transition from transience to recurrence will be established when the strength of the random perturbation decays fast enough.

**Jonty Rougier (University of Bristol, UK)**

*Server advantage in tennis matches.*

An on-the-blackboard argument proves that, in the obvious model, there is no advantage to serving first in tennis: in the tie-break, in the set, or in the match.

This was joint work with Iain MacPhee.

**Stas Volkov (Lund University, Sweden)**

*A polling system with 3 queues and 1 server in the overload regime.*

We consider a queueing system with three queues and just one server, in the regime when the system overall is eventually overloaded, but yet no individual queue is. The service discipline is as follows. Once the server is at node $j$, it stays there until it finishes serving all customers present in that queue. After this, the server moves to the "more expensive" of the two remaining queues. We will show that a.s. there will be periodicity in the order of services, as suggested by the behaviour of the corresponding dynamical systems; we also study the cases (of measure zero) when the dynamical system is chaotic, and prove that then the stochastic one cannot be periodic either. These results are quite surprising given the superficial "simplicity" of the model.

Based on joint work with Iain MacPhee, Mikhail Menshikov, and Serguei Popov.

**Andrew Wade (Durham University, UK)**

*Convex hulls of random walks.*

On each of $n$ unsteady steps, a drunken gardener drops a seed. Once the flowers have bloomed, what is the minimum length of fencing required to enclose the garden? Denote by $L_n$ the perimeter length of the convex hull of $n$ steps of a planar random walk whose increments have finite second moment. Snyder and Steele showed that $n^{-1}L_n$ converges almost surely to a deterministic limit, and proved an upper bound on the variance $\mathbb{V}\mathrm{ar}(L_n) = O(n)$.

Further study separates into two cases: (i) zero drift, in which the Brownian scaling limit of the walk entails a scaling limit for the convex hull, once one has set things up correctly; (ii) non-zero drift. I will describe recent work with Chang Xu (Strathclyde) on these problems. Our main result is in the case of non-zero drift, where we show that $n^{-1}\mathbb{V}\mathrm{ar}(L_n)$ converges, and give a simple expression for the limit, which is non-zero for walks outside a certain degenerate class. This answers a question of Snyder and Steele. Furthermore, we prove a central limit theorem for $L_n$ in the non-degenerate case.

**Dominic Yeo (University of Oxford, UK)**

*Exploring the acyclic random graph.*

Aldous considered the scaling limit of a depth-first walk to extract information about the asymptotic component sizes and cycle counts in a critical random graph. In this talk, I will present progress towards similar results for related processes where critical components are removed either at linear rate, or when they first contain a cycle. This is joint work with James Martin and Balázs Ráth.

## Participants

| Name | Affiliation |
|---:|---|
| Sayan Banerjee | Warwick |
| Ksenia Chernysh | Heriot-Watt |
| Stephen Connor | York |
| Frank Coolen | Durham |
| Codina Cotar | UCL |
| David Croydon | Warwick |
| James Cruise | Heriot-Watt |
| Denis Denisov | Manchester |
| Nicholas Georgiou[1] | Durham |
| Ben Hambly | Oxford |
| Ostap Hryniv[1] | Durham |
| Anshui Li | Utrecht |
| Malwina Luczak | QML |
| Mikhail Menshikov[1] | Durham |
| Lisa Müller | |
| Stephen Muirhead | UCL |
| Mathew Penrose | Bath |
| Dimitri Petritis | Rennes |
| Richard Pymar | UCL |
| Jonty Rougier | Bristol |
| Vsevolod Shneer | Heriot-Watt |
| Stanislav Volkov | Lund |
| Andrew Wade[1] | Durham |
| Chang Xu | Strathclyde |
| Dominic Yeo | Oxford |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[1] Organizers