

arXiv: 1412.3467 [hep-th]

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# Towards understanding the ultraviolet behavior of quantum loops in infinite- derivative theories of gravity

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# Collaborators

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## Other people's work

- E.T. Tomboulis, arXiv: 9702146 [hep-th]
- L. Modesto, arXiv: 1107.2403 [hep-th]
- D. Anselmi, arXiv:1302.7100 [gr-qc]

# Aim

- Our aim is to construct a UV-finite theory of quantum gravity that is not plagued by pathologies such as ghosts
- Towards that end, we consider a scalar field theory toy model
- Based on that, can we formulate a complete theory of quantum gravity?

# Degree of Divergence in GR

- The superficial degree of divergence in  $d$  dimensions is  $D = Ld + 2(V - I)$
- $L$  is the number of loops,  $V$  is the number of vertices and  $I$  is the number of internal propagators
- Use the topological relation  $L = 1 + I - V$
- In four dimensions, we get  $D = 2 + 2L$
- The superficial degree of divergence keeps increasing as  $L$  increases

# Renormalizability of GR

- Einstein-Hilbert action:

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} R$$

- Pure gravity is renormalizable at 1-loop order
- 1 new counterterm required at 2-loop order

# Renormalizability of GR

- Stelle (1977) has shown that fourth-order pure gravity is renormalizable!

$$S = - \int d^4x \sqrt{-g} (\alpha R_{\mu\nu} R^{\mu\nu} - \beta R^2 + \kappa^{-2} \gamma R)$$

where  $\gamma = 2$  &  $\kappa^2 = 32\pi G$

- We do not have to include  $\int d^4x \sqrt{-g} R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta}$  because of the Gauss-Bonnet topological invariance in four dimensions:

$$\int d^4x \sqrt{-g} (R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} - 4R_{\mu\nu} R^{\mu\nu} + R^2)$$

vanishes in Minkowski spacetime

# Ghosts

- Unfortunately, Stelle's theory, as higher-derivative theories generically do, contains ghosts (poles in the propagator with negative residue); specifically, a massive spin-2 ghost

$$\Pi(p^2) = \frac{1}{p^2(p^2 + m^2)} \sim \frac{1}{p^2} - \frac{1}{p^2 + m^2}$$

where  $m^2 > 0$

- **Unitarity is violated**
- **We want to get rid of the ghost**



# Non-local Higher-derivative Gravity

- Non-local means that we consider an infinite series of higher-derivative terms in the action
- The most general covariant action up to  $\mathcal{O}(h^2)$  (Biswas, Gerwick, Koivisto, Mazumdar, *Phys. Rev. Lett.* **108** (2012) 031101) is

$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{2} + R_{\mu_1 \nu_1 \lambda_1 \sigma_1} \mathcal{O}_{\mu_2 \nu_2 \lambda_2 \sigma_2}^{\mu_1 \nu_1 \lambda_1 \sigma_1} R^{\mu_2 \nu_2 \lambda_2 \sigma_2} \right]$$

- $\mathcal{O}$  is a differential operator containing covariant derivatives and  $\eta_{\mu\nu}$
- The quadratic curvature part of the action up to  $\mathcal{O}(h^2)$  can be written, after many simplifications, as

$$S_q = \int d^4x \left[ R \mathcal{F}_1(\square) R + R_{\mu\nu} \mathcal{F}_2(\square) R^{\mu\nu} + R_{\mu\nu\lambda\sigma} \mathcal{F}_3(\square) R^{\mu\nu\lambda\sigma} \right],$$

since the covariant derivatives take on the Minkowski values

# Non-local Higher-derivative Gravity

- As we shall see later, if we choose  $\mathcal{F}_3(\square) = 0$   
&  $\mathcal{F}_1(\square) = \frac{e^{-\frac{\square}{M^2}} - 1}{\square} = -\frac{\mathcal{F}_2(\square)}{2}$ , we obtain the ghost-free action (Biswas, Gerwick, Koivisto, Mazumdar, *Phys. Rev. Lett.* **108** (2012) 031101)

$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{2} + R \left[ \frac{e^{-\frac{\square}{M^2}} - 1}{\square} \right] R - 2R_{\mu\nu} \left[ \frac{e^{-\frac{\square}{M^2}} - 1}{\square} \right] R^{\mu\nu} \right]$$

# Linearized Action

- We perturb the metric fluctuations around the Minkowski background  $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
- We want to obtain the  $\mathcal{O}(h^2)$  part of the action
- If we perturb the metric fluctuations around the Minkowski background, we get (Biswas, Gerwick, Koivisto, Mazumdar, *Phys. Rev. Lett.* **108** (2012) 031101)

$$S_q = - \int d^4x \left[ \frac{1}{2} h_{\mu\nu} \square a(\square) h^{\mu\nu} + h_{\mu}^{\sigma} b(\square) \partial_{\sigma} \partial_{\nu} h^{\mu\nu} + h c(\square) \partial_{\mu} \partial_{\nu} h^{\mu\nu} \right. \\ \left. + \frac{1}{2} h \square d(\square) h + h^{\lambda\sigma} \frac{f(\square)}{\square} \partial_{\sigma} \partial_{\lambda} \partial_{\mu} \partial_{\nu} h^{\mu\nu} \right]$$

# Linearized Action

- We have the relations (Biswas, Gerwick, Koivisto, Mazumdar, *Phys. Rev. Lett.* **108** (2012) 031101)

$$a(\square) = 1 - \frac{1}{2} \mathcal{F}_2(\square)\square - 2\mathcal{F}_3(\square)\square,$$

$$b(\square) = -1 + \frac{1}{2} \mathcal{F}_2(\square)\square + 2\mathcal{F}_3(\square)\square,$$

$$c(\square) = 1 + 2\mathcal{F}_1(\square)\square + \frac{1}{2} \mathcal{F}_2(\square)\square,$$

$$d(\square) = -1 - 2\mathcal{F}_1(\square)\square - \frac{1}{2} \mathcal{F}_2(\square)\square,$$

$$f(\square) = -2\mathcal{F}_1(\square)\square - \mathcal{F}_2(\square)\square - 2\mathcal{F}_3(\square)\square$$

- If  $f(\square) = 0 \Rightarrow a(\square) = c(\square)$ , then we observe  
 $2\mathcal{F}_1(\square) + \mathcal{F}_2(\square) + 2\mathcal{F}_3(\square) = 0$

# Propagator in Non-local Higher-derivative Gravity

- As a consequence of the generalized Bianchi identities, we have

$$a + b = 0$$

$$c + d = 0$$

$$b + c + f = 0$$

- The field equations can be written in the form

$$\Pi_{\mu\nu}^{-1\lambda\sigma} h_{\lambda\sigma} = \kappa T_{\mu\nu}$$

- $\Pi_{\mu\nu}^{-1\lambda\sigma}$  is the inverse propagator

- The propagator is 
$$\Pi = \frac{P^2}{ak^2} + \frac{P_s^0}{(a - 3c)k^2}$$

- To recover GR in the IR, we must have  $a(0) = c(0) = -b(0) = -d(0) = 1$

As  $k^2 \rightarrow 0$ , we obtain the physical graviton propagator

$$\lim_{k^2 \rightarrow 0} \Pi^{\mu\nu}_{\lambda\sigma} = \frac{P^2}{k^2} - \frac{P_s^0}{2k^2}$$

# Ghosts in Non-local Higher-derivative Gravity

- If we apply the assumption  $f = 0 \Rightarrow a = c$ , then the propagator becomes

$$\Pi^{\mu\nu}_{\lambda\sigma} = \frac{1}{k^2 a(-k^2)} \left( P^2 - \frac{1}{2} P_s^0 \right) = \frac{1}{a(-k^2)} \Pi_{\text{GR}}$$

- We are left with a single arbitrary function  $a(\square)$   
since  $a = c = -b = -d$
- Provided  $a(\square)$  has no zeroes, only the graviton propagator is modified and ghosts are avoided (Biswas, Gerwick, Koivisto, Mazumdar, *Phys. Rev. Lett.* **108** (2012) 031101)
- Choosing  $a(-k^2)$  to be a suitable entire function, the ultraviolet behavior of the gravitons can be tamed
- One such choice is  $a(-k^2) = e^{k^2/M^2}$
- $M$  is a mass scale at which the non-local modifications become important

# Symmetries

- Field equations of GR satisfy the global scaling symmetry

$$g_{\mu\nu} \rightarrow \lambda g_{\mu\nu}$$

- Quadratic curvature actions of the form

$$S_q = \int d^4x \sqrt{-g} [R\mathcal{F}_1(\square)R + R_{\mu\nu}\mathcal{F}_2(\square)R^{\mu\nu} + R_{\mu\nu\lambda\sigma}\mathcal{F}_3(\square)R^{\mu\nu\lambda\sigma}],$$

where the  $\mathcal{F}_i$  's are analytic functions of  $\square$ ,

are invariant under the aforementioned symmetry

- When we expand the action around Minkowski space, the symmetry for  $h_{\mu\nu}$  becomes, infinitesimally,

$$h_{\mu\nu} \rightarrow (1 + \epsilon)h_{\mu\nu} + \epsilon\eta_{\mu\nu}$$

- Relates the free and interaction parts of the action (not a fundamental symmetry); it is useful to have a theory with propagators and vertices having opposing momentum dependence, which is a key feature of gauge theories
- We arrive at the shift-scaling symmetry

$$\phi \rightarrow (1 + \epsilon)\phi + \epsilon$$

- We can formulate scalar toy model whose quantum behavior resembles that of the full gravitational theory

# Degree of Divergence in Non-local Gravity

- Our modified superficial degree of divergence counting exponents is  $E = V - I$
- Use again the topological relation  $L = 1 + I - V$
- **We obtain**  $E = 1 - L$
- For  $L > 1$ ,  $E$  is negative, implying superficially convergent loop amplitudes
- Clear contrast with GR



# Scalar Field Theory Toy Model Action

- Our scalar field theory toy model action is

$$S = \frac{1}{2} \int d^4x (\phi \square a(\square) \phi) \\ + \frac{1}{M_p} \int d^4x \left( \frac{1}{4} \phi \partial_\mu \phi \partial^\mu \phi + \frac{1}{4} \phi \square \phi a(\square) \phi - \frac{1}{4} \phi \partial_\mu \phi a(\square) \partial^\mu \phi \right)$$

where  $a(\square) = e^{-\square/M^2}$

- M is a mass scale at which the nonlocal modifications become important
- Every propagator comes with an exponential suppression and every vertex comes with an exponential enhancement
- The superficial degree of divergence argument for non-local theories of gravity also holds true for the scalar field theory toy model

# Propagator

- Our propagator in Euclidean space is

$$\Pi(k^2) = \frac{-i}{k^2 e^{k^2/M^2}}$$

- The propagator is exponentially suppressed
- As  $k^2 \rightarrow 0$ , we obtain the  $k^{-2}$  momentum dependence of the propagator in GR, as it should be in the IR
- We define the propagator in Euclidean space since it is ill-defined in Minkowski space; the operator  $\exp(\square)$  diverges in Minkowski space

# Vertex Factors

- We have that

$$V(k_1, k_2, k_3) = iC \left[ 1 - e^{k_1^2/M^2} - e^{k_2^2/M^2} - e^{k_3^2/M^2} \right],$$

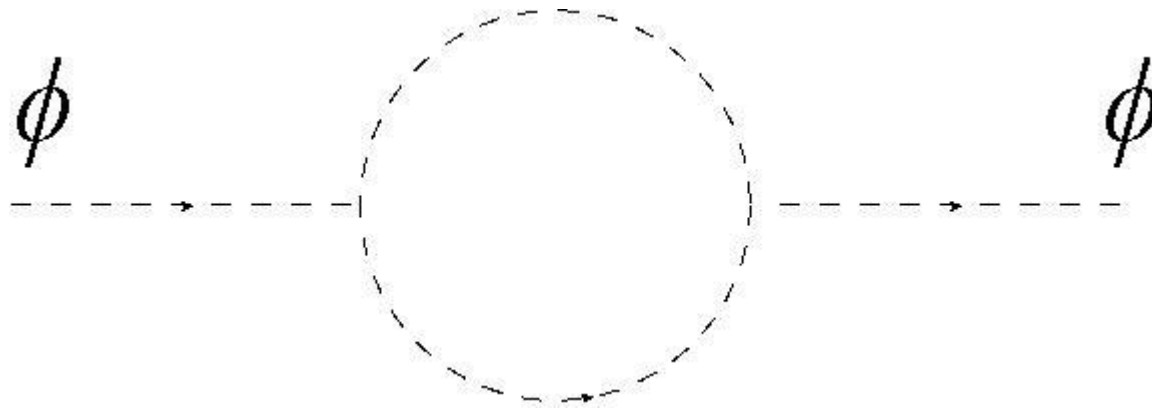
$$\text{where } C = \frac{1}{4} (k_1^2 + k_2^2 + k_3^2)$$

- The momenta are assumed to be incoming and satisfy the conservation law

$$k_1 + k_2 + k_3 = 0$$

# 1-loop, 2-point diagram with external momenta

- Here is the 1-loop, 2-point Feynman diagram with external momenta  $p, -p$ :



$$\Gamma_{2,1}(p^2) = \frac{i}{2i^2 M_p^2} \int \frac{d^4 k}{(2\pi)^4} \frac{V^2(-p, \frac{p}{2} + k, \frac{p}{2} - k)}{(\frac{p}{2} + k)^2 (\frac{p}{2} - k)^2 e^{(\frac{p}{2} + k)^2 / M^2} e^{(\frac{p}{2} - k)^2 / M^2}},$$

# 1-loop, 2-point diagram with external momenta

- There are three types of terms in  $\Gamma_{2,1}$ :
- i) With no exponential. It leads to a divergence ( $\frac{1}{\epsilon}$  pole) when dimensionally regulated.
- ii) With an exponential damping factor. They give rise to convergent results.
- iii) Terms involving  $e^{p \cdot k}$ . When dimensionally regulated, they give rise to no poles.

# 1-loop, 2-point diagram with external momenta

• We have that  $\Gamma_{2,1}(p^2) = \frac{iM^4}{M_p^2} f(x)$ ,

$$f(x) = \frac{x^4}{256\pi^2} \left( \frac{2}{\epsilon} - \log\left(\frac{x^2}{4\pi}\right) - \gamma + 2 \right)$$

$$+ \frac{e^{-x^2}}{512\pi^2 x^2} \left( (e^{x^2} - 1) \left( -2(x^4 + 3x^2 + 2) - e^{\frac{x^2}{2}} (2x^4 + 5x^2 + 4) \right) \right.$$

$$+ e^{x^2} (e^{x^2} - 1) x^6 \text{Ei}\left(-\frac{x^2}{2}\right) + e^{\frac{3x^2}{2}} (2x^4 + 5x^2 + 4) + 2e^{x^2} (7(x^4 + x^2) + 2) \left. \right)$$

$$- 2e^{x^2} (e^{2x^2} - 1) x^6 \text{Ei}(-x^2) \left. \right)$$

$$+ \frac{1}{128\pi} \int_0^1 dr e^{(1-2r)x^2} \left[ p(r, x) Y_0(2\sqrt{r-r^2}x^2) \right.$$

$$\left. + q(r, x) \sqrt{r-r^2} Y_1(2\sqrt{r-r^2}x^2) \right] \text{(DR)}$$

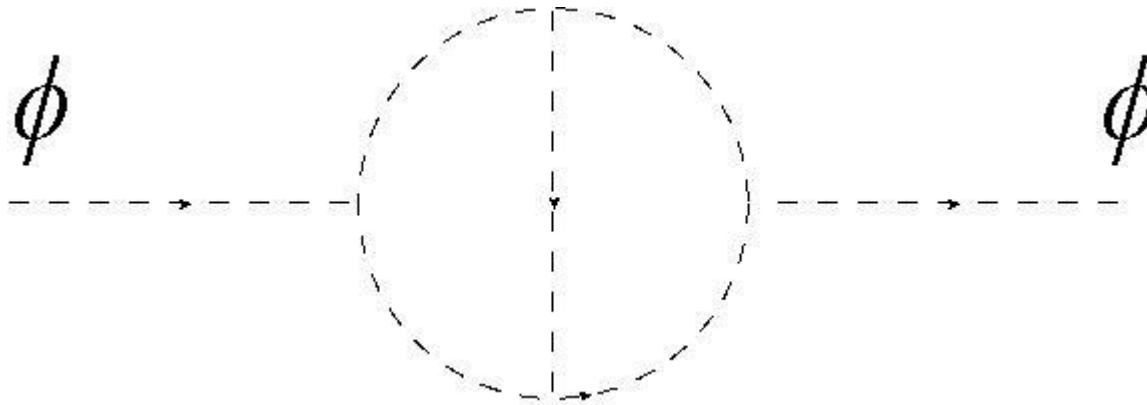
and  $x = \frac{p}{M}$  &  $p(r, x) = -16x^4 r^4 + (32x^4 + 8x^2)r^3 - (26x^4 + 12x^2)r^2 + (10x^4 + 4x^2)r - 2x^4$ ,

$$q(r, x) = -16x^4 r^3 + (24x^4 + 4x^2)r^2 - (16x^4 + 4x^2 - 8)r + 4x^4 + 3x^2 - 4$$

The  $\frac{1}{\epsilon}$  pole in DR is equivalent to a  $\Lambda^4$  divergence if we employ a hard cutoff  $\Lambda$

# 2-loop, 2-point diagram with zero external momenta

For simplicity, we have set the external momenta equal to zero.



$$\Gamma_{2,2a} = \frac{i^2}{4i^5 M_p^4} \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} \frac{V(k_1, -k_1, 0)V(k_2, -k_2, 0)V^2(k_1, k_2, k_3)}{k_3^2 k_2^4 k_1^4 e^{k_3^2} e^{2k_2^2} e^{2k_1^2}},$$

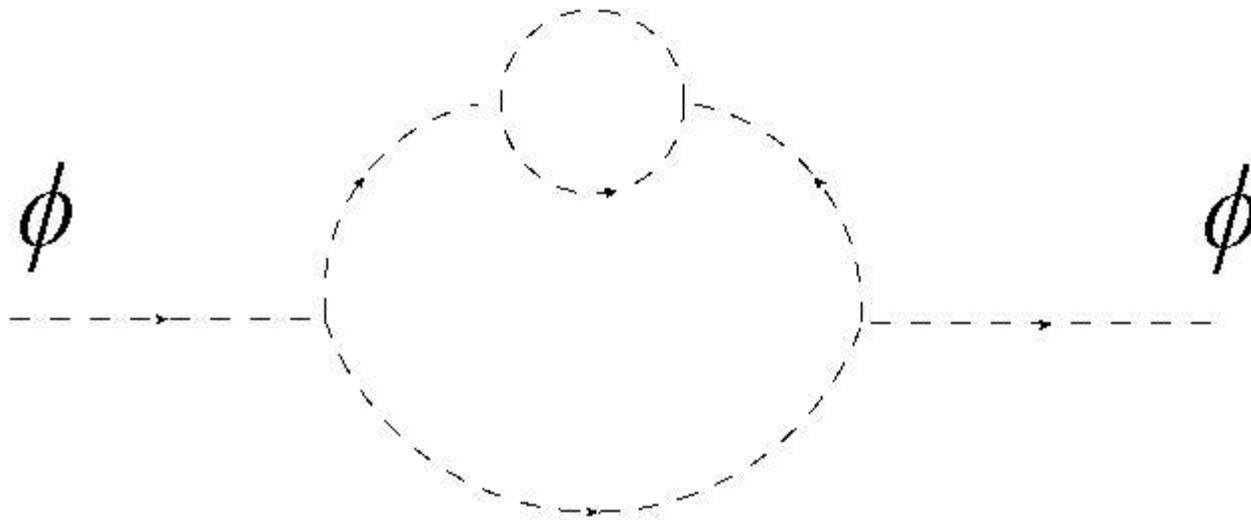
where  $k_3 = -k_1 - k_2$

# 2-loop, 2-point diagram with zero external momenta

- We can write the exponential part of  $\Gamma_{2,2a}$  as  $\sum_i \lambda_i \exp[E_i(k_1, k_2)]$ , where the  $E_i$ 's are quadratic polynomials of  $k_1, k_2$  and  $\lambda_i$ 's are constants taking on the values  $-2, -1, +1, +2$
- We can write  $E_i = \alpha_1 q_1^2 + \alpha_2 q_2^2$  ( $q_1, q_2$  are linear combinations of  $k_1, k_2$ )
- If both  $\alpha_1, \alpha_2$  are negative, we get convergent integrals; if one is negative and the other is positive, we again get convergent results after analytic continuation; if one is negative and the other is zero, we obtain a divergence



# The other 2-loop, 2-point diagram



$$\Gamma_{2,2b} = \frac{i^2}{4i^5 M_p^4} \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} \frac{V^2(k_1, -k_1, 0) V^2(k_1, -\frac{k_1}{2} + k_2, -\frac{k_1}{2} - k_2)}{k_1^6 (\frac{k_1}{2} + k_2)^2 (\frac{k_1}{2} - k_2)^2 e^{3k_1^2/M^2} e^{(\frac{k_1}{2} + k_2)^2/M^2} e^{(\frac{k_1}{2} - k_2)^2/M^2}}$$

Upon redefinition of the momenta, the two 2-loop diagrams give exactly the same result.

# 2-loop, 2-point diagrams with zero external momenta

- We have that

$$\Gamma_{2,2a} = \Gamma_{2,2b} = \frac{M^2}{4096M_p^4\pi^4} \left[ -M^4 \left( -12 \log \left( \frac{\Lambda}{M} \right) + 43 + 9i\pi + 3 \log(3) \right) + 15\Lambda^2 M^2 + 6\Lambda^4 \right]$$

using a hard cutoff  $\Lambda$

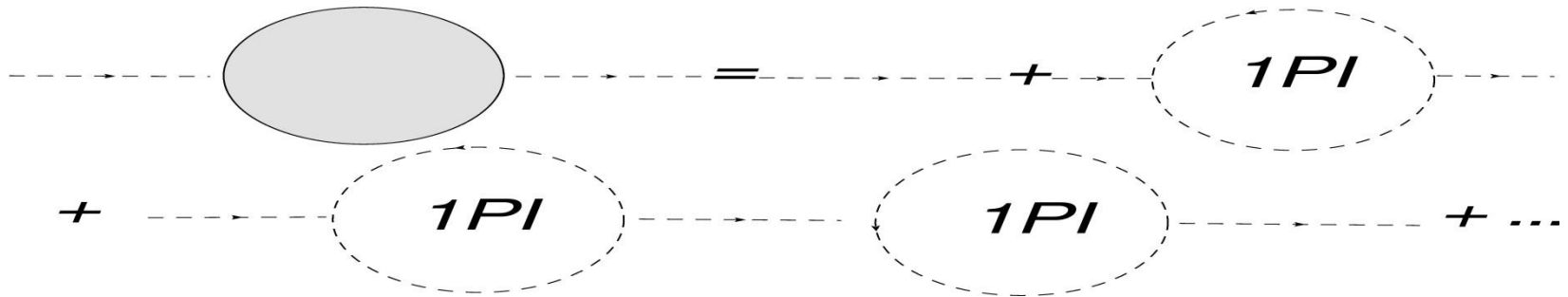
- We have a  $\Lambda^4$  divergence
- We observe that  $\Gamma_{2,1} \sim \Gamma_{2,2} \sim \Lambda^4$
- The degree of divergence stays the same

# Summary of Feynman diagram computations

- At 1-loop, the degree of divergence is  $\Lambda^4$  (hard cutoff)
- At 2-loop, the degree of divergence also stays  $\Lambda^4$
- Hence, we do not get higher divergences as we proceed from 1-loop to 2-loop
- Gives hope towards renormalizability

# Dressed Propagators

- If we sum the infinite geometric series of loop corrections to the propagator, we obtain the dressed propagator



- We have that  $\tilde{\Pi}(p^2) = \frac{\Pi(p^2)}{1 - \Pi(p^2)\Gamma_{2,1\text{PI}}(p^2)}$   
 where  $\Gamma_{2,1\text{PI}}(p^2)$  is the renormalized 1-loop, 2-point function
- We have that  $\tilde{\Pi}(p^2) \rightarrow \Gamma_{2,1\text{PI}}^{-1}(p^2) \sim e^{-\frac{3p^2}{2M^2}}$  in the UV

# Dressed Propagators

- We observe that the dressed propagator is more exponentially suppressed than the bare one
- If we replace the bare propagators with the dressed propagators, convergence of Feynman integrals is improved
- Higher-point 1-loop graphs & 2-loop graphs become finite in the UV
- Only 1-loop, 2-point function diverges
- Once we remove the aforementioned divergence, the theory at the 1-loop level is renormalized
- We believe that higher loops remain finite

# Heuristic argument for 2-point & 3-point diagrams

- We consider 2-point & 3-point diagrams which can be constructed out of lower-loop 2-point & 3-point ones

- Since  $\tilde{\Pi}(k^2) \xrightarrow{UV} e^{-\frac{3k^2}{2M^2}}$  &  $\Gamma_3 \xrightarrow{UV} \sum_{\alpha, \beta, \gamma} e^{\alpha \frac{k_1^2}{M^2} + \beta \frac{k_2^2}{M^2} + \gamma \frac{k_3^2}{M^2}}$ ,  
where  $\Gamma_3$  is the 3-point function &  $\alpha \geq \beta \geq \gamma$ ,

we have that the most divergent UV part of the 2-point diagram for zero external momenta is

$$\Gamma_{2,n} \xrightarrow{UV} \int \frac{d^4k}{(2\pi)^4} \frac{e^{(\alpha_1 + \alpha_2 + \beta_1 + \beta_2) \frac{k^2}{M^2}}}{e^{\frac{3k^2}{M^2}}}$$

# Heuristic argument for 2-point & 3-point diagrams

- Similarly, for the 3-point diagram,

$$\Gamma_{3,n} \xrightarrow{UV} \int \frac{d^4 k}{(2\pi)^4} \frac{e^{(\alpha_1 + \alpha_2 + \alpha_3 + \beta_1 + \beta_2 + \beta_3) \frac{k^2}{M^2}}}{e^{\frac{9k^2}{2M^2}}}$$

- We observe that both the 2- & 3-point diagrams become finite if  $\alpha_i + \beta_i < \frac{3}{2}$
- Even when one includes non-zero external momenta, finiteness is assured
- One can recursively check that  $\alpha_i + \beta_i < \frac{3}{2}$  for higher loops, which is as would be expected since the exponential suppression coming from the propagators is now stronger than the exponential enhancement originating from the vertices

# Conclusions

- Nonlocal gravity possesses many novel features
- Ghosts are avoided
- The degree of divergence stays the same as we proceed from 1-loop to 2-loop
- Dressed propagators improve the convergence at all loop orders
- Once we renormalize the 1-loop graphs, higher-loop graphs do not produce new divergences
- A renormalizable & ghost-free theory of quantum gravity may be within reach