Seiberg and level-rank duality from non-supersymmetric brane configurations

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- Such terms occur in condensed matter physics (fractional Quantum Hall effect) and useful in calculating 3D knot invariants in maths.

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- More evidence for its existence from string theory: brane configurations bearing such theories can be acted upon to reproduce this effect

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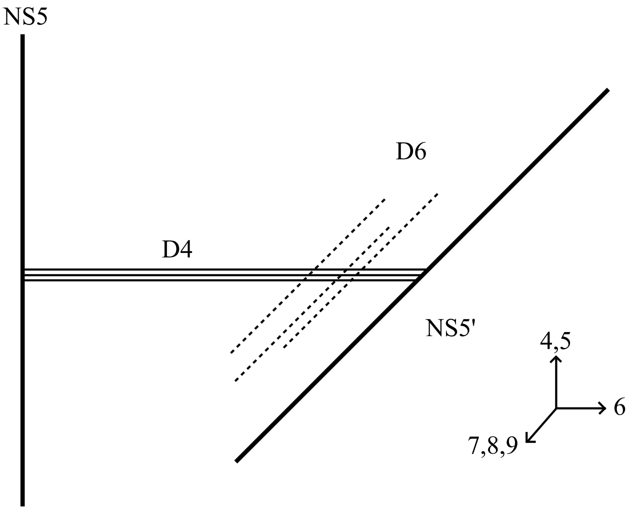
- Using this tool, Seiberg duality manifest itself when swapping fivebranes over, by conservation of linking number


NS5
D6
D4
NS5'
4,5
6
7,8,9

Statement of the result

We propose that using the same methods employed to justify previous results, two non-SUSY YM-CS theories are Seiberg-dual

$$Sp(2N)_{2k} + \text{ antisym. gauginos} \leftrightarrow Sp(2k-2N)_{2k} + \text{ antisym. gauginos} \quad (4)$$

For the principal reasons that this is supported by a consistent string theory interpretation and that their low-energy limits match precisely by Level-Rank duality.

Importantly: Taken a *4D, supersymmetric QCD* result and argue that its concepts should also apply to a *3D, non-SUSY, non-fundamental* theory! Quite a conceptual leap.

Brane pictures

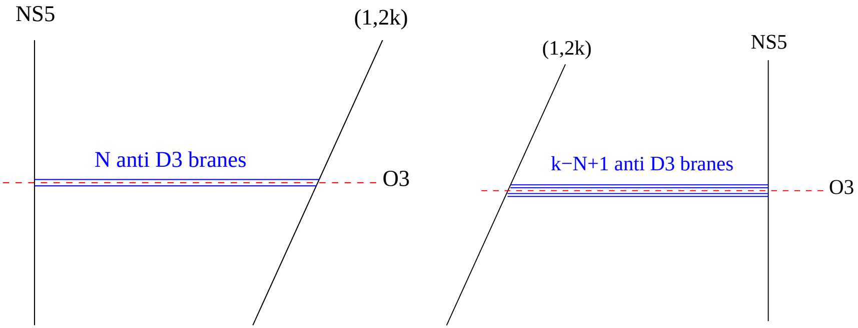


Figure: The Electric and Magnetic configurations

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- Crucial element: 3-Orientifold. Not a dynamical object, apply complicated projection operator on string modes. Pictorially represented by a 4D surface (anti-)symmetrising string modes above or below it. Carries multiple of Dp-brane charge so actively changes behaviour of Seiberg duality.

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- Orientifold reduces gauge group to $Sp(2N)$. Also breaks SUSY due to presence of anti-$D3$ branes (opposite set of preserved fermions)
- Matter content changes: adjoint (symmetric) gauge vector and scalar, but anti-symmetric fermions, hence explicit SUSY breaking.

CS-Seiberg duality

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- Swapping the five-branes sends one theory to the other. Hanany-Witten effect: every $D3$ connecting $NS5$ and $D5$ is destroyed, $D5$s not connected to $NS5$ reconnect. "Linking Number", quantity calculated from various brane charges is conserved in this process, but $O3$ has 3-brane charge, hence one extra anti-brane is generated in this effect. In total

$$N \leftrightarrow k - N + 1 \tag{5}$$

Level shifts and IR duality

- In the IR, the fermions can also be decoupled from the theory, being also massive. To do so consistently shifts the CS-level by a finite number (quadratic Casimir). For the electric theory:

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- Identity known to be exact for a variety of gauge groups including Sp.

Overview and Conclusion

- The result can be adapted to work in Type 0B, with an electric theory $U(2N)_{2k}$ with antisymmetric fermions, with minimal amount of work.
- Genuinely non-supersymmetric result. IR ranks exhibit signs of absence of UV supersymmetry.
- Result non-trivial for SO groups, different orientifold required which gives a tachyonic mass to gauge scalars. It repels the anti-branes, more work required to find the true vacuum.
- Further test of the validity of Seiberg duality and of the tools used to analyse it (brane constructions).
- Future research: generalise S-duality effects for such theories, harder because an exact duality. String theory effect so brane picture very useful there also.