Solving the Reconstruction Problem in Asymptotic Safety

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under the superivision of Tim Morris

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Overview

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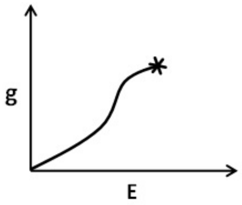

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- Approach of asymptotic safety based on the existence of a **UV fixed point for gravity** $\rightarrow$ physical quantities are safe from divergences.
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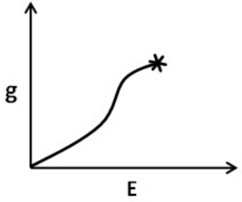
- **Non-perturbative** approach to quantum gravity.



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The Renormalization Group

- The beta function (from perturbation theory):

$$\beta(g_R) = \mu \frac{\partial g_R}{\partial \mu}$$

$\rightarrow$ Renormalized couplings, independent of Λ, take $\Lambda \rightarrow \infty$.

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- Bare action $S_\Lambda^{tot}[\phi]$.

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- Bare action $S_\Lambda^{tot}[\phi]$.
- Bare couplings are finite $g = g(\Lambda)$.

The Renormalization Group

- Wilson's picture leads to a deeper understanding of renormalizability.
- Arbitrarily complicated Lagrangian reduces to one containing only renormalizable terms as cutoff is lowered.
  $\rightarrow$ explains why QED is perturbatively renormalizable.
- Framework allows us to define flow equations.

The Renormalization Group

- Regulate in UV in **smooth** way by modifying propagators.

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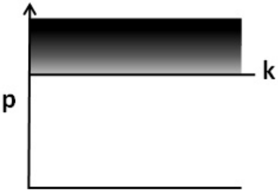
- Regulate in UV in **smooth** way by modifying propagators.
- Cutoff in UV at scale k (instead of Λ).

$$\Delta = \frac{1}{p^2} \to \Delta_{UV} = \frac{C_{UV}(p,k)}{p^2}$$



$$C_{UV}(p,k) \approx \begin{cases} 1 & \text{for } p < k - \epsilon \\ 0 & \text{for } p > k + \epsilon \end{cases}$$

$$Z[J] = \int \mathcal{D}\phi e^{-\frac{1}{2}\phi.\Delta_{UV}^{-1}.\phi - S_k[\phi] + J.\phi}$$

The Renormalization Group

- UV regulated theory

$$Z[J] = \int \mathcal{D}\phi e^{-\frac{1}{2}\phi.\Delta_{UV}^{-1}.\phi - S_k[\phi] + J.\phi}$$

- Polchinski's flow equation:

$$\frac{\partial Z[J]}{\partial k} = 0 \implies \frac{\partial S_k[\phi]}{\partial k} = \frac{1}{2}\frac{\delta S_k}{\delta \phi}.\Delta_{UV}.\frac{\delta S_k}{\delta \phi} - \frac{1}{2}\mathrm{Tr}\left[\frac{\partial \Delta_{UV}}{\partial k}.\frac{\delta^2 S_k}{\delta\phi\delta\phi}\right]$$

- Integral sum over spacetime indices

$$Tr\left[\Delta_{UV}.\frac{\delta^2 S}{\delta\phi\delta\phi}\right] = \int_{x,y} \Delta_{UV}(x,y)\frac{\delta^2 S}{\delta\phi(y)\phi(x)} = \int_p \Delta_{UV}(p,-p)\frac{\delta^2 S}{\delta\phi(-p)\phi(p)}$$

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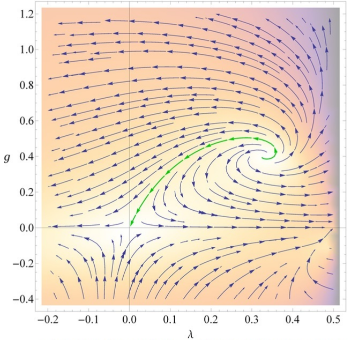

Author: Andreas Nink. Source: wikipedia.org/wiki/Asymptotic_safety_in_quantum_gravity

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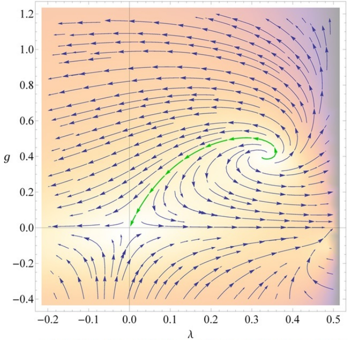
$$S_k[\phi] = \sum_i^{\infty} g_i(k)\mathcal{O}_i(\phi)$$

- Complete trajectory from UV fixed point to IR fixed point
  $\leftrightarrow$ divergence-free QFT
  $\leftrightarrow \{S_k, 0 \leq k < \infty\}$.



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Overview

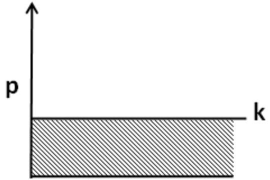
The Effective Average Action

- The effective average action $\tilde{\Gamma}_k[\varphi]$:

$$\tilde{\Gamma}_k[\varphi] + \frac{1}{2}\varphi.R(p,k).\varphi \ , \quad \varphi = \langle \phi \rangle$$

- Smooth **IR** cutoff function $R(p,k)$

$$R(p,k) \approx \begin{cases} k^2 - p^2 & \text{for } p < k - \epsilon \\ 0 & \text{for } p > k + \epsilon \end{cases}$$



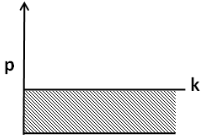
- $R(p,k)$ is an additive cutoff function

$$\frac{1}{p^2 + R} \approx \begin{cases} \frac{1}{k^2} & \text{for } p < k - \epsilon \\ \frac{1}{p^2} & \text{for } p > k + \epsilon \end{cases}$$

The Effective Average Action

- Flow equation for $\tilde{\Gamma}_k[\varphi]$:

$$\frac{\partial \tilde{\Gamma}_k[\varphi]}{\partial k} = \frac{1}{2}\mathrm{Tr}\left\{ \left[R_k + \frac{\delta^2 \tilde{\Gamma}_k}{\delta\varphi\varphi} \right]^{-1} \frac{\partial R_k}{\partial k} \right\}$$



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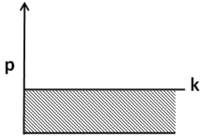

- No need for UV regulator.
- Complete set of solutions to flow equation $\leftrightarrow$ divergence-free QFT.

$$\{\tilde{\Gamma}_k, 0 \leq k < \infty\} \iff \text{complete QFT}$$

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    1. $\tilde{\Gamma}_k$ is the generator of 1PI Green's functions - directly related to scattering amplitudes.
    2. $\tilde{\Gamma}_k$ gives better approximation to a QFT.
    3. Don't have to construct a regulated path integral.

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- Conceptually nothing wrong with not having a path integral representation.
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  2. Some properties of QFT analysed more easily e.g. implementation of symmetries.
  3. Approximation schemes (e.g. perturbation theory, large N expansion) more naturally described.
  4. Theory that we put on the lattice is given by S_k.

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$$\tilde{\Gamma}_k[\varphi] \to Z[J] = \int\limits_{|p|<\Lambda} \mathcal{D}\phi e^{-S_\Lambda^{tot}[\phi]+J.\phi}$$

- Regulated with sharp cutoff Λ, take $\Lambda \to \infty$.

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- Regulated with sharp cutoff Λ, take $\Lambda \to \infty$.
- E. Manrique and M. Reuter (2008):

$$\boxed{\tilde{\Gamma}_{k=\Lambda}[\varphi] = S_\Lambda^{tot}[\phi] + \frac{1}{2} Tr_\Lambda ln\left\{ R_\Lambda + \frac{\delta^2 S_\Lambda^{tot}}{\delta\phi\delta\phi} \right\}}$$

$$\mathrm{Tr}_\Lambda\{...\} \equiv \mathrm{Tr}\{\theta(\Lambda^2 - p^2)[...]\}$$

- Derived by saddle point expansion - **approximate expression**.

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- Split modes $\phi \to \phi_> + \phi_<$ and propagators $\Delta \to \Delta_{IR} + \Delta_{UV}$.

$$Z[J] = \int_{|p|<\Lambda} \mathcal{D}\phi_> \mathcal{D}\phi_< e^{-\frac{1}{2}\phi_<.\Delta_{UV}^{-1}.\phi_< - \frac{1}{2}\phi_>.\Delta_{IR}^{-1}.\phi_> - S_{\Lambda_0}[\phi_< + \phi_>] + J.(\phi_< + \phi_>)}$$

$$\Delta_{IR} = \frac{C_{IR}(p,k)}{p^2}, \quad \Delta_{UV} = \frac{C_{UV}(p,k)}{p^2}$$

- Cutoff functions obey summation relation:

$$C_{IR} + C_{UV} = 1$$

Solving the Reconstruction Problem

- Compute integral over high momentum modes.

$$
\begin{aligned}
Z[J,\phi_<] &= \int \mathcal{D}\phi_> e^{-\frac{1}{2}\phi_>.\Delta_{IR}^{-1}.\phi_> - S_\Lambda[\phi_>+\phi_<]+J.(\phi_>+\phi_<)} \\
&= e^{\frac{1}{2}J.\Delta_{IR}.J+J.\phi_< - S_k[\Delta_{IR}.J+\phi_<]}
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$$J(p) = 0 \ \forall \ p > k \implies Z[J] = \int \mathcal{D}\phi_< e^{-\frac{1}{2}\phi_<.\Delta_{UV}^{-1}.\phi_< - S_k[\phi_<]+J.\phi_<}$$

- Recognise S_k as the interaction part of the bare action $S_k^{tot} = \frac{1}{2}\phi_<.\Delta_{UV}^{-1}.\phi_< + S_k$ regulated in the UV at scale k.

Solving the Reconstruction Problem

- Integral over high momentum modes:

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- Interpret as a functional integral for field $\phi_>$ regulated in the IR at scale k (in presence of background field $\phi_<$).
- Simply related to the generator of connected Green's functions W_k (cutoff in the IR):

$$Z[J,\phi_<] = e^{W_k[J,\phi_<]}$$

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$$\Gamma_k^{tot}[\varphi,\phi_<] = -W_k[J,\phi_<] + J.\varphi = \frac{1}{2}(\varphi-\phi_<).\Delta_{IR}^{-1}.(\varphi-\phi_<) + \Gamma_k[\varphi]$$

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- Relationship between Γ_k and S_k:

$$\boxed{\Gamma_k[\varphi] = S_k[\phi] + \frac{1}{2}(\varphi - \phi).\Delta_{IR}^{-1}.(\varphi - \phi)}$$

- Note that $\Gamma_k[\varphi]$ is interaction part of $\Gamma_k^{tot}[\varphi, \phi_<]$ and $S_k[\phi]$ is the interaction part of $S_k^{tot}[\phi]$.

Solving the Reconstruction Problem

- Compare our expression

$$\boxed{\Gamma_k[\varphi] = S_k[\phi] + \frac{1}{2}(\varphi - \phi).\Delta_{IR}^{-1}.(\varphi - \phi)}$$

to E. Manrique and M. Reuter's

$$\boxed{\tilde{\Gamma}_{k=\Lambda}[\varphi] = S_\Lambda^{tot}[\phi] + \frac{1}{2} Tr_\Lambda ln\left\{ R_\Lambda + \frac{\delta^2 S_\Lambda}{\delta\phi\delta\phi} \right\}}$$

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- How is Γ_k (Legendre effective action) related to $\tilde{\Gamma}_k$(effective average action)?

Solving the Reconstruction Problem

- Legendre effective action (without background field)

$$\Gamma_k^{tot}[\varphi] = \Gamma_k[\varphi] + \frac{1}{2}\varphi.\Delta_{IR}^{-1}.\varphi$$

- Flow equation for Γ_k

$$\frac{\partial \Gamma_k[\varphi]}{\partial k} = -\frac{1}{2} Tr\left\{ \left[1 + \Delta_{IR} \frac{\delta^2 \Gamma_k}{\delta\varphi\delta\varphi} \right]^{-1} \frac{1}{\Delta_{IR}} \frac{\partial \Delta_{IR}}{\partial k} \right\}$$

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- Flow equation for effective average action $\tilde{\Gamma}_k$

$$\frac{\partial \tilde{\Gamma}_k[\varphi]}{\partial k} = \frac{1}{2}\mathrm{Tr}\left\{ \left[R_k + \frac{\delta^2\tilde{\Gamma}_k}{\delta\varphi\varphi}\right]^{-1} \frac{\partial R_k}{\partial k} \right\}$$

Solving the Reconstruction Problem

- Split off kinetic part from $\tilde{\Gamma}_k$

$$\tilde{\Gamma}_k = \tilde{\Gamma}_k^{int} + \frac{1}{2}\varphi.p^2.\varphi$$

- Flow equation becomes

$$\frac{\partial \tilde{\Gamma}_k^{int}}{\partial k} = \frac{1}{2} Tr \left\{ \left[R_k + p^2 + \frac{\delta^2 \tilde{\Gamma}_k^{int}}{\delta\varphi\delta\varphi} \right]^{-1} \frac{\partial R_k}{\partial k} \right\}$$

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- Identify $p^2 + R_k = \frac{p^2}{C_{IR}} = \Delta_{IR}^{-1}$

$$\frac{\partial \tilde{\Gamma}_k^{int}[\varphi]}{\partial k} = -\frac{1}{2} Tr\left\{ \left[1 + \Delta_{IR} \frac{\delta^2 \tilde{\Gamma}_k^{int}}{\delta\varphi\delta\varphi} \right]^{-1} \frac{1}{\Delta_{IR}} \frac{\partial \Delta_{IR}}{\partial k} \right\}$$

- $\tilde{\Gamma}_k^{int}$ satisfies same flow equation as Γ_k!

$$\implies \tilde{\Gamma}_k^{int} = \Gamma_k$$

Overview

Summary and Conclusions

- Solved the reconstruction problem.
- Found an exact relationship between $\tilde{\Gamma}_k[\varphi]$ and $S_k[\phi]$.
- Making contact with E. Manrique & M. Reuter's formula:

$$\Gamma_k[\varphi] = S_k[\phi] + \frac{1}{2}(\varphi - \phi).\Delta_{IR}^{-1}.(\varphi - \phi)$$

vs

$$\tilde{\Gamma}[\varphi]_{k=\Lambda} = S_\Lambda^{tot}[\phi] + \frac{1}{2} Tr_\Lambda ln \left\{ R_\Lambda + \frac{\delta^2 S_\Lambda^{tot}}{\delta\phi\delta\phi} \right\}$$

- Next stage: use metric $g_{\mu\nu}$ as dynamical degree of freedom.

Thank you