Introduction to the gradient flow in QCD

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- Motivation
- Gradient flow in QCD
- D+1 dimensional theory and renormalisation
- Energy momentum tensor
- Outlook

Motivation

- Bare, local, composite operator

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- Dunne, Resurgence: Gradient flow for sign problem, steepest decent

The gradient flow in pure gauge theory

Euclidean action

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Flow equation

$$\partial_t B_\mu(t,x) = D_\nu G_{\nu\mu}$$

$$D_\nu = \partial_\nu + g_0 B_\nu, \quad G_{\nu\mu} = \frac{1}{g_0}[D_\nu, D_\mu]$$

The gradient flow in pure gauge theory - smearing

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$$B_\mu(t,x) = \int d^D y \; K_t(x-y)_{\mu\nu} A_\nu(y) + O(g_0)$$

$$K_t(x) = \frac{\mathrm{e}^{-\frac{x^2}{4t}}}{(4\pi t)^{D/2}}$$

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Exponential damping, smearing with radius $\sqrt{8t}$

Field Theory in D+1 dimensions and additional Feynman rules

$$S_{tot} = S + S_{fl}$$

Feynman rules

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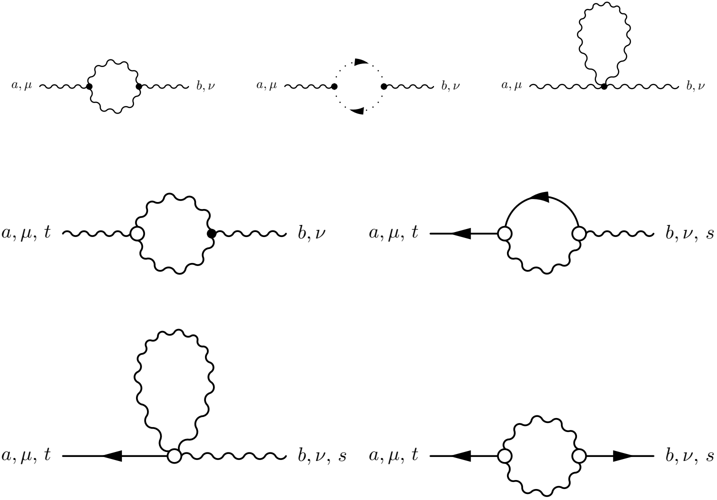
$$BL \quad \propto \quad \theta(t-s)\tilde{K}_t(p) \quad \propto \quad \theta(t-s) e^{-tp^2}$$

Pure gauge



Renormalisation of the bulk field

- Pure gauge: no additional renormalisation for bulk field required
- Prove to all orders with BRST (Lüscher)
- Special for gauge case; not trivial for fermions
- In general: divergencies $\propto \delta(t) \Rightarrow$ only at the boundary
- Seen from e^{-tp^2}: large momenta suppressed
- Fermions: similar treatment, multiplicative renormalisation
  $\chi = Z_\chi^{-1/2} \chi_R, \qquad L = Z_\chi^{1/2} L_R$

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- Interesting for many reasons:
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  - Quark gluon plasma
  - BSM: Scale invariance, conformal window
- Non-perturbative: lattice
- But: no translational invariance: need definition that recovers scale invariance in continuum limit
- EMT not finite anymore (Ward identity)
- Problem with subtraction of "infinities"
- Gradient flow might solve this last problem

Summary and Outlook

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- Define renormalisation independent observables and finite composite operators
- General aspects of non-Abelian field theories, e.g. scale invariance due to a well-defined EMT on lattice
- High precision measurements

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Thank you!