Can The Strong CP Problem Be Solved In The Free-Fermionic Setting?
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What Is The Strong CP Problem?

The $SU(3)$ gauge theory allows a CPV interaction term of the form

$$\mathcal{L}_{CP} = \frac{\overline{\theta}\alpha_s}{32\pi^2}\tilde{G}_{\mu\nu}G^{\mu\nu}$$

to be added to the QCD Lagrangian which contributes to the neutron electric dipole moment (nEDM).

Note.

$$\overline{\theta} \equiv \theta_0 + \theta_{weak}$$

with θ_0 being the angle given above the electroweak scale and θ_{weak} is the value introduced by the electroweak CP violation.

What Is The Strong CP Problem?

The current bound on the nEDM is

$$|d_N| < 2.9 \times 10^{-26} e\ cm$$

so that

$$|\bar{\theta}| < 10^{-10}\ \texttt{rad}$$

which is a strikingly small value for a dimensionless natural constant given that the CP violating phase, θ, in the CKM mixing matrix is of order one.

This smallness of $\bar{\theta}$ despite the large amount of CP violation in the weak sector is known as the strong CP problem.

Solutions To The Strong CP Problem

Axions are important because they are the most promising solution to the Strong CP problem.

Other solutions are ruled out or disfavoured by phenomenology:

- Calculable θ - The Nelson-Barr Mechanism (mimics CKM-type CP violation)

- Up Quark Mass Vanishing - Weinberg's famous up-down quark mass ratio

$$Z = \frac{m_u}{m_d} = \frac{5}{9}$$

What Are Axions?

Axions are the quanta of the axion field, $a(x)$, which is the phase of the PQ complex scalar field after the spontaneously breaking of the PQ symmetry gives it an absolute value f_a.

Simply put axions are pseudo-Nambu-Goldstone bosons related to the spontaneous breaking of the anomalous $U(1)$ global symmetry.

It is well-known that axions arise in string compactifications. There are two axions in superstring models. One is the model-independent axion (MI axion) and the other is the Peccei-Quinn type one (PQ axion) namely the global anomalous $U(1)$.

Cosmological Bound & The Axion Decay Constant

It is a well known fact that large f_a especially

$$f_a > 10^{12} \texttt{ GeV}$$

means axion energy density

$$\rho_a > \rho_{critical}$$

and therefore is unacceptable.

The **Axion Decay Constant Window** is

$$10^{9-10} \texttt{ GeV} < f_a < 10^{12} \texttt{ GeV}.$$

f_a smaller than 10^{9-10} GeV, will couple very weakly and f_a greater than 10^{12} GeV, will couple too strongly.

The Model-Independent Axion: References

Anomaly Cancellations In Supersymmetric $D = 10$ Gauge Theory And Superstring Theory,
Phys. Lett. 149B (1984),
M. B. Green, J. H. Schwarz

Harmful Axions In Superstring Models,
Phys. Lett. 154B (1985),
K. Choi, J. E. Kim

The Model-Independent Axion

In the Neveu-Schwarz sector, in four dimensions, there is always an antisymmetric tensor field

$$B_{\mu\nu}, \quad \mu, \nu = 0, ...3,$$

which has one physical degree of freedom and is crucial for anomaly cancellation, the gauge-invariant field strength for which is given by

$$H = \mathrm{d}B + \omega_{3L} - \omega_{3YM}, \; \mathrm{d}H = \frac{1}{16\pi^2}(\mathrm{Tr}\, R \wedge R - \mathrm{Tr}\, F \wedge F)$$

giving rise to a single scalar field in four dimensions with axion-like couplings.

The Model-Independent Axion Is Present In All Superstring Models Due To The Presence Of The Coupling.

The Axion Decay Constant Problem: Choi & Kim

$$M_a^{'} = 8\pi^2 M_a \Rightarrow M_a = \frac{M_a^{'}}{8\pi^2}$$

and

$$M_a^{'} = \frac{M_{Pl}}{12\sqrt{\pi}} \simeq 5.64 \times 10^{17}$$

The first relation gives

$$\frac{g^2\phi}{\kappa^2} = \frac{M_{Pl}^2}{8\pi}$$

which, upon substitution into the second relation, yields

$$M_a^{'2} = \frac{M_{Pl}^2}{144\pi} \Rightarrow M_a = \frac{1}{8\pi^2} \cdot \frac{M_{Pl}}{12\sqrt{\pi}} \Rightarrow M_a \simeq 7.15 \times 10^{15} \ \mathtt{GeV}$$

clearly violating the cosmological energy density upper bound on f_a.

The $U(1)_A$ Axion

The $U(1)_A$ axion is present in all the models in the free-fermionic setting which arises as the Nambu-Goldstone boson of the global anomalous U(1) in the theory. It is a formal linear combination of the $U(1)$s in the gauge symmetry of the theory.

The Free-Fermionic Setting

A general boundary condition basis vector is of the form

$$\alpha = \left\{ \psi^{1,2}, \chi^i, y^i, \omega^i | \overline{y}^i, \overline{\omega}^i, \overline{\psi}^{1,..,5}, \overline{\eta}^{1,2,3}, \overline{\phi}^{1,..,8} \right\}$$

where $i = 1, ..., 6$

- $\overline{\psi}^{1,..,5}$ - $SO(10)$ gauge group
- $\overline{\phi}^{1,..,8}$ - $SO(16)$ gauge group

I. Antoniadis, C.P. Bachas,
4D Fermionic Superstrings With Arbitrary Twists
Nuclear Physics B (1988), Volume 298, Issue 3, Page 586.

The Observable Gauge Group - $SO(10)$

- Edi Halyo (EH): The Standard-Like Model

$$SO(10) \to SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{Z'}$$

where

$$\begin{aligned} U(1)_Y &= \frac{1}{3}U(1)_C + \frac{1}{2}U(1)_L \\ U(1)_{Z'} &= U(1)_C - U(1)_L \end{aligned}$$

- Antoniadis, Leontaris, Rizos (ALR): The Pati-Salam Model

$$SO(10) \to SO(6) \times SO(4)$$

The Hidden Gauge Group - $SO(16)$

- EH: The Standard-Like Model

$$SO(16) \to SU(5) \times SU(3) \times U(1)^2$$

- ALR: The Pati-Salam Model

$$SO(16) \to SU(8) \times U(1)'$$

The Global Anomalous $U(1)$ & Traces

- EH: The Standard-Like Model
  $U(1)_A = 2(U(1)_1 + U(1)_2 + U(1)_3) - (U(1)_4 + U(1)_5 + U(1)_6)$
  with
  $\text{Tr}\, U(1)_A = 180$

  Note. In this case the $U(1)_A$ is color-anomalous. That is

$$\text{Tr}[SU(3)^2_{Obs} U(1)_A] \neq 0.$$

- ALR: The Pati-Salam Model

$$U(1)_A = U(1)_1 - U(1)_2 - U(1)_3, \ \text{Tr}\, U(1)_A = 72.$$

Aside

EH went on to show that the $U(1)_A$ axion is also a **harmful** one that is

$$\begin{aligned} \text{Tr}[SU(5)^2_{Hid} U(1)_A] &\neq 0, \\ \text{Tr}[SU(3)^2_{Hid} U(1)_A] &\neq 0. \end{aligned}$$

The cancellation mechanism generates a large Fayet-Iliopoulos D-term for the anomalous $U(1)_A$ which would break supersymmetry and destabilize the vacuum. However, in all known instances one can give VEVs to scalar fields charged under $U(1)_A$ along the F- and D- flat directions to cancel the Fayet-Iliopoulos D-term and restore supersymmetry.
Basically, we want

$$\boxed{\sum_i Q_A^i |\langle\phi_i\rangle|^2 < 0.}$$

Note. In general, all the local and global $U(1)$s will be spontaneously broken by the DSW mechanism.

The Fayet-Iliopoulos D-Terms

The general form of the anomalous D-term is

$$D_A = \sum_i Q_A^i |\phi_i|^2 + \frac{g^2 e^{\Phi_D}}{192\pi^2} \,\mathrm{Tr}\, Q_A$$

- EH: The Standard-Like Model
  $\Rightarrow \sum_i Q_A^i |\langle \phi_i \rangle|^2 (= -\frac{15g^2}{16\pi^2}) < 0$
- ALR: The Pati-Salam Model
  $\Rightarrow \sum_i Q_A^i |\langle \phi_i \rangle|^2 (= -\frac{3g^2}{8\pi^2}) < 0$

The scalar VEVs resulting from these are at the scale

$$\frac{M}{10} \sim 10^{17} \mathtt{GeV}.$$

Within the setting of String-Derived Models

- Exploring Hidden Sector Gauge Groups and Dark Matter
- Axion-Photon-Photon Coupling Computation

THANK YOU!!!