1/D Expansion of Quantum Gravity

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Outline

- Overview: Quantum Gravity(QG) & Non-Perturbative Renormalisation Group(NPRG) & Asymptotic Safety(AS)

- Background Field Technique

- Motivations for 1/D Expansion: large-D in the literature

- 1/D Results & Discussion

Overview: Quantum Gravity

Canonical dimensions of Newton's coupling: $[G_N] = 2 - D$

1. Gravitational interactions become significant at the Planck scale $\sim 10^{19}\mathrm{GeV} \rightarrow$ a scale much higher than our reach with current accelerators.

2. Perturbatively non-renormalisable due to negative mass dimensions.

- Some attempts to solve this problem,

non-QFT

  - Loop Quantum Gravity (discretise space)
  - Causal Sets (discretise space-time)
  - Causal Dynamical Triangulations (discretise space-time in terms of fractal triangles)
  - and of course String Theory

QFT

  - Effective Field Theory (below Planck scale)
  - Asymptotic Safety (UV completion)

Overview: NPRG

- Based on Wilson's idea of renormalisation.

- Define the scale dependent generating functional and the effective action; $Z[J] \to Z_k[J] \qquad \Gamma[\phi] \to \Gamma_k[\phi]$

- Such that when $k \to \Lambda$ (the inverse lattice spacing): Bare action (all fluctuations are frozen)

- when $k \to 0$: Full effective action (all fluctuations integrated out)

Overview: NPRG

- We obtain the flow equation (the Wetterich equation)

$$\partial_t \Gamma_k[\phi] = \mathrm{Tr}\left[(\Gamma_k^{(2)}[\phi] + R_k(q))^{-1}\partial_t R_k(q)\right] \qquad t = \ln k$$

- R_k is the IR regualator, controls the coarse graining. Typically taken as the optimised cutoff (D. Litim '00, '01):

$$R_k(q) = (k^2 - q^2)\Theta(k^2 - q^2)$$

- $\Gamma_k^{(2)}$ is the Hessian of the effective average action.

- For gravity, let's take a simple case: Einstein-Hilbert gravity, an action with only a first order Ricci scalar and a cosmological constant.

Background Field Technique

- Gravity is governed by the fluctuations in the metric. To apply QFT we need a background metric.

- Two ways of doing this: (i.e. the ones that we looked into)
  - Conventional Methods (single metric ansatz)
  - Bimetric Ansatz

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

$$\Gamma_k[g, \bar{g}] = \int d^D x \sqrt{g} \frac{1}{16\pi G_k} \left(R(g) - 2\Lambda_k \right)$$

$$\Gamma_k[\bar{g}, h] = \int d^D x \sqrt{\bar{g}} \frac{1}{16\pi G_k^B} \left(R(\bar{g}) - 2\Lambda_k^B \right) + \int d^D x \sqrt{g} \frac{1}{16\pi G_k} \left(R(g) - 2\Lambda_k \right)$$

- Plus the classical gauge and ghost actions.

Asymptotic Safety

- Recall asymptotic freedom: the running coupling constant reaches zero in the UV
- Similarly, an AS theory is where the running coupling reaches a constant and stops running in the UV => Fixed Points
- We test this by looking into the beta functions.
- For Einstein-Hilbert gravity, two couplings: gravitational constant and cosmological constant.

Large-D in Perturbation Theory

A. Strominger, DOI:10.1103/PhysRevD.24.3082

- Write the Green's Functions as:

$$G = \sum_{m,n} G_{mn} \kappa^m \left(\frac{1}{D}\right)^n \qquad \kappa = \frac{1}{8\pi G_N}$$

where D is the number of dimensions, gives Feynman diagram sum in powers of 1/D.

- D dependence in gravity is coming from both the dof of the diffeomorphism symmetry group and the loop calculations.

- Only graphs in which no two bubbles are touching each other survives the large-D limit. Nested graphs are higher in 1/D, hence dies.

- Quantum gravity simplifies, although *perturbative renormalisation of the theory is still unsuccessful*.

Large-D in Effective Field Theory

N. E. J. Bjerrum-Bohr, arXiv:hep-th/0310263v2

$$\mathcal{L}_{\text{effective EH}} = \int \mathrm{d}^D x \sqrt{-g} \left(\left(\frac{2R}{\kappa^2} + c_1 R^2 + c_2 R^{\mu\nu} R_{\mu\nu} + \cdots \right) + \mathcal{L}_{\text{eff. matter}} \right)$$

- Quantum gravity below Planck scale, investigated by using a 1/D expansion.

- Agrees with the previous results from A. Strominger except more graphs contribute.

- Quantum gravity simplifies in the large-D limit, *below Planck scale*.

Large-D in Classical GR

R. Emparan et. al. arXiv:1302.6382

- Interaction between the blackholes reduces with large-D, so the theory becomes non-interacting in the D goes to infinity limit.
- Large-D limit simplifies the theory even in the classical limit of general relativity

Large-D in Lattice Quantum Gravity

H.W. Hamber, R.M.Williams arXiv:hep-th/0512003

- Examination of a 1/D expansion in Lattice QG based on Regge's simplicial construction.
- Scaling exponent approaches 0 in the large-D.
- It is concluded that "The action simplifies considerably in the large-D limit."

Conventional Method (single metric)

(M. Reuter, arXiv:hep-th/9605030)

(D.Litim, arXiv:hep-th/0312114)

$$\beta_g = (D - 2 + \eta) g$$

$$\beta_\lambda = (2 - \eta)\lambda + \left(a_1(\lambda, D) + \eta a_2(\lambda, D)\right) g$$

$$\eta = \frac{b_1(\lambda, D) g}{1 - b_2(\lambda, D) g}$$

- where eta is the anomalous dimension.
- coefficient functions are functions of dimensions, D and lambda, gives us the fixed points in powers of 1/D.
- Note that, gauge is fixed and the gauge dependence is checked. (gauge dep. is only in the higher order terms.)

Fixed Points

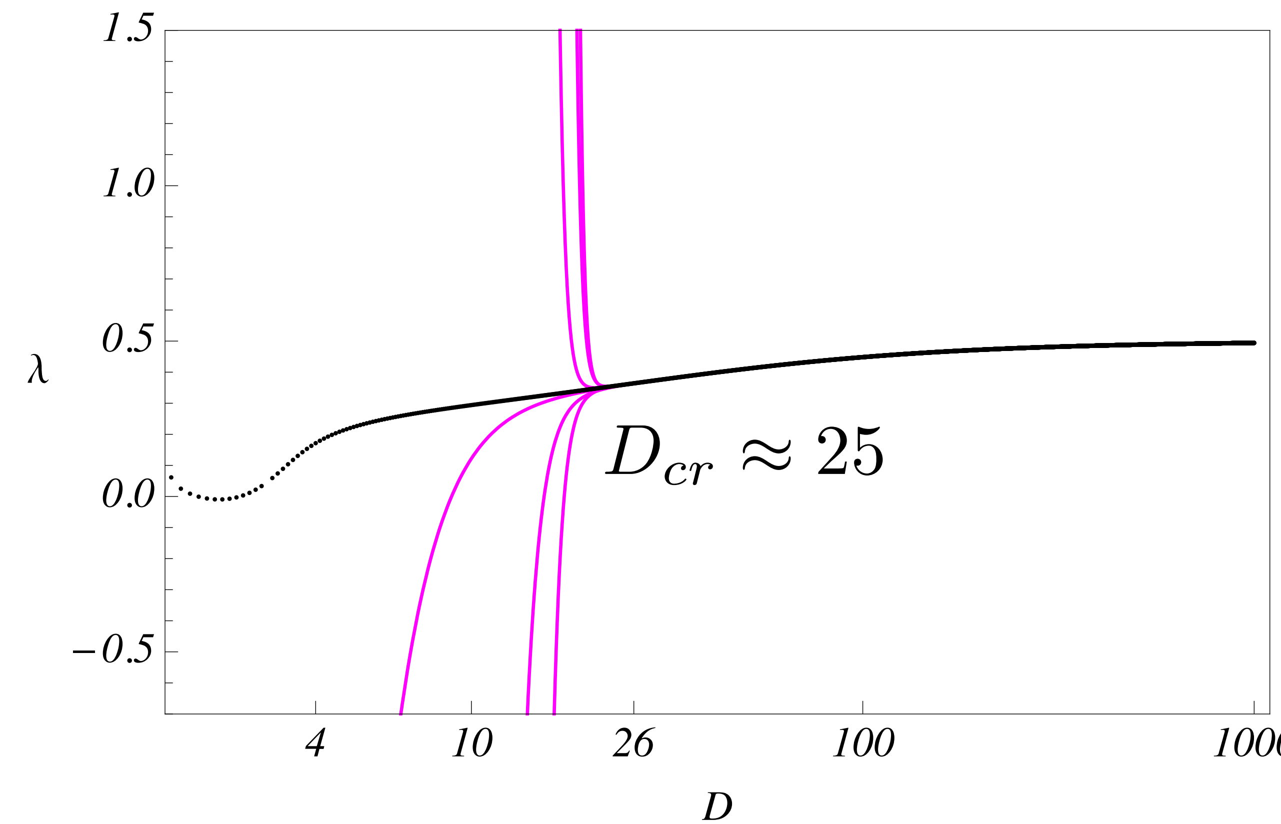
2 differential eqns. (beta functions) with 2 unknowns (lambda and g), solve simultaneously and expand in 1/D to get:

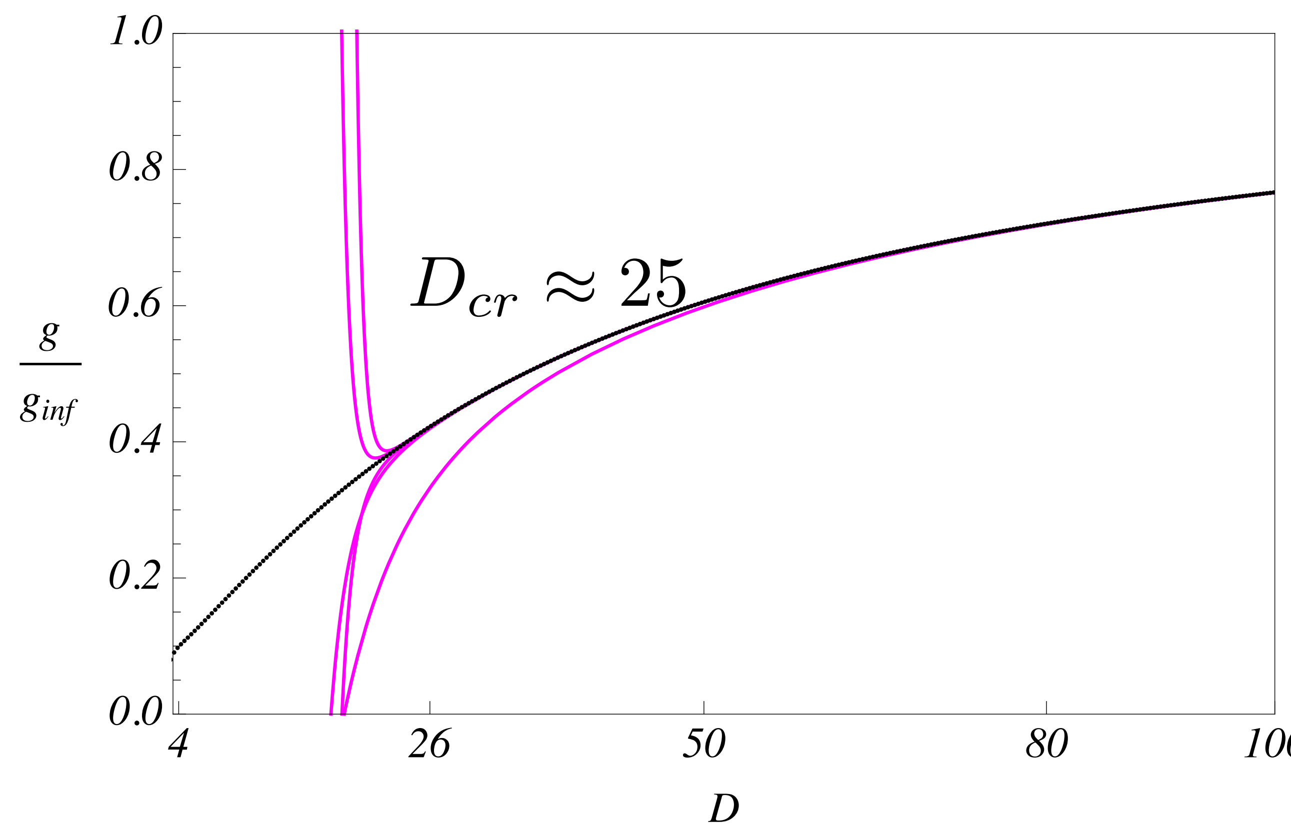
$$\lambda = \frac{1}{2} - \frac{6}{D} + \frac{90}{D^2} + \cdots$$

$$g = \frac{6c_D}{D^3}\left(1 - \frac{28}{D} + \cdots\right)$$

in Feynman gauge

$$c_D = (4\pi)^{D/2-1}\Gamma(D/2+2)$$





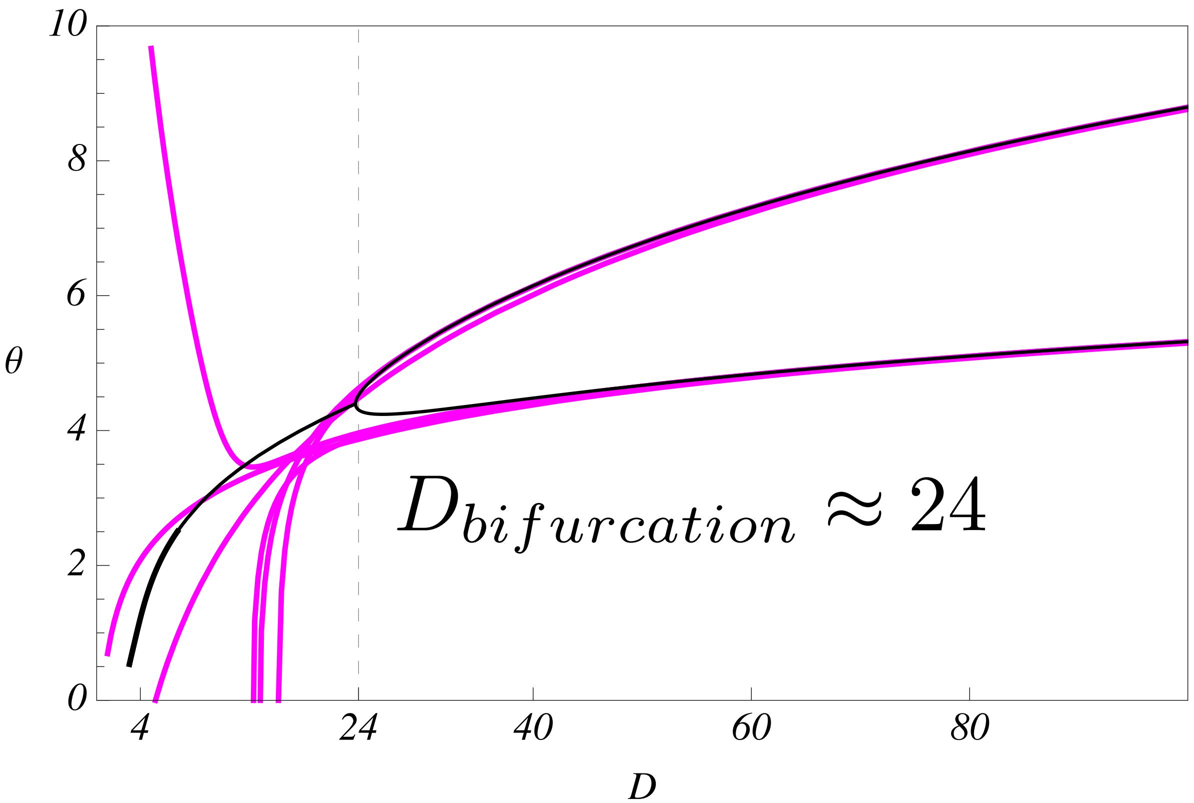
Scaling Exponents

- The universal quantities are the so-called scaling exponents defined as minus the eigenvalues of the stability matrix: $$\beta_{i,j} = \frac{\partial \beta_i}{\partial g_j}$$
- defines how the coupling approach the fixed point.

$$\theta_1 = \frac{D^3}{156}\left(1 + \frac{1}{13D} + \frac{50506}{D^2}\cdots\right)$$

$$\theta_2 = 2D\left(1 + \frac{1}{D} + \frac{98}{D^2}\cdots\right)$$

in Feynman gauge



Bimetric Truncation

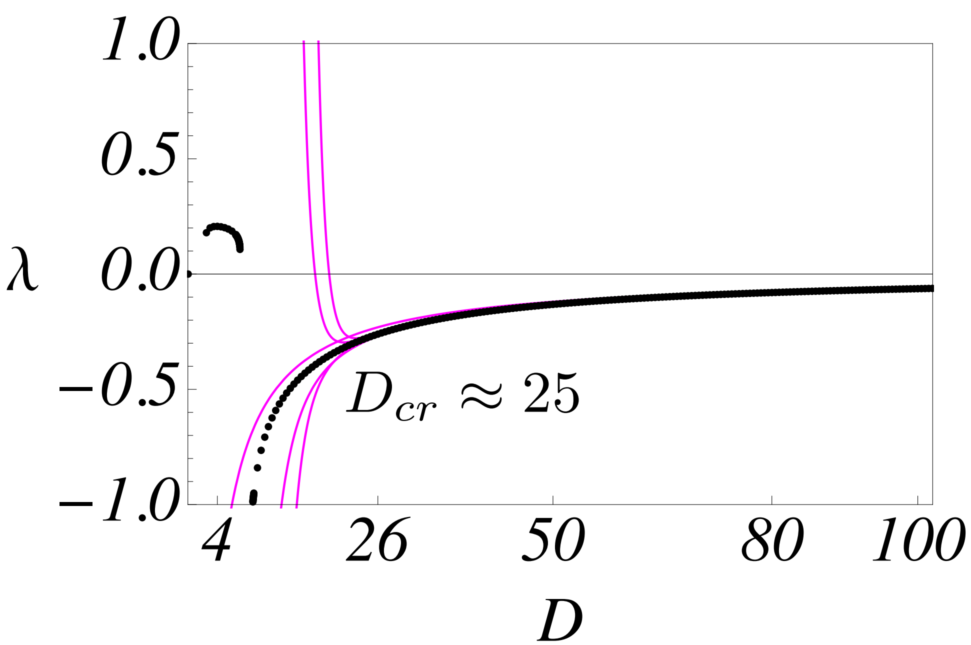
- Same form of beta functions as the previous case.
- a set of beta functions for the dynamical part of the metric and the background part of the metric. Dynamical beta functions are independent of the background couplings => background independence
- In this talk we will only look into the dynamical couplings as they are the equivalent couplings as the conventional case.
- Gauge fixing is taken as the Feynman gauge.

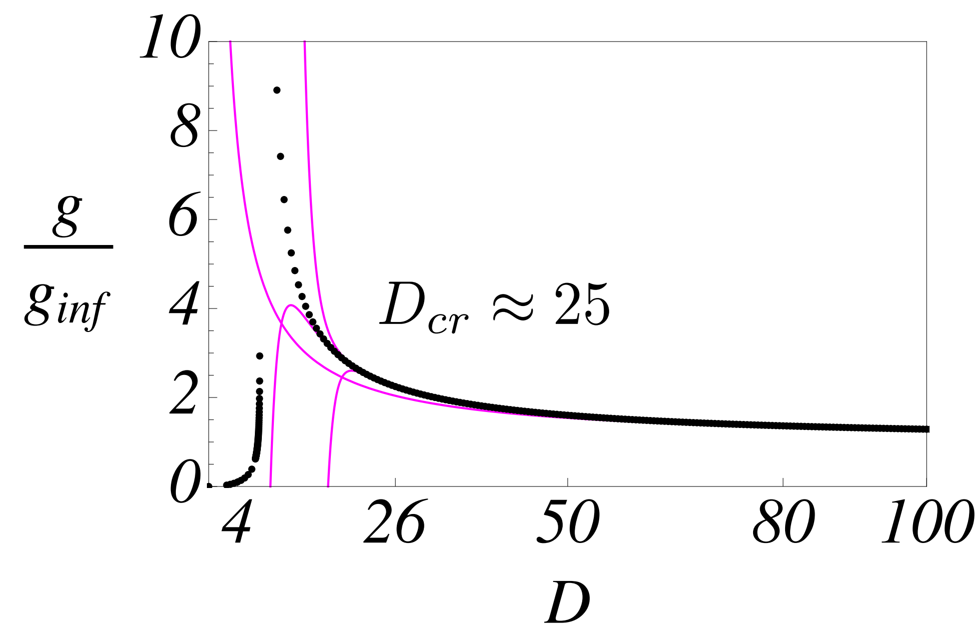
Fixed Points and Scaling Exponents

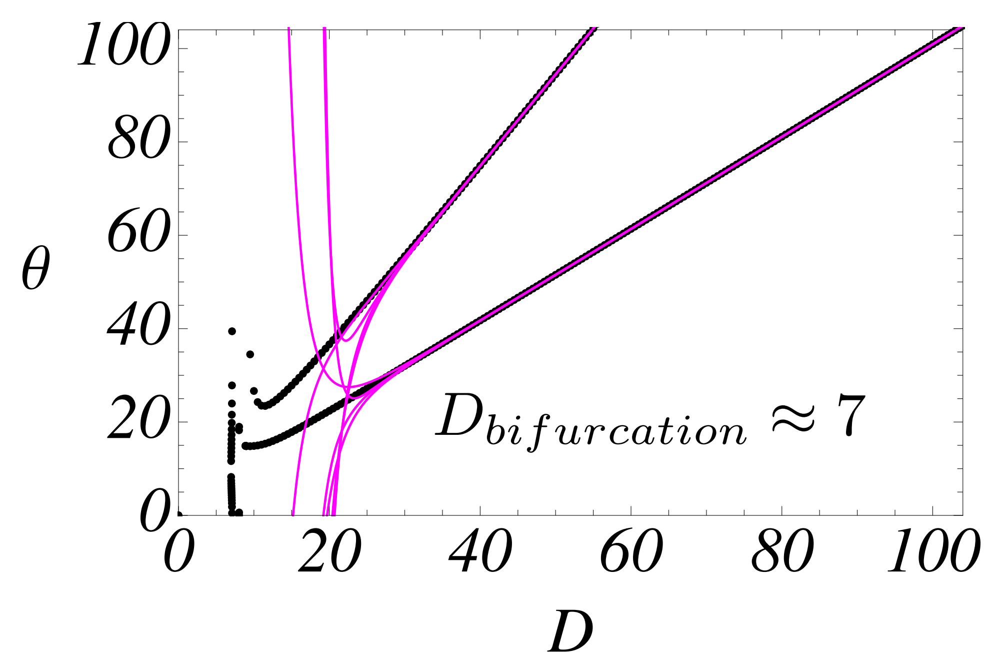
$$\lambda = -\frac{6}{D}\left(1 + \frac{7}{D} + \cdots\right)$$
$$g = \frac{6c_D}{D^3}\left(1 + \frac{27}{D} + \cdots\right)$$

and

$$\theta_1 = D + \frac{108}{D} + \cdots$$
$$\theta_2 = 2D - 8 + \frac{162}{D} + \cdots$$







Discussion

- Successful 1/D expansion of quantum gravity achieved for the first time. Gauge independent fixed points exist in very large dimensions with a consistent leading order behaviour.

- Radius of convergence is around D~25 from different approximations. Physical meaning to be explored.

- Scaling exponent (leading order) for the gravitational constant, “2D” is consistent with the literature. (lattice results, previous AS studies.)

- *Open questions:* What causes bifurcation? What causes the divergence in both cases?