Induced Chern-Simons Terms from 3d Field Theories with Massive Matter: A String Theorist's Perspective

Siraj R Khan

Supervisors: Dr Radu Tatar

University of Liverpool

December 17, 2014

What is Chern-Simons Theory

- $-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - A_\mu J^\mu$ terms allowed (Maxwell or Yang-Mills)
- Odd dimensions also allows for new type of gauge field terms
- Abelian: $\frac{k}{4\pi}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho - A_\mu J^\mu$
- Non-Abelian: $\frac{k}{4\pi}\epsilon^{\mu\nu\rho}\left(A_\mu\partial_\nu A_\rho + A_\mu A_\nu A_\rho\right) - A_\mu J^\mu$
- k is the 'level' of the theory
- We look at $N = 2$ (2+1)-dimensional theories with Yang-Mills **and** non-abelian Chern-Simons term

---

[Intriligator, Seiberg, 2013] [Dunne 1999]

Brane Construction

- In 10d maximal SUSY corresponds to 32 supercharges.
- Two supersymmetry generators Q_L and Q_R, each with 16 components.
- Put in 2 NS5-branes, each extending in $(x^1, x^2, x^3, x^4, x^5)$ and seperated in x^6:

- New equations for Q_L, Q_R reduce number of independent components by half $\rightarrow$ 16 supercharges

[Giveon, Kutasov, 1998] [Barbón, 1997]

Brane Construction

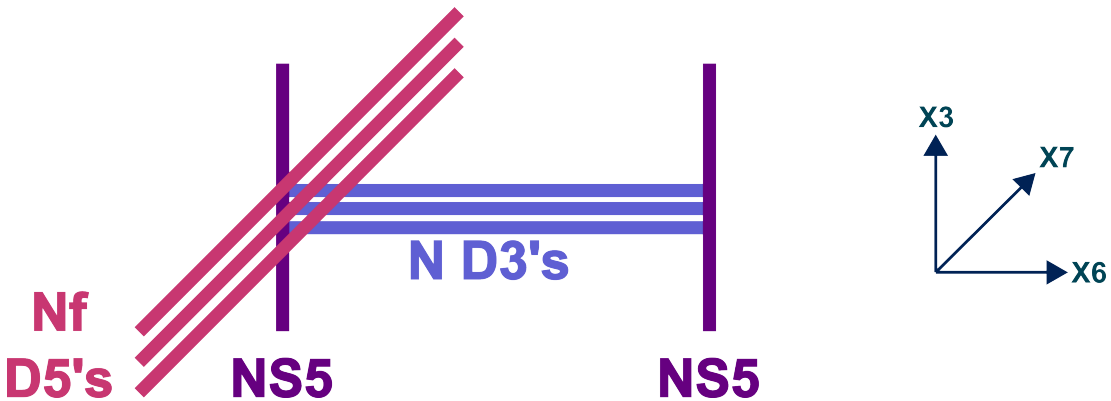
- Add in N_f D5-branes in $(x^1, x^2, x^7, x^8, x^9)$ and intersecting left-most NS5

- New equations for Q_L, Q_R relate half the remaining supercharges to the other half.
- $16 \rightarrow 8$ independent supecharges

[Giveon, Kutasov, 1998] [Barbón, 1997]

Brane Construction

- Add in N D3-branes extending in (x^1, x^2, x^6) and stretched between the two NS5-branes.



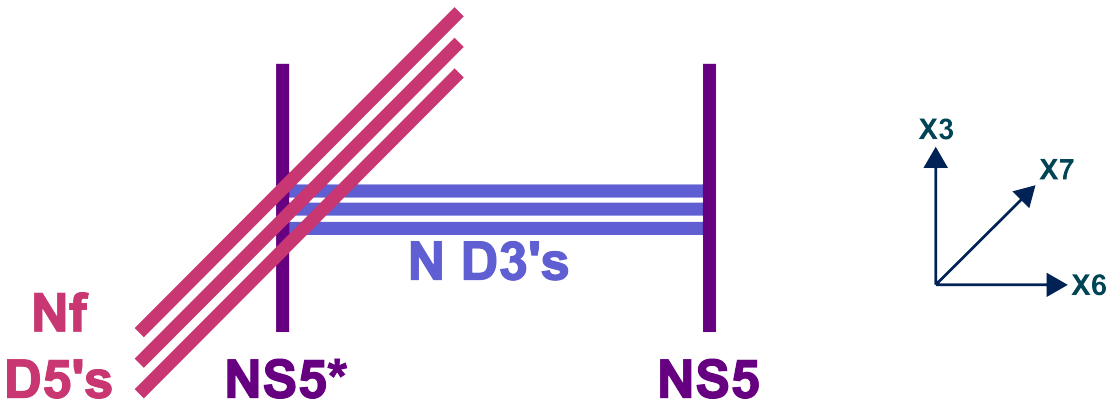
- These are the only branes that can be added without breaking SUSY completely.

- SUSY remains the same, 8 supercharges.

[Giveon, Kutasov, 1998] [Barbón, 1997]

Brane Construction

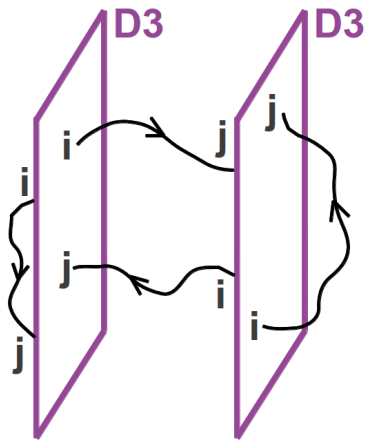
- Low energy theory is 3d when NS5-brane seperation is small
- Rotate the left-most NS5-brane from $(x^1, x^2, x^3, x^4, x^5)$ to $(x^1, x^2, x^3, x^8, x^9)$



- Number of independent supercharges $8 \to 4$
- Corresponds to making adjoint chiral multiplet in 3d $N = 4$ vector multiplet massive
- Breaks $N = 4$ to $N = 2$ in 3d.

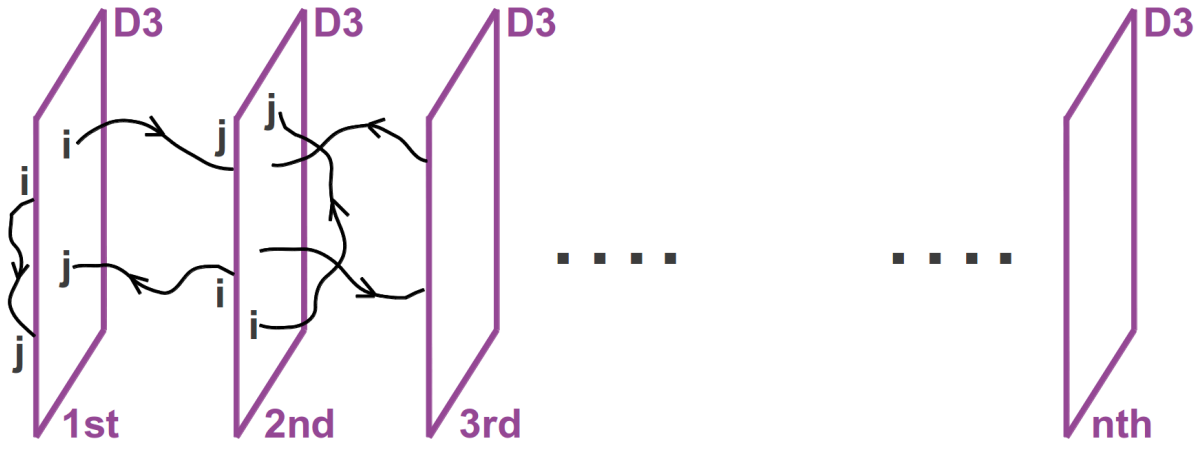
[Giveon, Kutasov, 1998] [Barbón, 1997]

Gauge Fields from D3-D3 Strings



- 4 combinations of end point configurations: $|1,1\rangle$, $|2,2\rangle$, $|1,2\rangle$, $|2,1\rangle$
- 4 gauge bosons
- U(2) gauge group has $2^2 = 4$ gauge bosons (corresponding to number of generators of group)

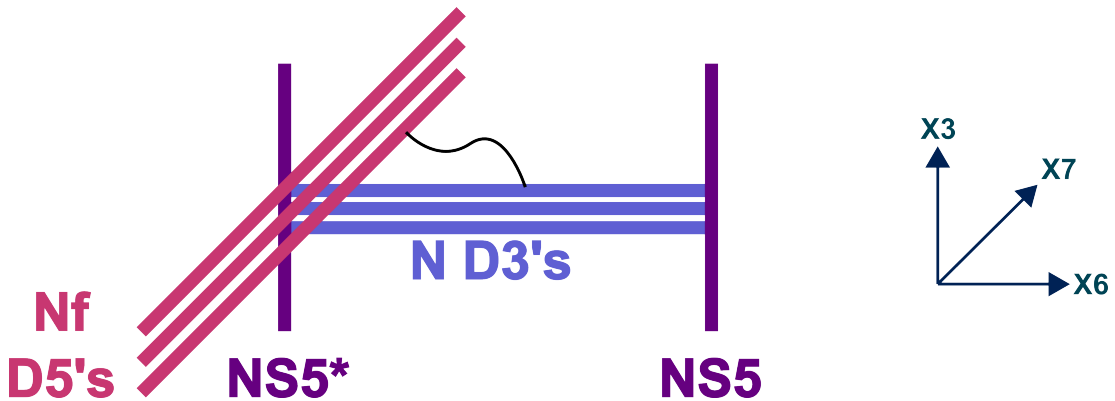
Gauge Fields from D3-D3 Strings



- N D3-branes has N^2 combinations of $|i,j\rangle$
- N^2 gauge bosons
- $U(N)$ gauge theory
- 2 separate stacks of branes would give $U(N) \otimes U(N)$

Quarks from D5-D3 Strings

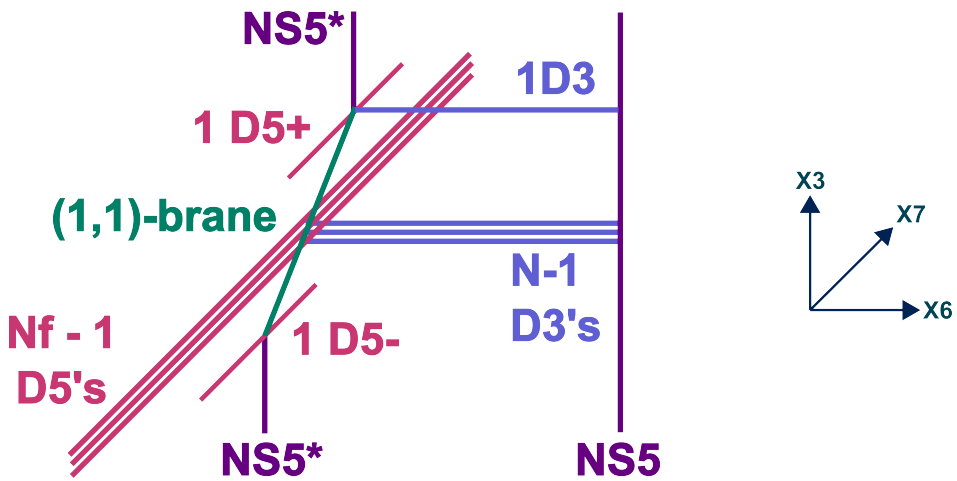
- N_f D5-branes also called 'flavour branes'.
- Strings between D5's and D3's give quarks transforming in fundamental of $U(N)$. [N = number of D3's.]



- In this case quarks are massless

Massive Quarks from D5-D3 Strings

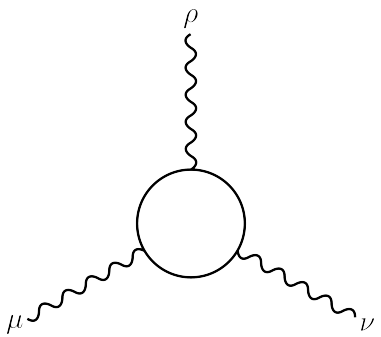
- Can't just move D5's along NS5* without breaking SUSY
- Need to create a (p,q)-brane at angle in (x^3, x^7)-plane
  p = no° of D5's at (D5, NS5, D3)-intersection
  q = no° of NS5's at (D5, NS5, D3)-intersection
- 4 supercharges ($N = 2$ in 3d) preserved.
- Massive quarks as strings stretch between D5's and D3's



[Aharony, Hanany, Kol, 1997]

Massive Quarks and Chern-Simons level k

- In 3d, calculating the effective action of a massive fermion to one-loop in perturbation theory results in a Chern-Simons term.
- In the non-abelian case this corresponds to the diagram:



- For a single massive quark we get:

$$\frac{1}{4\pi}\frac{m}{|m|}\epsilon^{\mu\nu\rho}\left(A_\mu\partial_\nu A_\rho + A_\mu A_\nu A_\rho\right)$$

[Redlich, 1984] [Dunne 1999] [Ojima, 1989]

Aharony Duality

- In 3d N=2 low energy theories with $k = 0$ we have 'Aharony duality'.
- $U(N)$ theory with N_f flavours is 'dual' to a $U(N_f - N)$ theory with N_f flavours
- The scalars of one behave the same as the scalars of the other.

  $\rightarrow$ vevs of scalars in one match the vevs of scalars in the other (same moduli spaces)
- Global symmetries also match for the two theories
- When one is weakly coupled other is strongly coupled (very useful!)

[Aharony, 1997]

Giveon-Kutasov Duality

- In 3d N=2 low energy theories with $k \neq 0$ we have 'Giveon-Kutasov duality'.
- $U(N)_k$ theory with N_f flavours is 'dual' to a $U(|k| + N_f - N)_{-k}$ theory with N_f flavours
- Again scalar vevs match up - same moduli
- Same Global symmetries
- When one is weakly coupled other is strongly coupled
- Other tests of Aharony and Giveon Kutasov Dualities exist

[Giveon, Kutasov, 2009]

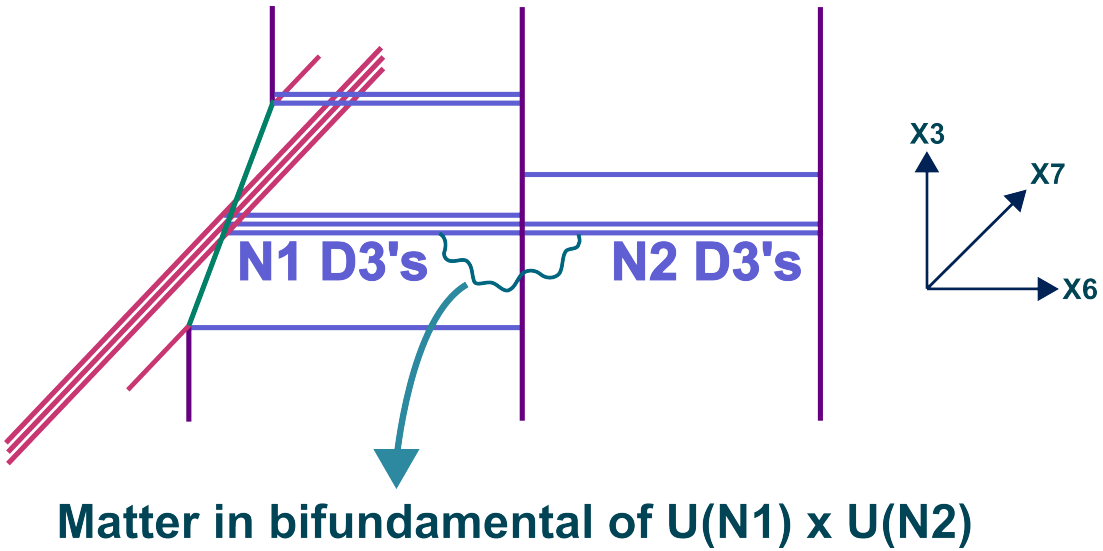
Flowing between Aharony and Giveon-Kutasov Dualities

- Aharony duality for $k = 0$
- Giveon-Kutasov duality for $k \neq 0$

We can change whether a theory is Aharony dual or Giveon-Kutasov dual at low energies by introducing massive matter accordingly!

Current Project

- Bifundamental quarks from strings between one stack of D3's and another stack of D3's separated by NS5:



- Bifundamental quarks transform in $(N_1, \bar{N}_2)$
- What are the resulting gauge theories from integrating out numerous massive bifundamentals?

References

K. Intriligator, N. Seiberg (2013)
Aspects of 3d N=2 Chern-Simons Matter Theories
arXiv:hep-th/1305.1633

G. V. Dunne (1999)
Aspects of Chern-Simons Theory
arXiv:hep-th/9902115.

A. Giveon, D. Kutasov (1998)
Brane Dynamics and Gauge Theory
arXiv:hep-th/9802067

J. L. F. Barbón (1997)
Rotated Branes and N=1 Duality
arXiv:hep-th/9703051

O. Aharony, A. Hanany, B. Kol (1997)
Branes and Supersymmetry Breaking in 3D Gauge Theories
arXiv:hep-th/9710116v3

Some more references

A. N. Redlich (1984)

Gauge Noninvariance and Parity Nonconservation of Three-Dimensional Fermions

Phys. Rev. Lett. 52, 18

S. Ojima (1989)

Derivation of Gauge and Gravitational Induced Chern-Simons Terms in Three Dimensions

Progress of Theoretical Physics. Vol.81, No.2

O. Aharony (1997)

IR Duality in $d = 3$ $N = 2$ Supersymmetric $USp(2N(c))$ and $U(N(c))$ Gauge Theories

arXiv:hep-th/9703215

A. Giveon, D. Kutasov (2009)

Seiberg Duality in Chern-Simons Theory

arXiv:hep-th/0808.0360

The End