

Toy models for Holographic Baryons

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- Introduce Topological Solitons

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- Holographic QCD

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- Baby Skyrmions and their variants

Topological Solitons

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- Examples include sine-Gordon kinks, sigma-model lumps, monopoles, vortices, Skyrmions, Yang-Mills instantons...

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- This leads to a topological charge

$$N = \frac{\phi_+ - \phi_-}{2\pi}$$

where $\phi(x) \rightarrow \phi_\pm$ as $x \rightarrow \pm\infty$.

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- Saturation of the bound implies a first order equation of motion. In this example we can solve it (for $N = 1$) to find analytic solutions, but this is not always possible.

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- Minimising this over λ gives a scale for the size of the soliton.

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- The static energy is invariant under spatial rescalings.

Yang-Mills Instantons

- In four spatial dimensions, consider the Lagrangian

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- YM instantons are highly symmetric and there is a wealth of mathematics concerning them!

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- Skyrmions possess interesting symmetries, although their binding energies are too large

Holographic QCD: The Sakai-Sugimoto Model

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- The Sakai-Sugimoto takes $p = \frac{2}{3}$ and we say the spacetime is AdS-like

Holographic QCD: The Sakai-Sugimoto Model

- The Lagrangian is, in suitable units:

$$\mathcal{L} = -\frac{1}{2} \text{Tr}(F_{\Gamma\Delta} F^{\Gamma\Delta}) + \frac{9\pi}{\lambda} \omega_5(A_\Gamma)$$

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- Topological solitons in the bulk correspond to Skyrmions on the boundary (coupled to a tower of vector mesons)
- This model is analytically and numerically very complicated

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- We can use these toy models to study bulk solitons and test predictions made in the full SS model
- A natural analogue for the Yang-Mills term is an $O(2) - \sigma$ term, since both are scale invariant
- We still have a choice to make regarding the analogue of the Chern-Simons term

Stabilization via the Baby Skyrme Term

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- One option is to represent the Chern-Simons term by a baby-Skyrme term

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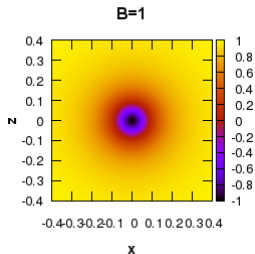
- For small κ (the analogue of large 't Hooft coupling) the solitons of this model are small compared to the curvature of spacetime
- We can approximate these solitons well by flat-space σ -model lumps
- The curvature of spacetime and the baby-Skyrme interaction pick out a preferred size

Static Solutions of the BS Model

- Here are some numerical static solutions to the baby-Skyrme model in this curved space, with parameter value $\kappa = 0.01$

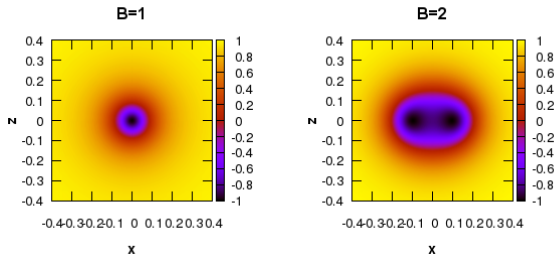
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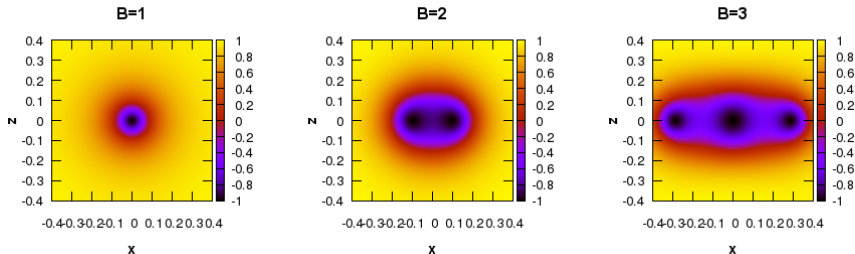
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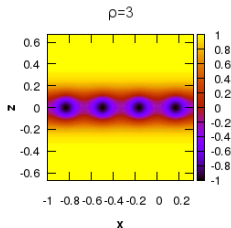


Finite Density Chains

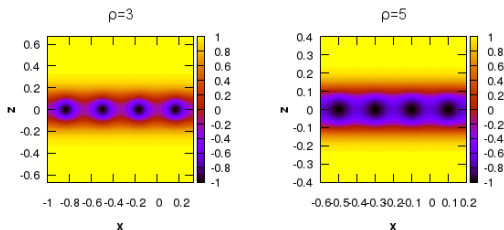
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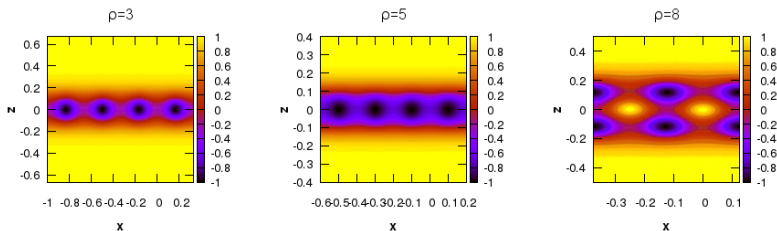
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- It can be shown that the Vector Meson model tends to the Baby Skyrme model in the limit

$$g, M \rightarrow \infty, \frac{g}{M} \propto \kappa \text{ constant}$$

Static Solutions of the VM Model

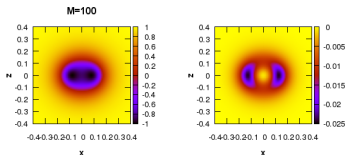
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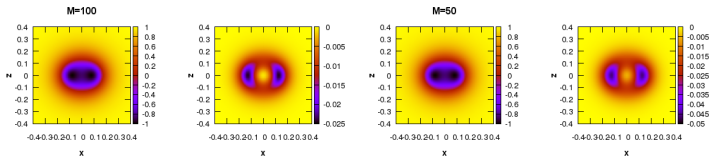
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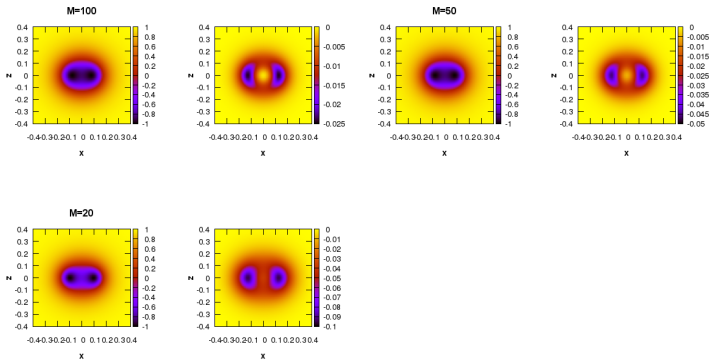
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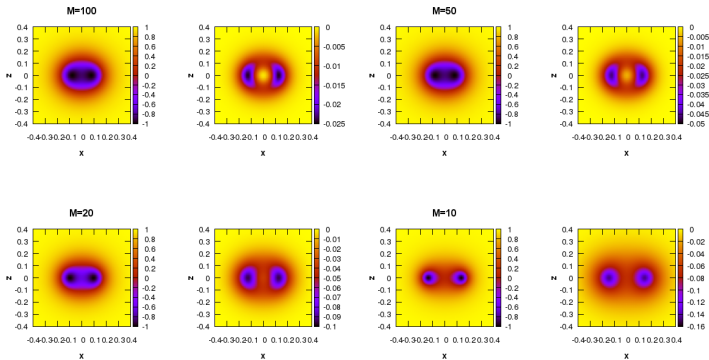
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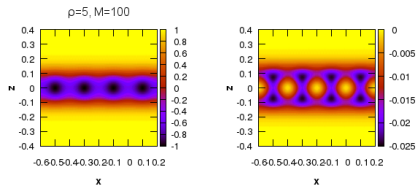
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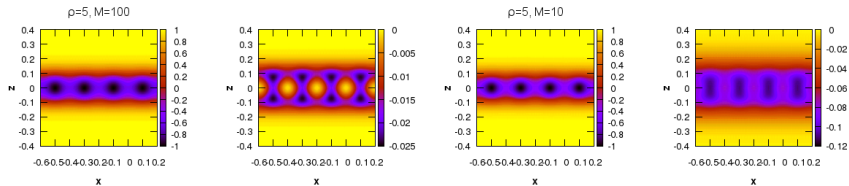
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- This requires that the optimal separation between two solitons is greater than the size of a single soliton
- Can we find a parameter regime in the Vector Meson model to reproduce this behaviour?

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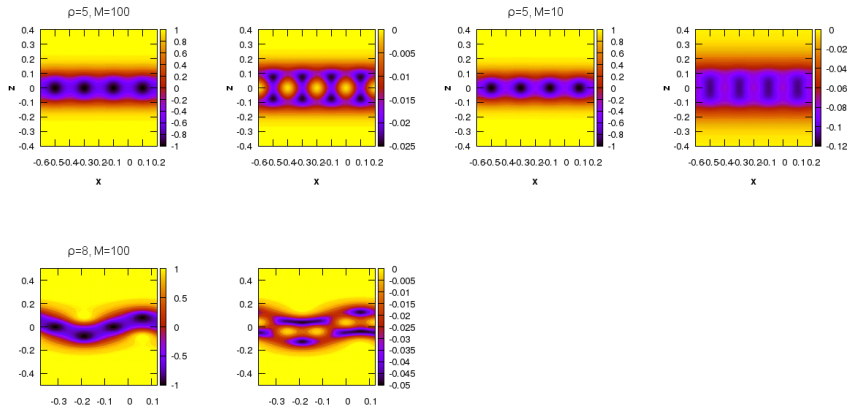
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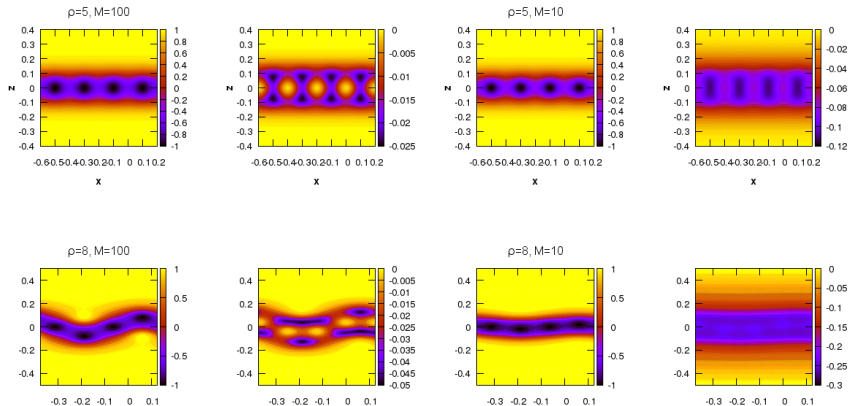
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- Topological solitons have been introduced and many examples have been mentioned
- Introduced the Sakai-Sugimoto model and looked at two lower-dimensional analogues
- Finite density chains of solitons have been investigated in both models
- At lower densities the chains in both models look similar, but at higher densities results are not yet conclusive
- Numerical results require further verification, and analogues of the dyon salt or popcorn phenomena are still to be found in the vector meson model

Thank you for listening.