# Writing your own Monte Carlo Integrator

Thomas Morgan

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# Disclaimer

What I won't be talking about...

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Sherpa Herwig++ Pythia

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# Ingredients

## Things we need

- VEGAS + Phase Space Generator
- Parton Distribution Functions (PDFs)
- Matrix elements
- Jet algorithm + observables

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- VEGAS + Phase Space Generator
- Parton Distribution Functions (PDFs)
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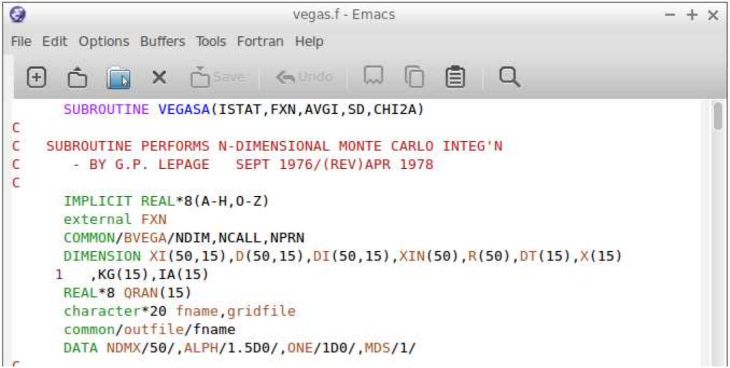
- Jet algorithm + observables
- Subtraction terms (beyond Leading Order (LO))

# VEGAS

Older than the known universe.



# VEGAS

- Use the idea of importance sampling.
- Generates a set of random numbers, these random numbers are fed into the phase space generator to generate a unique phase space point.
- Start with a uniform unit *n*-dimensional grid, where *n* is the required number of variables to define your phase space point.
- Once the initial sample is complete, VEGAS adapts the grid to focus on the dominant features.
- Rinse and repeat multiple times until you have a good convergence.

# from Protons to Partons

To compute cross sections we need to exploit the factorisation of QCD into low energy physics and high energy physics.

$$d\sigma = \sum_{i,j} \int \frac{d\xi_1}{\xi_1} \frac{d\xi_2}{\xi_2} \overbrace{f_i(\xi_1, \mu_F^2) f_j(\xi_2, \mu_F^2)}^{\text{Parton Distribution Functions}} \underbrace{d\hat{\sigma}_{ij}(\alpha_s(\mu_R), \mu_R, \mu_F)}_{\text{Partonic Cross Section}} \quad (1)$$

$\xi_1$ and $\xi_2$ are the momentum fractions of parton 1 and 2 respectively.

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# Parton Distribution Functions

Parton Distribution Functions (PDFs) give us the probability of finding a parton with a certain momentum fraction within a proton.
They describe low energy physics of the proton. This is impossible to model in any meaningful way, we fit PDFs using known results from previous collider experiments.

# Partonic Cross sections

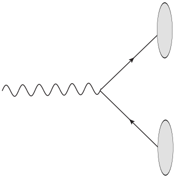
- We're interested in computing cross sections for massless QCD in a perturbative regime,

$$\mathrm{d}\hat{\sigma}_{ij} = \mathrm{d}\hat{\sigma}_{ij}^{LO} + \left(\frac{\alpha_s(\mu_R)}{2\pi}\right)\mathrm{d}\hat{\sigma}_{ij}^{NLO} + \left(\frac{\alpha_s(\mu_R)}{2\pi}\right)^2 \mathrm{d}\hat{\sigma}_{ij}^{NNLO} + \mathcal{O}(\alpha_s^3). \quad (2)$$

- What do we need to calculate a cross section for a given order?

# Matrix Elements

- Describes the high energy physics of the event (the interesting bit).

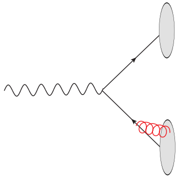


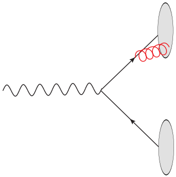
Numerous tools on the market for tree (helicity amplitudes, colour decompositions, recursion relations, . . . ) and one loop (integrand reduction, generalised unitarity, . . . ) scattering amplitudes.

# NLO Corrections

One extra power of $\alpha_s$. This implies we are left with two possibilities

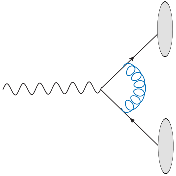
- Real corrections





Jet function maps 3 partons $\rightarrow$ 2 jets.

- Virtual corrections



# IR singularities in Real Corrections

Consider the real radiation corrections to the $\gamma^* \to 2$ jet process, the matrix element behaves like

$$|M(1_q, i_g, 2_{\bar{q}})|^2 \propto \frac{1}{E_g(1-\cos(\theta_{qg}))(1-\cos(\theta_{\bar{q}g}))} \quad (3)$$

## Singularities

- $E_g \to 0$, 'soft singularity'
- $\theta_{qg} \to 0$, 'collinear singularity'
- $\theta_{\bar{q}g} \to 0$, 'collinear singularity'

Singularities are bad!

# IR singularities in Virtual Corrections

An explicit pole structure appears from dimensional regularisation.

$$M_2^1(1_q, 2_{\bar{q}}) = 2\overbrace{I_{qq}(\epsilon, \mu; s_{12})}^{\text{Catani pole structure}} M_2^0(1_q, 2_{\bar{q}}) + \mathcal{O}(\epsilon^0) \tag{4}$$

Explicit pole structures are really bad . . .

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# Subtraction

$$\mathrm{d}\hat{\sigma}^{NLO}_{q\bar{q}} = \int_{\mathrm{d}\sigma_3} \left(\mathrm{d}\hat{\sigma}^{R,NLO}_{q\bar{q}} - \mathrm{d}\hat{\sigma}^{S,NLO}_{q\bar{q}}\right) + \int_{\mathrm{d}\sigma_2} \left(\mathrm{d}\hat{\sigma}^{V,NLO}_{q\bar{q}} - \mathrm{d}\hat{\sigma}^{T,NLO}_{q\bar{q}}\right), \quad (5)$$

where each set of brackets is free of IR poles. Also

$$\mathrm{d}\hat{\sigma}^{T,NLO}_{q\bar{q}} = -\int_{\mathrm{d}\sigma_1} \mathrm{d}\hat{\sigma}^{S,NLO}_{q\bar{q}}. \quad (6)$$

Unlike UV poles, your cross section and all IR-safe observables are *not* dependent on your subtraction scheme.

# NLO Subtraction

## Subtraction schemes

- Catani-Seymour (CS) dipole subtraction
- Frixione-Krunszt-Signer (FKS) subtraction
- Phase space slicing
- Sector Decomposition

# Antenna Subtraction

- Follows a very similar idea to CS dipole subtraction.
- Exploit the IR universal factorisation of QCD

$$M_3^0(1_q, i_g, 2_{\bar{q}}) \overset{i_g}{\underset{\text{unresolved}}{\longrightarrow}} \overbrace{A_3^0(1_q, i_g, 2_{\bar{q}})}^{\text{Antenna function}} \; \underbrace{M_2^0(\widetilde{(1i)}_q, \widetilde{(i2)}_{\bar{q}})}_{\text{reduced matrix element}} \,. \tag{7}$$

- The antenna function only depends on the momentum configuration and flavours of the unresolved parton and the hard radiators.
- By construction it contains all the unresolved limits between the two hard radiators and the unresolved parton.

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# Antenna Subtraction

$$\frac{M_3^0(1_q, 3_g, 2_{\bar{q}})}{M_2^0(\widetilde{(13)}_q, \widetilde{(32)}_{\bar{q}})} \equiv A_3^0(1_q, 3_g, 2_{\bar{q}}). \tag{8}$$

Once we have an antenna we can recycle this for arbitrarily complicated processes with the same unresolved limits.

## Problems

- The resulting antenna must be suitably simple such that we can integrate it analytically.
- We need to define a map from the $n+1 \to n$ phase space such that we correctly intepolate between the limits in your antenna.

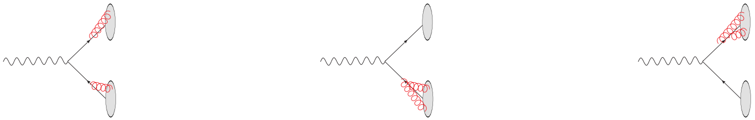
# Processes

- quark-antiquark antenna: $\gamma^* \to qg\bar{q}$
- quark-gluon antenna: $\tilde{\chi} \to \tilde{g}gg, \tilde{\chi} \to \tilde{g}q\bar{q}$
- gluon-gluon antenna: $H \to ggg, H \to gq\bar{q}$

# NNLO Corrections

Now we have two extra powers of $\alpha_s$ and 3 possible corrections.

- Real-Real corrections



- Real-Virtual corrections

- Virtual-Virtual corrections

# NNLO cross section

$$\begin{aligned} d\hat{\sigma}_{q\bar{q}}^{NNLO} &= \int_{d\sigma_4} \left( d\hat{\sigma}_{q\bar{q}}^{RR,NNLO} - d\hat{\sigma}_{q\bar{q}}^{S,NNLO} \right) \\ &+ \int_{d\sigma_3} \left( d\hat{\sigma}_{q\bar{q}}^{RV,NNLO} - d\hat{\sigma}_{q\bar{q}}^{T,NNLO} \right) \\ &+ \int_{d\sigma_2} \left( d\hat{\sigma}_{q\bar{q}}^{VV,NNLO} - d\hat{\sigma}_{q\bar{q}}^{U,NNLO} \right) \end{aligned} \tag{9}$$

# New Integredients for NNLO

## New Integredients

- $n+2$ parton phase space generator - trivial (ish)
- a numerically stable one loop matrix element - usually ok
- new subtraction terms - very hard
- two loop matrix element - very hard

# Conclusions

- The difference between an NLO and NNLO MC integrator is trivial once we have the subtraction scheme and two loop matrix elements.
- Antenna subtraction provides a numerically efficient and relatively simple approach to dealing with IR singularities at NNLO.
- Hopefully we'll be able to provide real physics results in the not too distant future.