Seiberg-Witten maps and sh-Lie-quasi-isomorphisms

based on arXiv:1806.10314, together with R.Blumenhagen, M.Brinkmann, V.Kupriyanov

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What?

- Present explicitly the connection between Seiberg-Witten maps and L_∞ quasi-isomorphisms
- Propose extension of Seiberg-Witten map for transformations of field equations

Outline

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Quasi-isomorphisms

Given two L_∞ algebras $(V, \{b_i\}), (W, \{\tilde{b}_j\})$ a morphism consists of multilinear, graded symmetric maps $\{F_n\} : V^{\otimes n} \to W$ of constant degree $|F_n| = 0$ such that

$$\sum_{\sigma \in \mathrm{Unsh}(k+l=n)} \epsilon(\sigma; x) F_{1+l}\Big(b_k(x_{\sigma(1)}, \dots, x_{\sigma(k)}), x_{\sigma(k+1)}, \dots, x_{\sigma(n)}\Big)$$
$$= \sum_{\sigma \in \mathrm{Unsh}(k_1+\cdots+k_j=n)} \frac{\epsilon(\sigma; x)}{j!} \tilde{b}_j(F_{k_1} \otimes \cdots \otimes F_{k_j})(x_{\sigma(K)}) \,.$$

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$\to$ Quasi-isomorphism if F_1 is an isomorphism on the homology of the chain complexes underlying the L_∞ algebras

Seiberg-Witten map

NC space with Moyal Weyl star product $f \star g = e^{\frac{i}{2}\Theta^{ij}\partial_i \otimes \partial_j} f \otimes g$

- Gives non-commutative gauge theory with gauge parameters $\hat{\lambda}$ and fields $\hat{A}$
- Is there a Seiberg-Witten map $\hat{A}(A)$, $\hat{\lambda}(\lambda, A)$?

$$\hat{A}(A + \delta_\lambda A) = \hat{A} + \hat{\delta}_{\hat{\lambda}} \hat{A}$$

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Inspecting the gauge closure in the original SW-map gives conjecture:

$$\hat{A}\left(A + \delta_{[\lambda_1,\lambda_2]} A\right) = \hat{A}(A) + \hat{\delta}_{[\hat{\lambda}_1,\hat{\lambda}_2]_\star} \hat{A}(A) + \hat{\delta}_{\hat{\lambda}(\lambda_1, \delta_{\lambda_2} A)} \hat{A}(A) - \hat{\delta}_{\hat{\lambda}(\lambda_2, \delta_{\lambda_1} A)} \hat{A}(A) \, .$$

Gauge variation, closure etc

Building on the dictionary obtained by Hohm & Zwiebach arXiv:1701.08824

- Graded vector space $X = X_1 \oplus X_0 \oplus X_{-1}$
- X_1:gauge parameters, X_0: fields, X_{-1}: field equations

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- Graded vector space $X = X_1 \oplus X_0 \oplus X_{-1}$
- X_1:gauge parameters, X_0: fields, X_{-1}: field equations
- Gauge variation: $\delta_\lambda A = \sum_{n=0}^{\infty} \frac{1}{n!} b_{n+1}(\lambda, A^n)$
- Gauge closure: $[\delta_{\lambda_1}, \delta_{\lambda_2}]A = \delta_{C(\lambda_2,\lambda_1,A)} A$
- Field equations: $\mathcal{F} = \sum_{n=1}^{\infty} \frac{1}{n!} b_n(A^n)$.
- Covariance of eom: $\delta_\lambda \mathcal{F} = \sum_{n=0}^{\infty} \frac{1}{n!} b_{n+2}(\lambda, \mathcal{F}, A^n)$.

Outline

General case

Let $(V = V_1 \oplus V_0 \oplus V_{-1}, \{b_i\})$ and $(W = W_1 \oplus W_0 \oplus W_{-1}, \left\{\tilde{b}_j\right\})$ be two L_∞ algebras underlying classical gauge theories. Suppose there is a Seiberg-Witten map. Then this can be recast into:

- Field map: $\hat{A}(A) := \sum \frac{1}{n!} F_n(A^n)$
- Parameter map: $\hat{\lambda}(\lambda, A) := \sum \frac{1}{n!} F_{n+1}(\lambda, A^n)$
- Eom map: $\hat{E} := \sum \frac{1}{n!} F_{n+1}(E, A^n)$

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with $F_n : V^{\otimes n} \to W$ are graded symmetric maps of degree 0. Then $\hat{A}(A + \delta_\lambda A) = \hat{A} + \hat{\delta}_{\hat{\lambda}} \hat{A} \Leftrightarrow F_n$ satisfy morphsim eq. for inputs (λA^n) on shell.

SW QISO

$\rightarrow$ Turn logic around and see what it gives on shell:

Morphism input $(\lambda_1, \lambda_2 A^n) \Leftrightarrow$

$$\hat{A}\left(A + \delta_{C(\lambda_2,\lambda_1,A)}A\right) = \hat{A}(A) + \hat{\delta}_{C(\hat{\lambda}_2,\hat{\lambda}_1,A)}\hat{A}(A) + \hat{\delta}_{\hat{\lambda}(\lambda_1,\delta_{\lambda_2}A)}\hat{A}(A) - \hat{\delta}_{\hat{\lambda}(\lambda_2,\delta_{\lambda_1}A)}\hat{A}(A)$$

Morphism input $(A^n) \Leftrightarrow \hat{E}(A) = \hat{\mathcal{F}} = \hat{\mathcal{F}}(\mathcal{F}, A)$.

Input $(\lambda, E, A^n) \Leftrightarrow \hat{E}(A + \delta_\lambda A, E + \delta_\lambda A) = \hat{E}(E, A) + \hat{\delta}_{\hat{\lambda}(\lambda,A)}\hat{E}(E, A)$.

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Thanks for your attention!