D-Brane Potentials in the Warped Resolved Conifold & Natural Inflation

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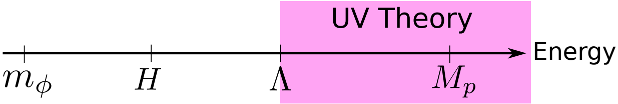
Based on arXiv:1409.1221
by ZK and Steven Thomas

Young Theorists' Forum, Annual High Energy Physics Conference
17/12/2014

Effective Field Theory



Two approaches:

- Bottom Up:
  - Begin only with light fields
  - Parametrize ignorance of heavier fields
- Top Down:
  - Begin with full UV theory
  - Derive low energy theory by integrating out heavier fields

Outline

1. Bottom Up: Inflation as an Effective Field Theory
2. Top Down: Inflation from String Theory
3. Summary

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   - Overview
   - Flux Compactifications
   - Calabi-Yau Manifolds

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Inflation

FRW metric

$$ds^2 = -dt^2 + a(t)^2 dx^i dx^j \delta_{ij}$$

$$H = \frac{\dot{a}}{a}$$

Inflation

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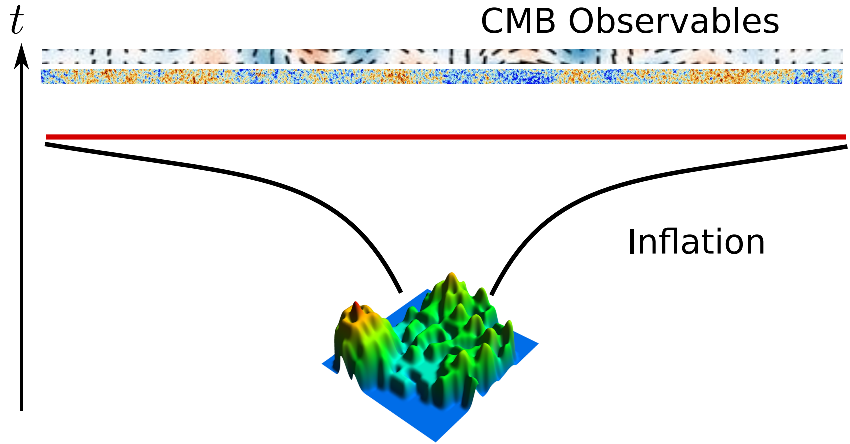
$$H = \frac{\dot{a}}{a}$$

Definition of inflation:

$$-\dot{H} \ll H^2$$

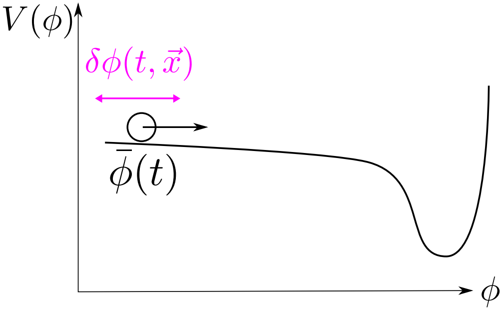
Stretching Quantum Fluctuations

- At least two light fields: inflaton and graviton
- Light fields have quantum fluctuations during inflation
- Inflation stretches these to classical observable scales



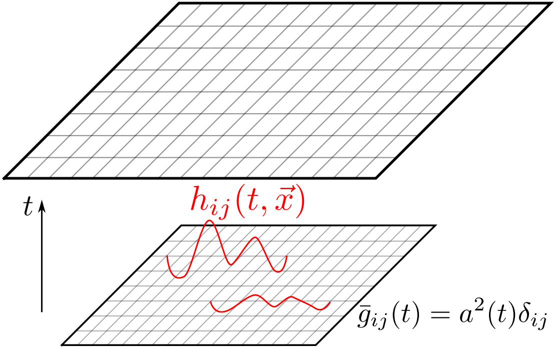
Simplest: Single Field, Slow Roll

- Simplest: single slowly rolling scalar field ϕ.
- Need a flat potential to get $-\dot{H} \ll H^2$.
- Fluctuations $\delta\phi$



Scalar Power Spectrum: $P_s(k) \sim \langle|\delta\phi_k|^2\rangle$

Gravitational Waves



Tensor Power Spectrum: $P_t(k) \sim \langle |h_k|^2 \rangle$

Detectable $r \Rightarrow$ Large Field Inflation

Tensor-to-scalar ratio r

$$r \equiv \frac{P_t(k)}{P_s(k)} \sim \frac{\langle |h_k|^2 \rangle}{\langle |\delta\phi_k|^2 \rangle}$$

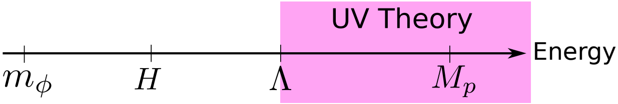
≈ 0.16 from BICEP2?

Lyth bound implies Large Field Inflation for detectable r

$$\frac{\Delta\phi}{M_p} \gtrsim \left(\frac{r}{0.01}\right)^{1/2} \Rightarrow \Delta\phi \gtrsim M_p.$$

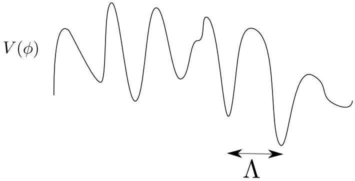
[Lyth,1996]

Large Field Inflation as an EFT

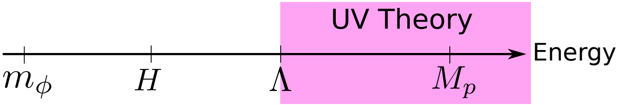


- UV Theory of gravity requires extra fields of mass $\Lambda < M_p$
- How do these fields couple to the inflaton?
- Parametrize ignorance by EFT potential

$$V(\phi) = V_{\text{S.R.}}(\phi) + \phi^4 \sum_n c_n \left(\frac{\phi}{\Lambda}\right)^n$$



Large Field Inflation as an EFT



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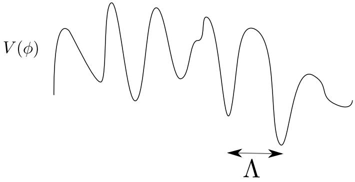

- Disaster for large-field inflation $\Delta\phi \gtrsim M_p$

Natural Inflation

[Freese, Freeman, Olinto, 1990]

- Maybe symmetry disallows these coupling terms
- Shift symmetry $\phi \to \phi +$ const.
- Natural Inflation: class of models with potential

$$V(\phi) = V_0^4 \left[1 + \cos\left(\frac{\phi}{f}\right) \right]$$

- Detectable r: $\quad V_0 \sim M_{GUT}, \quad f \sim M_p$
- Hard to achieve $f \sim M_p$ from stringy models, like axion inflation
- ZK & ST achieve these parameters in a brane model.

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Shopping List

- Open String: Brane Inflation
  - Type of brane: e.g. Dp-brane or NS5 or M5.
  - Warped
  - Unwarped
  - Relativistic (DBI)
  - Single or multiple
  - Wrapped brane?
  - Flux on brane?
  - Direction of motion
- Closed String: Moduli Inflation
  - Complex-structure moduli
  - Kähler moduli
  - p-form axions

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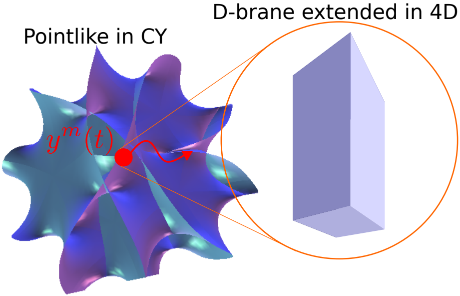
Embed D-brane



- Identify the coordinates of a **probe** D-brane moving in the extra dimensions with scalar fields.
- One of these coordinates $y^m(t)$ could be an inflaton.
- D-brane feels a potential $V(y)$ coming from the DBI & CS action for a brane in a SUGRA Flux Compactification background.

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Flux Compactifications: IIB Action

$$S_{\text{IIB}} = -\frac{1}{2\kappa_{10}^2}\int_{M_{10}} d^{10}X\sqrt{|g|}\left(R - \frac{|\partial\tau|^2}{2(\text{Im}\tau)^2} - \frac{|G_3|^2}{2\text{Im}\tau} - \frac{|\tilde{F}_5|^2}{4\cdot 5!}\right)$$
$$-\frac{1}{2\kappa_{10}^2}\frac{1}{4i}\int_{M_{10}}\frac{C_4\wedge G_3\wedge\overline{G}_3}{\text{Im}\tau} + S_{\text{branes}}$$

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Form fields:

$$\tilde{F}_5 \equiv F_5 - \frac{1}{2}C_2\wedge H_3 + \frac{1}{2}B_2\wedge F_3,$$
$$G_3 \equiv F_3 - \tau H_3$$

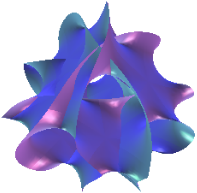
where

$$F_5 = dC_4, \qquad F_3 = dC_2, \qquad H_3 = dB_2$$
$$\tau = C_0 + ie^{-\Phi} \text{ axiodilaton}$$

Flux Compactifications: Ansatz

Ansatz: Warped Spacetime, parametrized by warp factor $\mathcal{H}(y)$

$$ds^2 = \underbrace{\mathcal{H}^{-1/2}(y)}_{\text{warping}} \underbrace{g_{\mu\nu}^{\text{FRW}} dx^\mu dx^\nu}_{\text{4D FRW}} + \mathcal{H}^{1/2}(y) \underbrace{g_{mn}^{\text{CY}} dy^m dy^n}_{\text{6D Calabi-Yau}}$$



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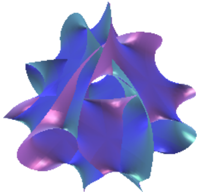

Ansatz: Flux, parametrized by $\alpha(y)$

$$C_4 = \alpha(y)\sqrt{g^{\text{FRW}}}dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 .$$

Flux Compactifications: Solution

Equation of motion is then

$$\nabla_{CY}{}^2\Phi_- = \frac{\mathcal{H}^{-2}(y)}{6\mathrm{Im}\tau}|G_-|^2 + \mathcal{H}(y)|\partial\Phi_-|^2 + \underbrace{\text{branes}}_{\geq 0}$$

where $\Phi_- \equiv \mathcal{H}^{-1}(y) - \alpha(y), \qquad G_- \equiv \star_6 G_3 - iG_3.$

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ISD Solution:

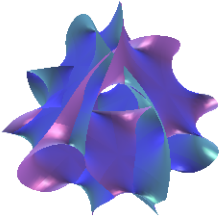
$$\Phi_- = 0, \qquad G_- = 0.$$

Brane feels a potential related to Φ_-

$$V_{D3}(y) = T_3\Phi_-(y)$$
$$V_{D5}(y) = \underbrace{\varphi(y)}_{\sim 0} + \lambda\Phi_-(y)$$

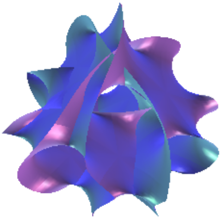
Outline

Calabi-Yau Manifolds



- Why Calabi-Yau? Preserve $1/4$ of the supercharges.
- 3-complex dimensions $(z^a, \bar{z}^{\bar{a}})$, i.e. as 6-real dimensions y^m.

Calabi-Yau Manifolds



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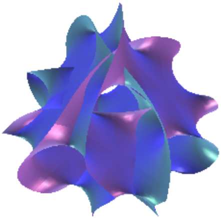
- **Ricci flat**: $R_{mn} = 0$
- **Kähler**: The Kähler form $J \equiv i g_{a\bar{b}} dz^a \wedge d\bar{z}^{\bar{b}}$ is closed $dJ = 0$.

Calabi-Yau Manifolds



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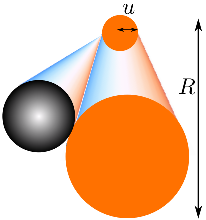
- **Ricci flat**: $R_{mn} = 0$
- **Kähler**: The Kähler form $J \equiv i g_{a\bar{b}} dz^a \wedge d\bar{z}^{\bar{b}}$ is closed $dJ = 0$.
- We don't know the explicit metric on any **compact** Calabi-Yau.

The Resolved Conifold

- The Resolved Conifold is Ricci Flat and Kähler
- But noncompact
- We **know the metric** explicitly



- At base looks like a cone over an $S^3 \times S^2$ base, with angles $\{\theta_i\}$
- At tip smoothed out, not conical
- **Warped** Resolved Conifold: Warp factor $\mathcal{H}$ sourced by a stack of $N \gg 1$ D3-branes at tip of Resolved Conifold.

Gluing the Resolved Conifold & Potentials

$$\nabla_{RC}{}^2\Phi_- = \frac{\mathcal{H}^{-2}(y)}{6\mathrm{Im}\tau}|G_-|^2 + \mathcal{H}(y)|\partial\Phi_-|^2$$

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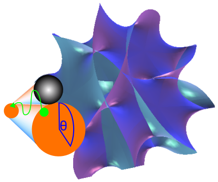
$$\nabla_{RC}{}^2\Phi_- = \frac{\mathcal{H}^{-2}(y)}{6\text{Im}\tau}|G_-|^2 + \mathcal{H}(y)|\partial\Phi_-|^2$$

- We compactify by gluing the Resolved Conifold to a bulk CY



- Gluing induces perturbations $\Phi_- = \mathcal{O}(\delta), G_- = \mathcal{O}(\delta), \delta \ll 1$
- Potential then satisfies Laplace on RC to leading order in δ

$$\nabla_{RC}{}^2\Phi_- = 0$$

- Can solve this exactly for Resolved Conifold. One solution is

$$\Phi_- \propto \cos\theta$$

Natural Inflation from a brane in the RC

- θ coordinate but not canonical scalar field
- Canonical field $\Theta = f\theta$.

$$V = V_0^4 \cos\theta = V_0^4 \cos\left(\frac{\Theta}{f}\right)$$

- BICEP2 needs: $V_0 \approx M_{GUT}, \qquad f \approx 5M_p$
- V_0 and f depend on choice of brane

D3-brane

- D3 is simplest, but can't achieve these values for V_0 and f.

$$S_{D3} = -T_3 \int_{M_4} d^4\xi \sqrt{P_4[g_{MN}]} + T_3 \int_{M_4} P_4[C_4]$$

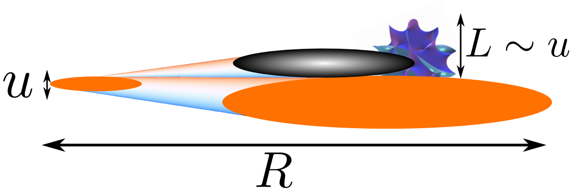
$$\Rightarrow \mathcal{L} \approx \frac{1}{2} T_3 g^{RC}{}_{mn} \dot{y}_m \dot{y}_n - T_3 \Phi_-(y) \qquad \text{(slowly moving brane)}$$

$$\approx \frac{1}{2} T_3 u^2 \dot{\theta}^2 - T_3 \Phi_-(\theta) \qquad \text{(motion in } \theta \text{ direction)}$$

$$\approx \frac{1}{2} \dot{\Theta}^2 - T_3 \Phi_-(\Theta/f) \qquad \text{for } f = u\sqrt{T_3}$$

D3-brane

Can $f = u\sqrt{T_3}$ ever be $\mathcal{O}(M_p)$?



- Planck mass is only computable if we have a long throat $R \gtrsim L$, giving $M_p \gtrsim \sqrt{N}R$, for $N \gg 1$ from the warping.
- Expect $L \sim u$ when glue RC to bulk, natural lengthscale should be about the same in gluing region.
- So should have that $M_p \sim \sqrt{N}R \gg R \gtrsim L \sim u$, i.e. $M_p \gg u$
- So we can't get $f \sim M_p$ for a D3

D5-brane

- D5 can be wrapped p times around 2-cycle Σ_2 in compact dimensions
- Turn on F_2 flux on the probe brane quantized by q
- Have the D5 near the tip at $r = r_{\text{min}} \sim u/50$

$$\mathcal{L} \approx \frac{1}{2} 4\pi p T_5 \left(\frac{u}{r_{\text{min}}}\right)^2 l_s^2 \sqrt{4\pi g_s N} u^2 \dot{\theta}^2 - 12\pi^2 l_s^2 p q T_5 \Phi_-(\theta)$$

$$\approx \frac{1}{2} \dot{\Theta}^2 - M_{\text{GUT}}^4 \cos(\Theta/f) \qquad \text{(choose } q \text{ to get } M_{\text{GUT}})$$

$$\text{for } \Theta = f\theta, \qquad f^2 = 4\pi p T_5 \left(\frac{u}{r_{\text{min}}}\right)^2 l_s^2 \sqrt{4\pi g_s N} u^2$$

$$\text{Use } p \sim \left(\frac{r_{\text{min}}}{u}\right)^2 \sqrt{N} \qquad \text{to set } f \sim M_p \sim \sqrt{N} R$$

- Can show p is not so large that backreaction is a problem.

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   - Observable $r \Rightarrow$ Large Field Inflation
   - Large Field Inflation is sensitive to UV completion of gravity.
2. Top Down: Inflation from String Theory
   - Can get Natural Inflation from string theory model
   - We used a wrapped D5-brane with flux at tip of the warped RC.

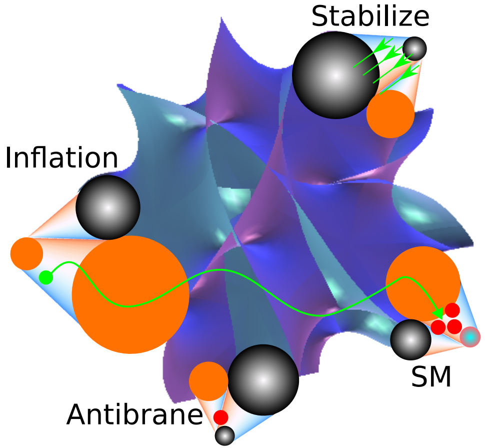
Thank you

Special thanks to my supervisor Prof. Steven Thomas

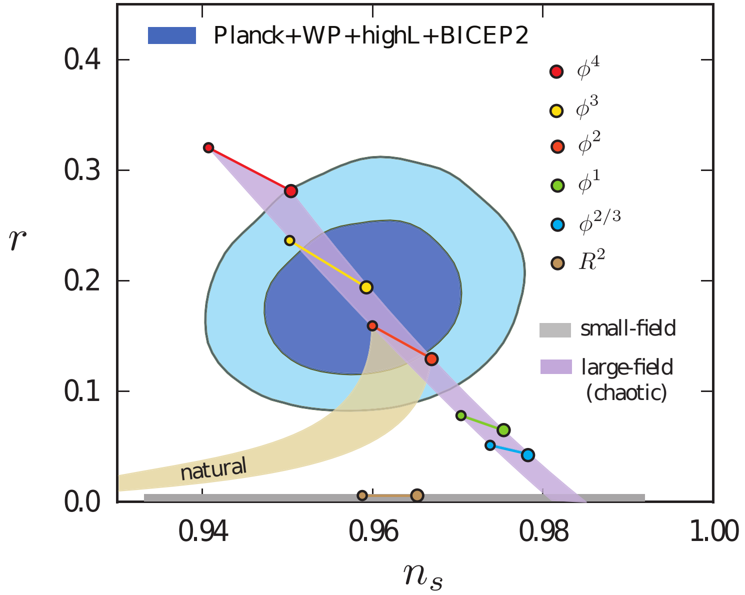
Too Good to be True?

- Bulk can have many moduli - massless scalar fields - which can couple to the inflaton.
- Moduli stabilization achieved for the Deformed Conifold - cousin of the Resolved Conifold. [Giddings, Kachru, Polchinski, 2002]
- However, no analytic solution to Laplace on DC - can't probe the tip.
- Would be good to stabilize moduli using the RC - future work!
- Alternative is to have multiple throats. [Chen, 2005]

Many Throats?



Maybe there are multiple throats?



Planck+WP+highL+BICEP2
ϕ^4
ϕ^3
ϕ^2
ϕ^1
$\phi^{2/3}$
R^2
small-field
large-field
(chaotic)
natural
0.4
0.3
0.2
0.1
0.0
r
0.94
0.96
0.98
1.00
n_s