High Energy Jets - Pure Jet Processes at the LHC

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Outline

- Pure Jets with High Energy Jets (HEJ)
  - Idea
  - Amplitude
  - Resummation
- Extensions to HEJ at 4 jet multiplicity
  - $q\bar{q}$ outgoing states - new diagrams
  - New amplitude
- Leading Log and Next-to-Leading Log
- Conclusions and Outlook

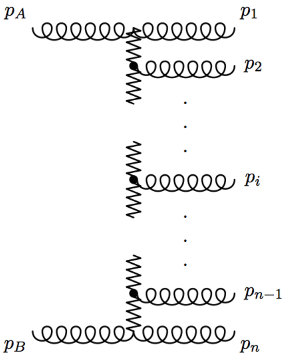
HEJ - Idea

- HEJ is inspired by the simple (factorised) form QCD amplitudes take in the high energy (Multi-Regge Kinematic, or MRK) limit. For example:

$$|M^{MRK}_{gg\to g..g}|^2 = \frac{4s^2}{N_C^2-1}\frac{g^2 C_A}{|p_{1\perp}|^2}\left(\prod_{i=2}^{n-1}\frac{4g^2 C_A}{|p_{i\perp}|^2}\right)\frac{g^2 C_A}{|p_{n\perp}|^2} \qquad (1)$$

  Other incoming parton flavours differ only by colour factors
- In this limit, all jets are well separated in rapidity and the dominant diagrams are given by graphs with a t-channel gluon exchange (in the above, $t = p_\perp$)
- With HEJ, we aim to keep a simple structure but expand the phase space where it is applicable

HEJ - Idea



HEJ - Amplitude

- An amplitude can be built up by starting with $qQ \to qQ$ scattering (we'll ignore colour and coupling):

$$M_{q^- Q^- \to q^- Q^-} = \langle 1|\mu|a\rangle \frac{\eta^{\mu\nu}}{t} \langle 2|\nu|b\rangle \tag{2}$$

- Adding a gluon emission to 5 possible sites:

$$M^{HEJ}_{q^- Q^- \to q^- g Q^-} = \langle 1|\mu|a\rangle \frac{\eta^{\mu\nu}}{t_1} \frac{V^{\rho}\varepsilon^*_{\rho}}{t_2} \langle 2|\nu|b\rangle \tag{3}$$

Where V is the Lipatov vertex, describing the effect of the 5 possible emissions

HEJ - Amplitude

- Generalising to n-jets and helicity and color sums/averages:

$$|M^{HEJ}_{qQ\to qg..gQ}|^2 = \frac{1}{4(N_C^2-1)}||S_{qQ\to qQ}||^2 \left(g^2 C_F \frac{1}{t_1}\right) \left(g^2 C_F \frac{1}{t_{n-1}}\right) \prod_{i=1}^{n-2} \left(\frac{-g^2 C_A}{t_i t_{i+1}} V^\mu V_\mu\right) \quad (4)$$

- Factorised and can be fairly easily shown to reduce to MRK result in that limit

- Within the MRK limit, the virtual corrections can be obtained to all orders via the Lipatov ansatz:

$$\frac{1}{t_i} \rightarrow \frac{1}{t_i} exp(\hat{\alpha}(q_i)\Delta y) \tag{5}$$

- When combined with the corresponding real corrections, we have an all-order, resummed and regularised (by some IR cut-off) amplitude essentially by the replacement:

$$\frac{1}{t_i} \rightarrow \frac{1}{t_i} exp(\omega^0 \Delta y) \tag{6}$$

HEJ - In Five Lines

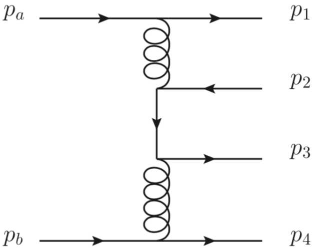
- Marvel at the simplicity of high-energy amplitudes
- Create a t-channel factorised matrix element at LO
- Use the Lipatov ansatz to derive an all-order result
- Monte Carlo that beast up!
- Importantly, everything so far has been for extra jets emitted as gluons

Extensions - New States

- Can we extend our description?
- Gluons are not the only way to create jets...
- At 4 jet multiplicity, we can start thinking of other final states - for example, $qQ \to q\bar{q}'q'Q$
- This process in analogous to $qQ \to qQ$ in that it only occurs in t-channel exchanges so it is a good place to start

Extensions - New Diagrams

- 7 diagrams we can consider. One such:



- Currently, we do not include these processes (sub-leading in 4 jet cross section calculations)

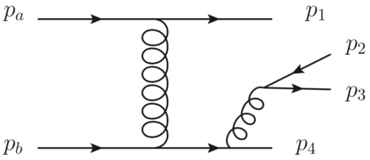
Extensions - New Amplitude

- Let's try and include them!
- If I want to include these diagrams in HEJ, what do I require?
  - Currents - factorisation requires we 'untangle' the diagram
  - Explicit t-channel poles - our virtual corrections rely on replacing t-channel gluon propagators
- So our amplitude would look something like:

$$M^{HEJ} \sim \langle 1|\mu|a\rangle \frac{V^{\mu\nu}}{t_1 t_2 t_3} \langle 4|\nu|b\rangle \tag{7}$$

Extensions - New Amplitude

- Given this form, the processes that are already t-channel factorised can be trivially and exactly included. So that's 3 diagrams down already
- Deal with 'eikonal' diagrams:



Extensions - New Amplitude

- Let's take a closer look at such diagrams (take negative helicity extremal quark lines)...

$$\langle 4|\mu|2\rangle \langle 2|\rho|b\rangle + \langle 4|\mu|3\rangle \langle 3|\rho|b\rangle + \langle 4|\mu|4\rangle \langle 4|\rho|b\rangle \qquad (8)$$

- One of these terms will be identically zero, as it is contracted with the on-shell quark current 2-3. If we take a positive helicity anti-quark and negative helicity quark, then it will be term 1. Vice versa, it would be term 2
- In any case, the only term with the 'unbroken' current is term 3 - can we fairly extract this alone? i.e. can we take the eikonal limit?

Extensions - New Amplitude

- Just about - this term goes like $\hat{s}$ which is the largest invariant in the problem
- You can justify it a bit more by requiring s_{23} small - where these types of diagrams are more important anyway
- From now on, I will assume this - it allows for some further simplifications. For example, the off-shell quark propagator goes like $\frac{1}{s_{23}+s_{42}+s_{43}}$ and I can drop the first invariant in comparison to the other two

Extensions - New Amplitude

- Making the same approximation for the other three diagrams, we end up with expressions for all graphs in the 'right' form
- We can play a further trick to make the expression simpler
- Temporarily let $p_a \sim p_1$ and $p_b \sim p_4$. Such a correspondence is inspired by the MRK limit and doing it allows us to use the colour algebra to combine graphs

Extensions - New Amplitude

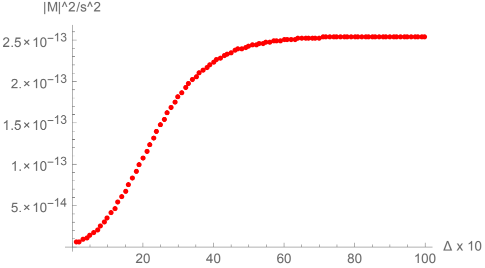
- Doing this, our effective vertex takes the form:

$$V^{\mu\nu} = \frac{t_2 C_1}{s_{23}} \left(\eta^{\mu\nu} V_{eik} + V_{3g}^{\mu\nu} \right) + C_2 V_{qprop}^{\mu\nu} + C_3 V_{qprop'}^{\mu\nu} \quad (9)$$

- We can attempt to put back in more of the original process by reinstating the symmetry between p_a, p_1 and p_b, p_4 in V_{eik}
- Mod-square it, do the colour/helicity sum/average and we're done!

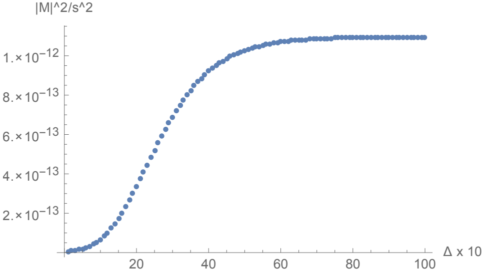
Extensions - New Amplitude

- Generate momentum vectors with rapidities chosen as $\Delta, 0.2, -0.2, -\Delta$ and vary Δ
- MadGraph:



Extensions - New Amplitude

- My formalism:



- Work is ongoing...

Leading Log and Next-To-Leading Log

- What was the point of all that?
- Two reasons - leading log prediction for the process $qQ \to q\bar{q}qQ$ but *part* of the next-to-leading log prediction for 4 jet processes
- What does that actually mean?

Leading Logs

- The basic idea of perturbation theory is that we expand in some small parameter (α_s, say) and perform a fixed order calculation in the assumption that higher orders are suppressed by powers of this small parameter and so contribute less
- Sometimes, these 'higher order corrections' are not as small as we'd hope
- The MRK limit is one such region

Leading Logs

- The MRK limit can be formulated by requiring a fixed momentum transfer t and a centre of mass energy tending to infinity
- In the limit where rapidity differences between jets are large, then we can show:

$$\Delta y \approx \ln\left(\frac{\hat{s}}{-\hat{t}}\right) \tag{10}$$

- Hence, if we see these logs anywhere, they're going to be important

Leading Logs

- These logs pop out when considering virtual corrections to jet processes - so, even though they are suppressed by another factor of α_s, they are 'un-supressed' to an extent by the large log
- When the divergence arising from these corrections is cancelled by corresponding real emissions, there is a remnant left over which is enhanced by Δy, i.e., this large log
- We should capture this behaviour somehow - this is where the mysterious Lipatov ansatz came from

Leading Logs

- This process resums the 'leading log' - terms that go like $\alpha_s^n \ln^n \left(\frac{\hat{s}}{-\hat{t}}\right)$ in the perturbative expansion
- What about terms like $\alpha_s^n \ln^{n-1} \left(\frac{\hat{s}}{-\hat{t}}\right)$? Do they appear?
- Well, yes. These are the next-to-leading logs
- How do we probe them?

Next-to-Leading Logs

- We have to relax the strict MRK limit - the amplitudes contain these logs because of the large rapidity separation
- So, if we systematically allow each pair of jets to become close to each other, we can investigate these sub-leading contributions
- Another way - if we replace a gluon propagator with a quark, we don't use the Lipatov ansatz on the propagator and lose a log (though the quark does still reggeize - beyond the scope of this talk)
- Overall - calculation is LL in that subprocess, but NLL in 4 jet process

Conclusions and Outlook

- HEJ continues to perform admirably and so we look to extend it
- This is just one of the many improvements we're looking into
- Being able to describe as much as we can at the LHC is obviously beneficial - SM and BSM studies
- To come - hopefully, a claim at full NLL accuracy