Solving the Reconstruction Problem in Asymptotic Safety

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Overview

1. Introduction and Motivation
2. The Renormalization Group
3. RG Flows in Theory Space
4. The Effective Average Action
5. The Reconstruction Problem
6. Solving the Reconstruction Problem
7. Summary and Conclusions

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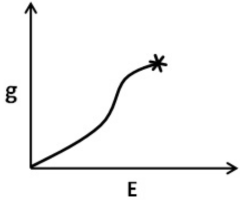

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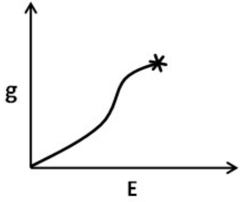

- **Non-perturbative** approach to quantum gravity.

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- The beta function (from perturbation theory):

$$\beta(g_R) = \mu \frac{\partial g_R}{\partial \mu}$$

$\rightarrow$ Renormalized couplings, independent of Λ, take $\Lambda \rightarrow \infty$.

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- Bare action $S_\Lambda^{tot}[\phi]$.

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- Λ is physical e.g. Planck scale, inverse lattice spacing...
- Bare action $S_\Lambda^{tot}[\phi]$.
- Bare couplings are finite $g = g(\Lambda)$.

The Renormalization Group

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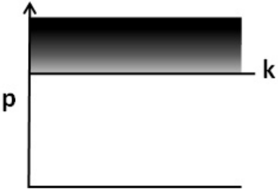
- Regulate in UV in **smooth** way by modifying propagators.
- Cutoff in UV at scale k (instead of Λ).

$$\Delta = \frac{1}{p^2} \to \Delta_{UV} = \frac{C_{UV}(p,k)}{p^2}$$



$$C_{UV}(p,k) \approx \begin{cases} 1 & \text{for } p < k - \epsilon \\ 0 & \text{for } p > k + \epsilon \end{cases}$$

$$Z[J] = \int \mathcal{D}\phi e^{-\frac{1}{2}\phi.\Delta_{UV}^{-1}.\phi - S_k[\phi] + J.\phi}$$

The Renormalization Group

- UV regulated theory

$$Z[J] = \int \mathcal{D}\phi e^{-\frac{1}{2}\phi.\Delta_{UV}^{-1}.\phi - S_k[\phi] + J.\phi}$$

- Polchinski's flow equation:

$$\frac{\partial Z[J]}{\partial k} = 0 \implies \frac{\partial S_k[\phi]}{\partial k} = \frac{1}{2}\frac{\delta S_k}{\delta \phi}.\Delta_{UV}.\frac{\delta S_k}{\delta \phi} - \frac{1}{2}\mathrm{Tr}\left[\frac{\partial \Delta_{UV}}{\partial k}.\frac{\delta^2 S_k}{\delta\phi\delta\phi}\right]$$

- Integral sum over spacetime indices

$$Tr\left[\Delta_{UV}.\frac{\delta^2 S}{\delta\phi\delta\phi}\right] = \int_{x,y} \Delta_{UV}(x,y)\frac{\delta^2 S}{\delta\phi(y)\phi(x)} = \int_p \Delta_{UV}(p,-p)\frac{\delta^2 S}{\delta\phi(-p)\phi(p)}$$

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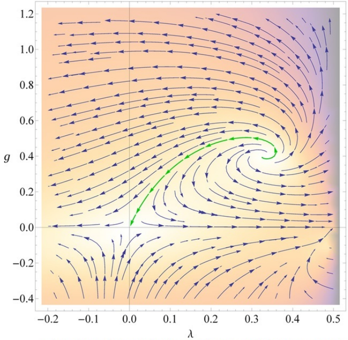

Author: Andreas Nink. Source: wikipedia.org/wiki/Asymptotic_safety_in_quantum_gravity

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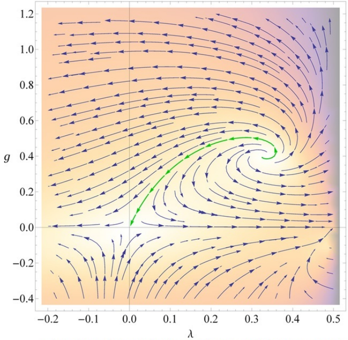
$$S_k[\phi] = \sum_i^{\infty} g_i(k)\mathcal{O}_i(\phi)$$

- Complete trajectory from UV fixed point to IR fixed point
  $\leftrightarrow$ divergence-free QFT
  $\leftrightarrow \{S_k, 0 \leq k < \infty\}$.



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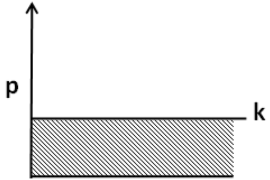
The Effective Average Action

- The effective average action $\tilde{\Gamma}_k[\varphi]$:

$$\tilde{\Gamma}_k[\varphi] + \frac{1}{2}\varphi.R(p,k).\varphi \ , \quad \varphi = \langle\phi\rangle$$

- Smooth **IR** cutoff function $R(p,k)$

$$R(p,k) \approx \begin{cases} k^2 - p^2 & \text{for } p < k - \epsilon \\ 0 & \text{for } p > k + \epsilon \end{cases}$$



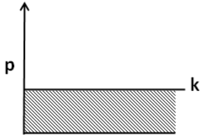
- $R(p,k)$ is an additive cutoff function

$$\frac{1}{p^2 + R} \approx \begin{cases} \frac{1}{k^2} & \text{for } p < k - \epsilon \\ \frac{1}{p^2} & \text{for } p > k + \epsilon \end{cases}$$

The Effective Average Action

- Flow equation for $\tilde{\Gamma}_k[\varphi]$:

$$\frac{\partial \tilde{\Gamma}_k[\varphi]}{\partial k} = \frac{1}{2}\mathrm{Tr}\left\{ \left[R_k + \frac{\delta^2 \tilde{\Gamma}_k}{\delta\varphi\varphi} \right]^{-1} \frac{\partial R_k}{\partial k} \right\}$$



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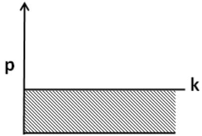

- No need for UV regulator.
- Complete set of solutions to flow equation $\leftrightarrow$ divergence-free QFT.

$$\{\tilde{\Gamma}_k, 0 \leq k < \infty\} \iff \text{complete QFT}$$

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- Why $\tilde{\Gamma}_k$ and not S_k?
  1. $\tilde{\Gamma}_k$ is the generator of 1PI Green's functions - directly related to scattering amplitudes.
  2. $\tilde{\Gamma}_k$ gives better approximation to a QFT.
  3. Don't have to construct a regulated path integral.

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  1. Can find the classical system whose quantization gave rise to the complete QFT.
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  4. Theory that we put on the lattice is given by S_k.

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$$\tilde{\Gamma}_k[\varphi] \to Z[J] = \int_{|p|<\Lambda} \mathcal{D}\phi e^{-S_\Lambda^{tot}[\phi]+J.\phi}$$

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- Regulated with sharp cutoff Λ, take $\Lambda \to \infty$.
- E. Manrique and M. Reuter (2008):

$$\boxed{\tilde{\Gamma}_{k=\Lambda}[\varphi] = S_\Lambda^{tot}[\phi] + \frac{1}{2} Tr_\Lambda ln\left\{ R_\Lambda + \frac{\delta^2 S_\Lambda^{tot}}{\delta\phi\delta\phi} \right\}}$$

$$\mathrm{Tr}_\Lambda\{...\} \equiv \mathrm{Tr}\{\theta(\Lambda^2 - p^2)[...]\}$$

- Derived by saddle point expansion - **approximate expression**.

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- Split modes $\phi \to \phi_> + \phi_<$ and propagators $\Delta \to \Delta_{IR} + \Delta_{UV}$.

$$Z[J] = \int_{|p|<\Lambda} \mathcal{D}\phi_> \mathcal{D}\phi_< e^{-\frac{1}{2}\phi_<.\Delta_{UV}^{-1}.\phi_< - \frac{1}{2}\phi_>.\Delta_{IR}^{-1}.\phi_> - S_{\Lambda_0}[\phi_< + \phi_>] + J.(\phi_< + \phi_>)}$$

$$\Delta_{IR} = \frac{C_{IR}(p,k)}{p^2}, \quad \Delta_{UV} = \frac{C_{UV}(p,k)}{p^2}$$

- Cutoff functions obey summation relation:

$$C_{IR} + C_{UV} = 1$$

Solving the Reconstruction Problem

- Compute integral over high momentum modes.

$$
\begin{aligned}
Z[J,\phi_<] &= \int \mathcal{D}\phi_> e^{-\frac{1}{2}\phi_>.\Delta_{IR}^{-1}.\phi_> - S_\Lambda[\phi_>+\phi_<]+J.(\phi_>+\phi_<)} \\
&= e^{\frac{1}{2}J.\Delta_{IR}.J+J.\phi_< - S_k[\Delta_{IR}.J+\phi_<]}
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$$J(p) = 0 \;\forall\; p > k \implies Z[J] = \int \mathcal{D}\phi_< e^{-\frac{1}{2}\phi_<.\Delta_{UV}^{-1}.\phi_< - S_k[\phi_<]+J.\phi_<}$$

- Recognise S_k as the interaction part of the bare action $S_k^{tot} = \frac{1}{2}\phi_<.\Delta_{UV}^{-1}.\phi_< + S_k$ regulated in the UV at scale k.

Solving the Reconstruction Problem

- Integral over high momentum modes:

$$Z[J,\phi_<] = \int \mathcal{D}\phi_> e^{-\frac{1}{2}\phi_>.\Delta_{IR}^{-1}.\phi_> - S_\Lambda[\phi_> + \phi_<] + J.(\phi_> + \phi_<)}$$
$$= e^{\frac{1}{2}J.\Delta_{IR}.J + J.\phi_< - S_k[\Delta_{IR}.J + \phi_<]}$$

- Interpret as a functional integral for field $\phi_>$ regulated in the IR at scale k (in presence of background field $\phi_<$).
- Simply related to the generator of connected Green's functions W_k (cutoff in the IR):

$$Z[J,\phi_<] = e^{W_k[J,\phi_<]}$$

Solving the Reconstruction Problem

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$$\Gamma_k^{tot}[\varphi,\phi_<] = -W_k[J,\phi_<] + J.\varphi = \frac{1}{2}(\varphi-\phi_<).\Delta_{IR}^{-1}.(\varphi-\phi_<) + \Gamma_k[\varphi]$$

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- Relationship between Γ_k and S_k:

$$\boxed{\Gamma_k[\varphi] = S_k[\phi] + \frac{1}{2}(\varphi-\phi).\Delta_{IR}^{-1}.(\varphi-\phi)}$$

- Note that $\Gamma_k[\varphi]$ is interaction part of $\Gamma_k^{tot}[\varphi,\phi_<]$ and $S_k[\phi]$ is the interaction part of $S_k^{tot}[\phi]$.

Solving the Reconstruction Problem

- Compare our expression

$$\Gamma_k[\varphi] = S_k[\phi] + \frac{1}{2}(\varphi - \phi).\Delta_{IR}^{-1}.(\varphi - \phi)$$

to E. Manrique and M. Reuter's

$$\tilde{\Gamma}_{k=\Lambda}[\varphi] = S_\Lambda^{tot}[\phi] + \frac{1}{2} Tr_\Lambda ln \left\{ R_\Lambda + \frac{\delta^2 S_\Lambda}{\delta\phi\delta\phi} \right\}$$

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- How is Γ_k (Legendre effective action) related to $\tilde{\Gamma}_k$(effective average action)?

Solving the Reconstruction Problem

- Legendre effective action (without background field)

$$\Gamma_k^{tot}[\varphi] = \Gamma_k[\varphi] + \frac{1}{2}\varphi.\Delta_{IR}^{-1}.\varphi$$

- Flow equation for Γ_k

$$\frac{\partial \Gamma_k[\varphi]}{\partial k} = -\frac{1}{2} Tr \left\{ \left[1 + \Delta_{IR} \frac{\delta^2 \Gamma_k}{\delta\varphi\delta\varphi} \right]^{-1} \frac{1}{\Delta_{IR}} \frac{\partial \Delta_{IR}}{\partial k} \right\}$$

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- Flow equation for effective average action $\tilde{\Gamma}_k$

$$\frac{\partial \tilde{\Gamma}_k[\varphi]}{\partial k} = \frac{1}{2}\mathrm{Tr}\left\{ \left[R_k + \frac{\delta^2 \tilde{\Gamma}_k}{\delta\varphi\varphi} \right]^{-1} \frac{\partial R_k}{\partial k} \right\}$$

Solving the Reconstruction Problem

- Split off kinetic part from $\tilde{\Gamma}_k$

$$\tilde{\Gamma}_k = \tilde{\Gamma}_k^{int} + \frac{1}{2}\varphi . p^2 . \varphi$$

- Flow equation becomes

$$\frac{\partial \tilde{\Gamma}_k^{int}}{\partial k} = \frac{1}{2} Tr \left\{ \left[R_k + p^2 + \frac{\delta^2 \tilde{\Gamma}_k^{int}}{\delta\varphi\delta\varphi} \right]^{-1} \frac{\partial R_k}{\partial k} \right\}$$

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$$\frac{\partial \tilde{\Gamma}_k^{int}}{\partial k} = \frac{1}{2} Tr \left\{ \left[R_k + p^2 + \frac{\delta^2 \tilde{\Gamma}_k^{int}}{\delta\varphi\delta\varphi} \right]^{-1} \frac{\partial R_k}{\partial k} \right\}$$

- Identify $p^2 + R_k = \frac{p^2}{C_{IR}} = \Delta_{IR}^{-1}$

$$\frac{\partial \tilde{\Gamma}_k^{int}[\varphi]}{\partial k} = -\frac{1}{2} Tr \left\{ \left[1 + \Delta_{IR} \frac{\delta^2 \tilde{\Gamma}_k^{int}}{\delta\varphi\delta\varphi} \right]^{-1} \frac{1}{\Delta_{IR}} \frac{\partial \Delta_{IR}}{\partial k} \right\}$$

- $\tilde{\Gamma}_k^{int}$ satisfies same flow equation as Γ_k!

$$\implies \tilde{\Gamma}_k^{int} = \Gamma_k$$

Overview

1. Introduction and Motivation
2. The Renormalization Group
3. RG Flows in Theory Space
4. The Effective Average Action
5. The Reconstruction Problem
6. Solving the Reconstruction Problem
7. Summary and Conclusions

Summary and Conclusions

- Solved the reconstruction problem.
- Found an exact relationship between $\tilde{\Gamma}_k[\varphi]$ and $S_k[\phi]$.
- Making contact with E. Manrique & M. Reuter's formula:

$$\Gamma_k[\varphi] = S_k[\phi] + \frac{1}{2}(\varphi - \phi).\Delta_{IR}^{-1}.(\varphi - \phi)$$

vs

$$\tilde{\Gamma}[\varphi]_{k=\Lambda} = S_\Lambda^{tot}[\phi] + \frac{1}{2} Tr_\Lambda ln\left\{ R_\Lambda + \frac{\delta^2 S_\Lambda^{tot}}{\delta\phi\delta\phi} \right\}$$

- Next stage: use metric $g_{\mu\nu}$ as dynamical degree of freedom.

Thank you