Resumming Threshold Logarithms at Hadron Colliders

YTF 2014

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December 17, 2014

Outline

Conventions and Processes

Focus on $t\bar{t}$ production at hadron colliders.

$$s = (P_{h1} + P_{h2})^2$$

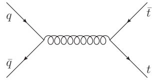
$$\hat{s} = (p_1 + p_2)^2$$

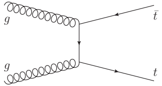
$$M_{t\bar{t}}^2 = (p_3 + p_4)^2$$

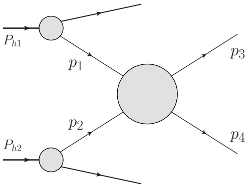
$$\tau = \frac{M_{t\bar{t}}^2}{s} \quad z = \frac{M_{t\bar{t}}^2}{\hat{s}}$$

True threshold: $\tau \to 1$

Partonic threshold: $z \to 1$







Cross-section and Divergences

QCD factorisation allows us to write the cross section for such processes as

$$\frac{d^2\sigma(\tau)}{dM\,d\cos\theta} = \frac{8\pi\beta_t}{3sM}\sum_{ij}\int_\tau^1 \frac{dz}{z}\mathcal{L}_{ij}(\tau/z,\mu_f)C_{ij}(z,M,m_t,\cos\theta,\mu_f)$$

where

$$\mathcal{L}_{ij}(\tau/z,\mu_f) = \int_{\tau/z}^1 \frac{dx}{x}\phi_{i/N_1}(x,\mu_f)\phi_{j/N_2}(\frac{\tau}{zx},\mu_f)$$

However on calculating C_{ij} we find it contains distributions, singular in $z \to 1$. At leading order C_{gg} (for example) is proportional to a delta function. At n^{th} order,

$$C \propto \left(\frac{\alpha_s}{4\pi}\right)^n \left(\frac{\ln^m(1-z)}{1-z}\right)_+ \qquad m = 0...2n-1$$

Real and Virtual diagrams separately infinite, but cancellations occur in the sum.
However, as $z \to 1$, phase space for gluons is restricted - *soft gluons*.
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Factorisation theorem in $z \to 1$ limit from Soft Collinear Effective theory (SCET).

Factorisation

$$C_{ij} = \mathrm{Tr}[\mathbf{H}_{ij}(M_{t\bar{t}}, \mu_f, ..)\mathbf{S}_{ij}(\sqrt{\hat{s}}(1-z), \mu_f, ...)] + \mathcal{O}(1-z)$$

$\mathbf{H}_{ij}$ - Hard Function. Related to virtual corrections
$\mathbf{S}_{ij}$ - Soft Function. Related to real emission of soft gluons.
Contains distributions singular in $(1-z)$.

Renormalisation Group Evolution & Resummation

$$\mathbf{H}(M_{t\bar{t}}, \mu_f, ..) \sim \ln \frac{M}{\mu_f} \qquad \mathbf{S}(\sqrt{\hat{s}}(1-z), \mu_f, ..) \sim \left[\frac{\ln\left(\frac{M_{t\bar{t}}(1-z)}{\mu_f}\right)}{1-z} \right]_+$$

No ideal choice for μ_f
Can use RG equations and pick $\mu_h = M$, $\mu_s = M(1-z)$

$$\mathbf{H}(\mu) = U_H(.., \mu, \mu_h, ..)\mathbf{H}(\mu_h)U_H^\dagger(.., \mu, \mu_h, ..)$$

where

$$U_H = \exp\left\{ 2S(\mu_h, \mu) - a_\Gamma(\mu_h, \mu)\left(\ln \frac{M^2}{\mu_h^2} - i\pi\right)\right\}\mathbf{u}$$

$$S(\mu_h, \mu) = -\int_{\alpha_s(\mu_h)}^{\alpha_s(\mu)} d\alpha \frac{\Gamma_{\text{cusp}}(\alpha)}{\beta(\alpha)} \int_{\alpha_s(\mu_h)}^{\alpha} \frac{d\alpha'}{\beta(\alpha')}$$

Performing a similar procedure for the soft function we can arrive at,

$$\frac{d^2\sigma(\tau)}{dM\, d\cos\theta} \sim \int_\tau^1 \frac{dz}{z} \,\mathrm{Tr}[U_H H U_H^\dagger U_s S U_s^\dagger]\, \mathcal{L}\left(\frac{\tau}{z}\right)$$
$$\sim \int_\tau^1 \frac{dz}{z} \,\mathrm{Tr}[U H U^\dagger S]\, \mathcal{L}\left(\frac{\tau}{z}\right)$$

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There is an alternative method...

Mellin Space

In Mellin (or moment) space, convolutions become products

$$\frac{d^2\sigma(N)}{dM\, d\cos\theta} = \int_0^1 d\tau\, \tau^{N-1} \int_\tau^1 \frac{dz}{z} \mathcal{L}(\frac{\tau}{z}) C(z)$$
$$= \mathcal{L}(N) C(N)$$

where

$$f(N) = \int_0^1 dx\, x^{N-1} f(x)$$

Here our soft function changes

$$S \sim \ln\left(\frac{M}{\bar{N}\mu_s}\right) \quad \bar{N} = Ne^{\gamma_E}$$

So really, want to pick $\mu_s = \frac{M}{\bar{N}}$

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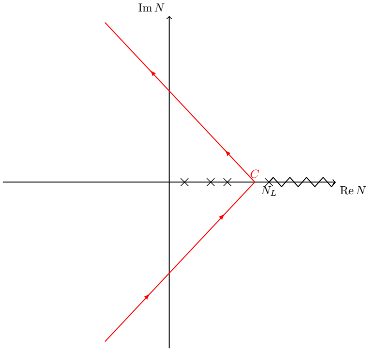


What's the difference?

Momentum Space,

$$U = \exp\left\{2S(\mu_h, \mu_s) - a_\Gamma(\mu_h, \mu_s)\left(\ln\frac{M^2}{\mu_h^2} - i\pi\right)\right\}\mathbf{u}$$

Resums logs of the form,

$$\left[\frac{\ln^n(1-z)}{1-z}\right]_+$$

Mellin Space

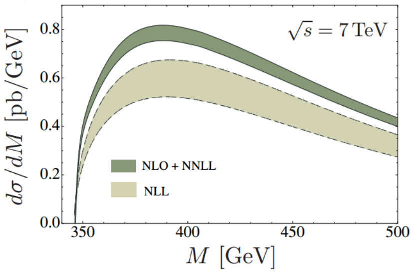
$$U = \exp\left\{\ln N\, g_0 + g_1 + \frac{\alpha_s}{4\pi}g_2 + \ldots\right\}\mathbf{u}$$

Resums logs of the form,

$$\left[\frac{\ln^n(-\ln z)}{-\ln z}\right]_+$$

Results and Comparison

Momentum Space



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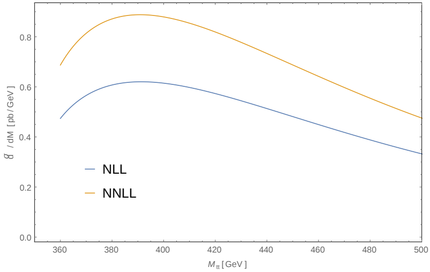

Summary & Outlook

- Divergent threshold logarithms appear in perturbative calculations
- Factorisation in SCET allows a separation of scales
- The RG evolution of resulting terms resums logarithms
- Can be performed in Mellin or momentum space
- Next steps: Investigate contributions from the non-threshold limit, perform more phenomenology calculations, match to NLO, boosted tops?...