New NNLL Resummation of Event Shapes in e^+e^- Annihilation

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Why Study Soft-Gluon Resummation?

Want to learn from collider experiments to test fundamental theories and search for new ones.

So need to calculate observables motivated by experimental (easily measurable quantities) and theoretical convenience (IRC safe).

Event-shape observables provide us with precisely determined values of α_s, insight into the structure of QCD objects.

Perturbation theory is the tool we use to compute kinematic distributions of relevant processes.
α_s is our expansion parameter.

And in particular, we study e+e- annihilation to QCD degrees of freedom by calculating **event-shape observables**.
Event-event shapesshapes characterise the geometric energy-momentum flow in an event.

(These methods can be generalised to include initiating partons.)

Our Observables

We choose to calculate the thrust, τ, and its relatives the C parameter and the heavy jet mass m_H.

$$T \equiv \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{Q} \,, \qquad \tau \equiv 1 - T \,,$$

as well as the total and wide broadenings, B_T and B_W, and their relatives oblateness, O, and the thrust major, T_M.

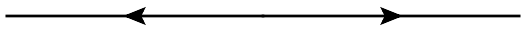
$$B_T \equiv B_L + B_R,$$

where

$$B_L \equiv \sum_{i \in \mathcal{H}^{(1)}} \frac{|\vec{p}_i \times \vec{n_T}|}{2Q} \,, \quad B_R \equiv \sum_{i \in \mathcal{H}^{(2)}} \frac{|\vec{p}_i \times \vec{n_T}|}{2Q} \,.$$

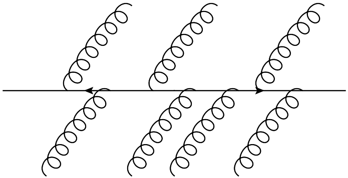
What do event shapes look like?

For dijet events,



Two back-to-back particles, $V = 0$

In reality, soft and collinear gluon emission will occur, resulting in a value for the event shape $V \approx 0$:



The quasi back-to-back region: a pair of hard quarks emit soft/collinear gluons

This is the region in which most events lie, and so the region in which we are interested.

In the soft-collinear region of phase space, α_s is modified:

$$\sigma \propto \alpha_s \int_v \frac{dk_t}{k_t} \int_v \frac{d\theta}{\theta} = \alpha_s \log^2\left(\frac{1}{v}\right)$$

When the value of the observable becomes small, the logarithms become large.

$\alpha_s log(\frac{1}{v}) \approx 1$ is no longer a perturbative expansion parameter.

In order to calculate cross-sections in this region, we rearrange the expansion: $1 + \alpha_s + \alpha_s^2 + \alpha_s^3 + \cdots \rightarrow$

$$1+(\alpha_s L+\alpha_s L^2)+(\alpha_s^2 L+\alpha_s^2 L^2+\alpha_s^2 L^3+\cdots)+(\alpha_s^3 L+\alpha_s^3 L^2+\alpha_s^3 L^3 \cdots)+\cdots$$

$$= e^{Lg_1(\alpha_s L)}(G_2(\alpha_s L) + \alpha_s G_3(\alpha_s L) + \alpha_s^2 G_4(\alpha_s L) + \cdots)$$

$$= e^L(1 + \alpha_s + \alpha_s^2 + \alpha_s^3 + \cdots)$$

$$(L \equiv log(\frac{1}{v}))$$

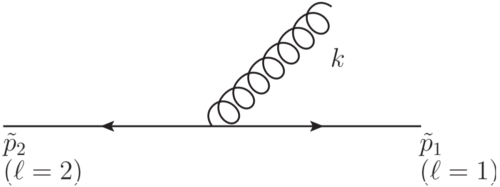
g_1 resums all of the leading logarithmic terms (LL); G_2 the next-to-leading terms (NLL), and so on.

Resummation to NLL

Our starting point is the generic observable used by the Computer Automated Semi-Analytical Resummer (CAESAR) [1].

$$V(\{\tilde{p}\}, k) = d_\ell \left(\frac{k_t^{(\ell)}}{Q} \right)^{a_\ell} e^{-b_\ell \eta^{(\ell)}} g_\ell(\phi^{(\ell)})$$

All event shapes that have been resummed at NLL can be expressed in this way.
($a = b_\ell = 1$ for thrust-type observables, and $a = 1, b_\ell = 0$ for broadening-type.)



[1] A. Banfi, G. P. Salam and G. Zanderighi, JHEP 0503 (2005) 073 [hep-ph/0407286]

For CAESAR observables, the resummed cumulative distribution has the form (for $V < v$)

$$\Sigma(v) = \frac{1}{\sigma}\int_0^v dv' \frac{d\sigma(v')}{dv'}$$

$$= e^{Lg_1(\lambda)+g_2(\lambda)}\mathcal{F}_{\rm NLL}(\lambda) = \exp\left(-\int_v [dk]|M^2(k)|\right)\mathcal{F}_{NLL}(\lambda)$$

The Sudakov form factor, containing all the virtual corrections as double logs (a LL contribution).

The $\mathcal{F}$ function, containing single logs coming from soft-collinear real emissions that are widely separated in rapidity and independent from one another (an NLL contribution).

$(L = \ln(1/v),\ \lambda = \alpha_s(Q)\beta_0 L)$

To NLL order, emissions are:

- soft and collinear
- widely separated in rapidity
- emitted independently (QCD coherence effect)
- event shapes: emissions have arbitrary rapidity fraction

All of this is encoded in the Sudakov exponent (virtual corrections) and the $\mathcal{F}$-function (real resolved+unresolved emissions).

Resummation to NNLL

In e^+e^- processes - to gain a **measurement of** α_s at ‰ level accuracy.

(More generally, in hadronic processes - to allow uncertainty in theoretical predictions to match experimental uncertainty.)

Start with the NLL formulation and relax any one of the NLL assumptions at a time.
This will result in contributions at NNLL accuracy (and beyond).

Manipulate the result to attain a pure NNLL correction.

At NNLL the resummed cumulative distribution is now

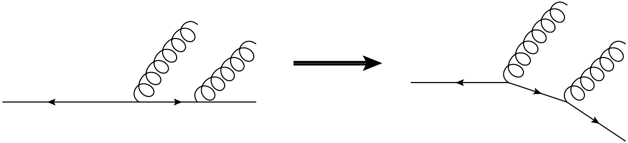
$$\Sigma(v) = \frac{1}{\sigma}\int_0^v dv' \frac{d\sigma(v')}{dv'}$$
$$= e^{Lg_1(\lambda)+g_2(\lambda)+\frac{\alpha_s(Q)}{\pi}g_3(\lambda)}\left[\mathcal{F}_{\mathrm{NLL}}(\lambda) + \frac{\alpha_s(Q)}{\pi}\delta\mathcal{F}_{\mathrm{NNLL}}(\lambda)\right]$$

(Recall that $\lambda = \alpha_s(Q)\beta_0 L$)

Sources of $\delta\mathcal{F}_{\mathrm{NNLL}}(\lambda)$

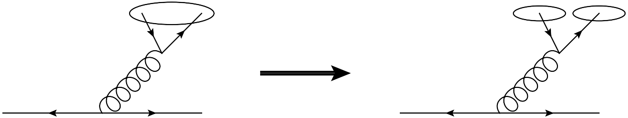
Emissions at the edges of phase space:
An emission is soft but not collinear: $\delta\mathcal{F}_{wa}$
An emission is collinear but not soft: $\delta\mathcal{F}_{hc}$

Also $\delta\mathcal{F}_{rec}$, correctly treating the recoil of one particle ($\delta\mathcal{F}_{rec} \to 0$ as $k_t \to 0$)



Sources of $\delta\mathcal{F}_{\mathrm{NNLL}}(\lambda)$

Subsequent gluon splitting can happen non-inclusively: $\delta\mathcal{F}_{correl}$



An emission having its exact kinematic rapidity bounds: $\delta\mathcal{F}_{sc}$

N.B: *all other emissions* obey the assumptions in $\mathcal{F}_{NLL}$.

The function g_3

We have not yet derived the g_3 function in terms of the generic parameters $(a, b_\ell, d_\ell, g_\ell(\phi))$.

In this work, however, we are resumming a subset of event shapes, all commonly-studied.
Two observables with the same soft-collinear behaviour will have the same Sudakov exponent: so we can use g_3 functions from the literature.
Thus the real and virtual singularities will be safely cancelled.

Since we only require a similar soft-collinear behaviour, our Sudakov factor does not accurately describe the hard-collinear or soft-wide-angle regions of phase space. These are fully accounted for in the real correction pieces $\delta\mathcal{F}_{wa}$, $\delta\mathcal{F}_{hc}$ and $\delta\mathcal{F}_{rec}$.

Semi-numerical Resummation

E.g. for the thrust, a fully analytic resummation is possible.

For other observables, however, we need to manipulate the five $\delta\mathcal{F}$ functions so they can be more easily computed using a Monte Carlo.

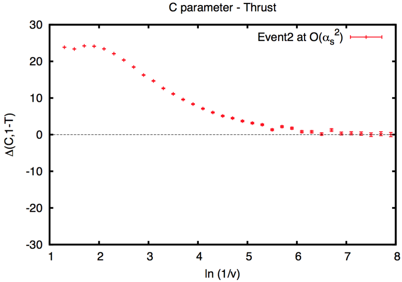
Our Monte Carlo program computes the real corrections to each event-shape using a generic procedure for each correction.

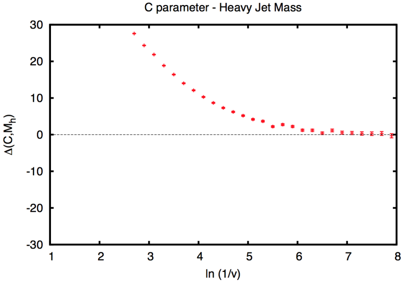
Our results for τ, B_T, B_W and m_H agree to NNLL@NNLO (i.e. up to terms $\alpha_s^3 L^2$) with those already analytically resummed to NNLL. [2,3,4,5]

The three new results (C-parameter**, O, T_M) are checked by expanding our result to α_s^2 and comparing with Monte-Carlo event generator predictions.
For this we used `Event2` [6] with 10^{11} events.

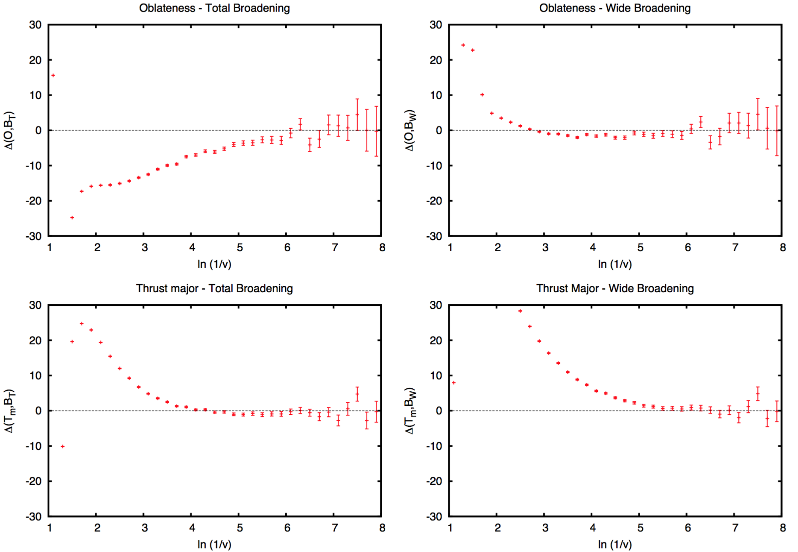
$$\Delta(v_1, v_2) = \left(\frac{1}{\sigma_0} \frac{d\sigma^{\rm NLO}}{d \ln \frac{1}{v_1}} - \frac{1}{\sigma_0} \frac{d\sigma^{\rm NNLL}|_{\rm expanded}}{d \ln \frac{1}{v_1}} \right) - \{ v_1 \to v_2 \} \, .$$

[2] T. Becher and M. D. Schwartz, JHEP 0807 (2008) 034 [arXiv:0803.0342 [hep-ph]]
[3] P. F. Monni, T. Gehrmann and G. Luisoni, JHEP 1108 (2011) 010 [arXiv:1105.4560 [hep-ph]]
[4] Y. T. Chien and M. D. Schwartz, JHEP 1008 (2010) 058 [arXiv:1005.1644 [hep-ph]]
[5] T. Becher and G. Bell, JHEP 1211 (2012) 126 [arXiv:1210.0580 [hep-ph]]
[6] S. Catani and M. H. Seymour, Nucl. Phys. B 485 (1997) 291 [Erratum-ibid. B 510 (1998) 503] [hep-ph/9605323]
[**] S. Alioli et al, JHEP 1309 (2013) 120 [arXiv:1211.7049 [hep-ph]]





In the region $v \to 0$ the distribution is dominated by the large logs which require resumming, so the difference between the expansion of our results and the full result must go to zero as $\ln(1/v) \to$ large positive values.


Oblateness - Total Broadening
Oblateness - Wide Broadening
Thrust major - Total Broadening
Thrust Major - Wide Broadening
$\Delta(O,B_T)$
$\Delta(O,B_W)$
$\Delta(T_m,B_T)$
$\Delta(T_m,B_W)$
ln (1/v)
30
20
10
0
-10
-20
-30
1 2 3 4 5 6 7 8

We have carried out calculations of event shapes in e+e- to NNLL accuracy using a novel and general method.

Our checks with analytic results in the literature and with Monte Carlo event generators allow us to be confident of our methods.

We aim to extend our work to include further event-shape variables, and become even more general.

This framework will allow us to calculate observables in a range of processes relevant for probing physics at colliders.

Bibliography

[1] A. Banfi, G. P. Salam and G. Zanderighi, JHEP 0503 (2005) 073 [hep-ph/0407286]

[2] T. Becher and M. D. Schwartz, JHEP 0807 (2008) 034 [arXiv:0803.0342 [hep-ph]]

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[6] S. Catani and M. H. Seymour, Nucl. Phys. B 485 (1997) 291 [Erratum-ibid. B 510 (1998) 503] [hep-ph/9605323]

[7] S. Alioli, C. W. Bauer, C. J. Berggren, A. Hornig, F. J. Tackmann, C. K. Vermilion, J. R. Walsh and S. Zuberi, JHEP 1309 (2013) 120 [arXiv:1211.7049 [hep-ph]]