Axiverse-induced Dark Radiation Problem

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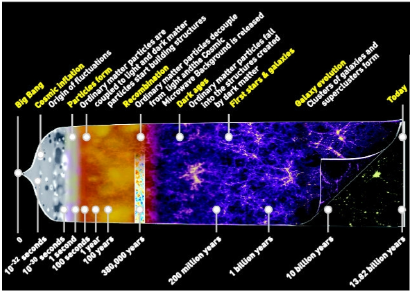
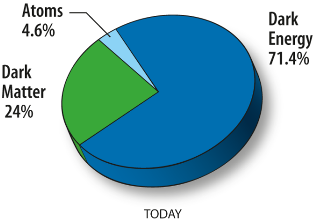
Overview

- Experimental and Theoretical motivation
  - What is dark radiation?
  - Axions and Moduli history of early universe
- Moduli-induced axion problem
  - Explicit examples: LVS, KKLT
  - Axiverse
- Axiverse dark radiation problem
  - Decay width + constraint on manifold
  - Condition on moduli mass matrix
  - Detail analysis on particular example

What is dark radiation





- Most of matter content of the universe are dark matter
- Beyond standard model **massive** particles $\rightarrow$ Dark matter
- If DM exists, why not dark radiation?
- Beyond standard model **massless** particles $\rightarrow$ Dark radiation

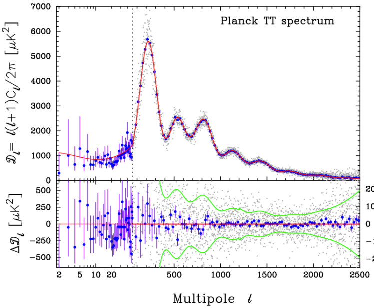
What is dark radiation

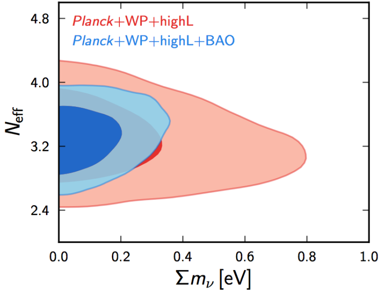
- Additional radiation decoupled from standard model thermal bath
- From CMB/BBN spectrum, we can measure **"effective number of neutrino species"**, N_{eff}
- During recombination,

$$\rho_{rad} = \rho_\gamma \left(1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{eff}\right)$$

- Standard model gives $N_{eff} = 3.046$
- Any excess $\Delta N_{eff} = N_{eff}^{measured} - 3.046$ would represent dark radiation

Experimental hints

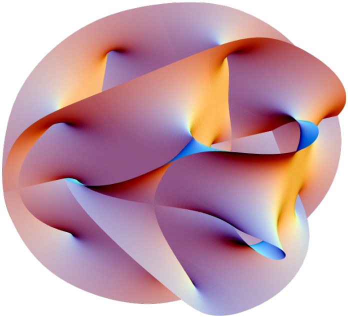




- Plank 2013: $N_{eff} = 3.55 \pm 0.60 \; (2\sigma)$
- WMAP 9: $N_{eff} = 3.30 \pm 0.27 \; (2\sigma)$

Moduli

- Moduli = light scalar fields from compactification of extra dimensions in String theory
- They usually appear in higher dimension metric and control size or shape of hidden manifold
- Their vev governs all coupling of low energy physics $\rightarrow$ Moduli stabilisation



Moduli Cosmology

- Evolution of scalar field in expanding background

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0$$

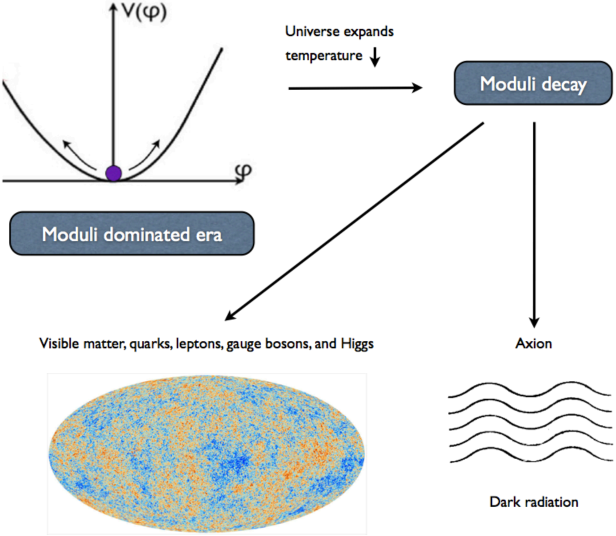
- At $H \sim m_\phi$, moduli begin to oscillate $\rightarrow$ moduli condensate
- Gravitational interaction $\rightarrow$ Moduli lifetime

$$\tau \sim (\frac{m_\phi^3}{M_{Pl}^2})^{-1} \sim 1\text{sec}\left(\frac{10\text{TeV}}{m_\phi}\right)^3$$

- BBN constraint $\rightarrow$ heavy moduli $\rightarrow$ Cosmological moduli problem

(de Carlos, Casas, Quevedo, Roulet, 1993, Banks, Kaplan, Nelson, 1993)

Cosmological history



Axions

- Axions appear as low energy spectrum of higher dimension antisymmetric tensor fields
- Axion = complex partner of moduli
- The moduli space of X is parameterised by moduli s_i and axion fields t_i

$$z_i = t_i + is_i$$

- Because of shift symmetry $z_i \to z_i + \alpha$, axions are expected to be massless or light
- One of them can be used to solve the strong CP problem (QCD axion)

Dark radiation problem

- Axion relativistic production from moduli decay gives

$$\Delta N_{eff} = \frac{43}{7}\frac{\rho_{axion}}{\rho_{\gamma}}\left(\frac{g^*}{g^*_{reheat}}\right)^{1/3}$$
$$= \frac{43}{7}\frac{\Gamma_{axions}}{\Gamma_{visible}}\left(\frac{g^*}{g^*_{reheat}}\right)^{1/3}$$

- This leads to constraint on decay width of moduli fields

- Let's see some examples

Large Volume Scenario

- Kahler and Superpotential (Swiss-Cheese CY)

$$
\begin{aligned}
K &= -2\ln(V+\xi) \\
W &= W_0 + Ae^{-2\pi T} \\
V &= S_b^{3/2} - S_s^{3/2}
\end{aligned}
$$

- Moduli decay rate

$$
\begin{aligned}
\Gamma_{axion} &= \frac{1}{48\pi}\frac{m_\phi^3}{M_{Pl}^2} \\
\Gamma_{Higgs} &= \frac{z^2}{24\pi}\frac{m_\phi^3}{M_{Pl}^2}
\end{aligned}
$$

- $\Delta N_{eff} < 0.3 \rightarrow z \geq 2 \rightarrow$ many Higgs doublet model

(Cicoli, Conlon, Quevedo, 2012; Angus, Conlon, Haisch, Powell, 2013)

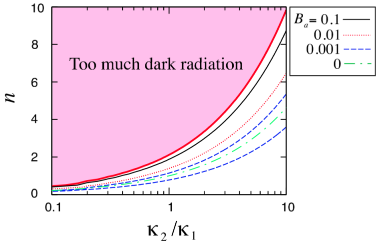
QCD axion in KKLT

- Kahler and Superpotential

$$
\begin{aligned}
K &= -2\ln(V) \\
W &= W_0 + Ae^{-\alpha Z_0} + Be^{-\beta(Z_1+nZ_2)} \\
V &= S_0^{3/2} - \kappa_1 S_1^{3/2} - \kappa_2 S_2^{3/2}
\end{aligned}
$$

- Moduli decay rate

$$
\begin{aligned}
\Gamma_{axion} &= \frac{(n^2\kappa_1^2 - \kappa_2^2)^2}{768\pi\kappa^3}\frac{m_\phi^3}{M_{Pl}^2} \\
\Gamma_{gauge} &= \frac{\kappa_2}{8\pi}\frac{m_\phi^3}{M_{Pl}^2}
\end{aligned}
$$



(Higaki, Nakayama, Takahashi, 2013)

Axiverse

- Number of scalar field from two-form (three-form) is equal to number of non-equivalent two (three)-cycles on compactified manifold
- Compactified manifold is complicated $\rightarrow$ *The number of moduli/axions is expected to be large* $(N \sim 100 - 1000)$
- A plenitude of axions = **String Axiverse**
- They range in mass and coupling $\rightarrow$ many ways to detect/constrain them

(Arvanitaki, Dimopoulos, Dubovsky, Kaloper, Marsh-Russell, 2009)

Axiverse-induced dark radiation problem

- A model with only one (or two) species of axion (LVS, KKLT) $\rightarrow$ tight constraint on couplings
- $\Gamma_{axions} \propto N \rightarrow \Delta N_{eff} \propto N$
- Axiverse with $O(100 - 1000)$ axions $\rightarrow$ very tight constraint on number of axions

Our goal: study a (more generic) solution with large number of axions

Analysis: decay width coefficients

- The interaction Lagrangian can be written as following

$$\mathcal{L} = \frac{1}{2}K_{ij}\partial^\mu s^i \partial_\mu s^j + \frac{1}{2}K_{ij}\partial^\mu t^i \partial_\mu t^j - \frac{1}{4}\left(\sum_{i=1}^{N} N_i z_i\right) F_{\mu\nu}F^{\mu\nu} + K_{\alpha\beta}D_\mu f^\alpha D^\mu f^\beta + K_{\alpha\beta}\tilde{f}^\alpha \not{D} \tilde{f}^\beta$$

- Moduli decay width is straightforward

$$\Gamma_{\text{axions}} = \sum_{j=1}^{N}\left(\sum_{i=1}^{N} C_{ij}U_{ik}\right)^2 \frac{m_{s_k}^3}{M_{PL}^2}$$

$$\Gamma_{\text{gauge particles}} = n_G\left(\sum_{i=1}^{N} B_i U_{ik}\right)^2 \frac{m_{s_k}^3}{M_{PL}^2}$$

$$\Gamma_{\text{fermions/sfermions}} = n_f\left(\sum_{i=1}^{N} D_i U_{ik}\right)^2 \frac{m_{s_k}^3}{M_{PL}^2}$$

- where $C_{ij} = \frac{1}{\sqrt{K_{ii}^D}}\frac{\partial \ln K_{ii}^D}{\partial s_j}, B_i = \frac{\alpha}{\sqrt{K_{ii}^D}}N_i, D_i = \frac{1}{\sqrt{K_{ii}^D}}\frac{\partial \ln K_{\alpha\alpha}^D}{\partial s_i}$

Analysis: decay width coefficients

- Assume that all couplings are roughly the same order, $C_{ij} \sim \langle C \rangle \sim B_j \sim \langle B \rangle \sim D_j \sim \langle D \rangle$
- As one would expect, increasing number of axions is equivalent to increasing dark radiation.

$$\langle \Delta N_{eff} \rangle \propto \frac{\Gamma_{axions}}{\Gamma_{visible}} \propto \frac{N \langle C \rangle^2}{n_G \langle B \rangle^2 + n_f \langle D \rangle^2} \propto N$$

where the unity in mean value of $(\sum_{i=1}^{N} U_{ik})^2$ is used.

- Very bad situation for axiverse, $\Delta N_{eff} \sim N \sim 100 - 1000$

Analysis: decay width coefficients

- Simplest way out → force each modulus to interact with only its partner → **Dark radiation is N independent**
- if we impose condition on coefficients:

$$C_{ij} = \frac{1}{\sqrt{K_{ii}^D}} \frac{\partial \ln K_{ii}^D}{\partial s_j} \equiv C_i \delta_{ij}$$

- on average, dark radiation becomes

$$\langle \Delta N_{eff} \rangle \propto \frac{\langle C \rangle^2}{n_G \langle B \rangle^2 + n_f \langle D \rangle^2}$$

where the orthogonality of rotation matrix, $(\sum_{i=1}^{N} U_{ik}^2) = 1$, is introduced.

- The strong correlation is relaxed → which manifold exhibits this property?

Constraints on internal manifold: Examples

- Calabi-Yau

$$V_X = \sum_{i,j,k}^{N} d_{ijk} S_i S_j S_k, \quad K = -\gamma \ln V_X$$

generically $C_{ij} \neq C_i \delta_{ij}$ unless the coefficients is carefully chosen, for example $V_X = d\, S_1^3$ or $V_X = d\, S_1 S_2 S_3$

- G_2 manifold

$$V_X = \prod_{i=1}^{N} S_i^{a_i}, \quad K = -\gamma \sum_{i=1}^{N} a_i \ln S_i, \quad K_{ij} = \frac{\gamma a_i \delta_{ij}}{s_i^2}$$

Only non-zero component is

$$\partial_i K_{ii} = \frac{-2\gamma a_i}{s_i} \to C_{ij} = \frac{-2\langle s_i \rangle^2}{\sqrt{\gamma a_i}} \delta_{ij}$$

Condition on moduli mass matrix

- Can we do better?

- We found important relation between moduli mass matrix and eigenvalue of Kahler metric which exhibit $\frac{1}{N}$ behaviour of dark radiation.

$$\sqrt{K_i} \propto U_{ij}$$

  where K_i is eigenvalues of Kahler metric and U_{ij} is rotation matrix of moduli fields.

- In this case, correlation becomes

$$\langle \Delta N_{eff} \rangle \propto \frac{N \langle C \rangle^2}{n_G N^2 \langle B \rangle^2 + n_f N^2 \langle D \rangle^2} \propto \frac{1}{N}$$

- How can we realise this in practice? → **M-theory on G_2 manifold**

G_2 compactified M theory

- M theory on manifold with G_2 holonomy - G_2 MSSM (Acharya, Bobkov, Kane, Shao, Kumar, 2008; Acharya, Kane, Kumar, 2012)
- The Kahler potential arising from G_2 manifold compactification is controlled by moduli fields and given by the function

$$K = -3\ln\left(\prod_{i=1}^{N} s_i^{a_i}\right), \quad \sum_{i=1}^{N} a_i = \frac{7}{3}$$

- It has been shown that a setup with a hidden sector with two gauge groups where first group is non-abelian with 1 flavour of quarks and second group is pure glue non-abelian leads to dS vacua.
- In this realistic setup, superpotential is written as

$$W = A_1\phi^a e^{ib_1\sum_i^N N_i S_i} + A_2 e^{ib_2\sum_i^N N_i S_i}$$

G_2 compactified M theory

- It is straightforward to work out rotation matrix from moduli stabilisation.

$$U_{kj} = \sqrt{\frac{a_{j+1}}{(\sum_{i=1}^{j} a_i)(\sum_{i=1}^{j+1} a_i)}}\sqrt{a_k}, \quad , \quad k \leq j$$

$$U_{kj} = -\sqrt{\frac{\sum_{i=1}^{j} a_i}{\sum_{i=1}^{j+1} a_i}} \quad , \quad k = j+1$$

$$U_{kN} = \sqrt{\frac{3a_k}{7}}$$

where $i = 1 \ldots N-1$ are degenerated lightest moduli and $i = N$ is heavy modulus.

- Notice that except $k = j+1$, $U_{kj} \propto \sqrt{a_k} \propto \sqrt{K_k}$. Therefore, we can suppress the element $U_{j+1,j}$ by setting

$$\sum_{i=1}^{j} a_i \ll a_{j+1}$$

We expect $\frac{1}{N}$ suppression on dark radiation under this condition.

G_2 compactified M theory

- In stead of scanning N parameters space satisfying condition, $\sum_{i=1}^{N-1} a_i \ll a_N$, we will give toy models which can be parametrised by single parameter.

- n-moduli dominated configuration

$$a = \{\underbrace{\epsilon\bar{a}, \ldots, \epsilon\bar{a}}_{N-n}, \underbrace{\frac{7}{3n} - \frac{\epsilon\bar{a}(N-n)}{n}, \ldots, \frac{7}{3n} - \frac{\epsilon\bar{a}(N-n)}{n}}_{n}\}$$

- geometric sequence configuration

$$a = \{a_0, a_0, a_0 r, a_0 r^2, \ldots, a_0 r^{N-2}\}$$

Results

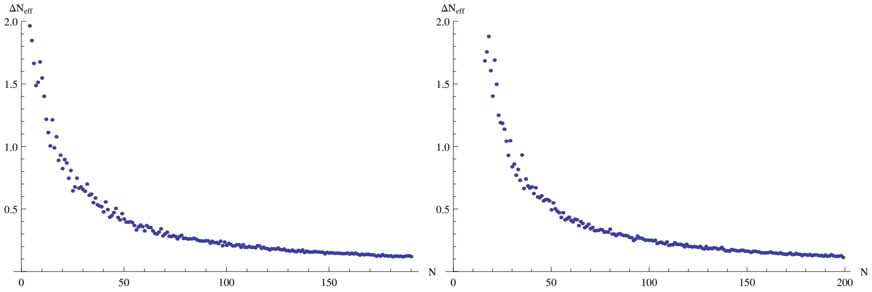


Figure: Result from geometric sequence and double moduli dominated configurations showing ΔN_{eff} as a function of N, where $r = 2$

Results

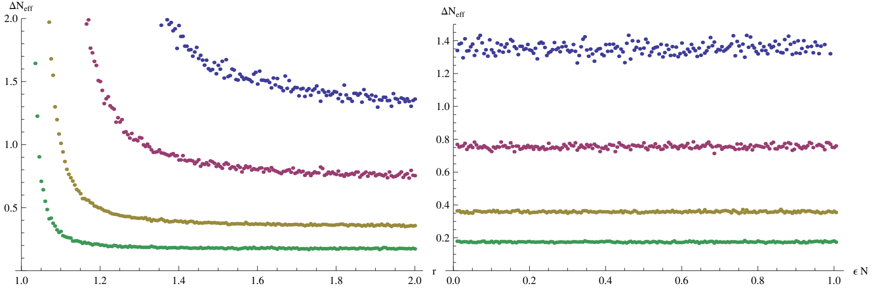


Figure: Result from geometric sequence and double moduli dominated configurations showing ΔN_{eff} as a function of ϵN, where points in blue, red, yellow, green are $N = 30, 50, 100, 200$ respectively

Conclusions

- Generic Axiverse scenario predicts too much dark radiation – $\Delta N_{eff} \propto N$
- Using effective lagrangian approach, we show the following:
- G_2 manifold naturally relaxes strong correlation between number of moduli/axions and dark radiation.
- It is possible to suppress $(\frac{1}{N})$ dark radiation by proposing relation between moduli couplings and moduli mass matrix.

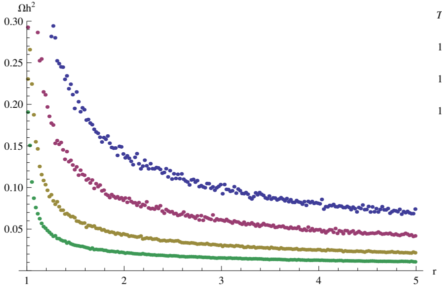
Back up slides

G_2MSSM, Reheating temperature and dark matter relics

$$\Omega_{LSP}h^2 = 0.15\left(\frac{1}{D_{\text{total}}}\right)^{1/2}\left(\frac{10.75}{g_\star}\right)^{1/4}\left(\frac{m_{LSP}}{100\text{GeV}}\right)\left(\frac{\sigma_0}{\langle\sigma v\rangle}\right)\left(\frac{100\text{TeV}}{m_X}\right)^{3/2}$$

$$T_r = \left(\frac{40}{\pi g_\star}\right)^{1/4}\sqrt{\frac{D_{\text{total}}m_X^3}{M_{Pl}}}$$

We use $m_{LSP} = 100MeV$, $\langle\sigma v\rangle \sim \sigma_0 \sim 3\times 10^{-24}\ cm^3 s^{-1}$.



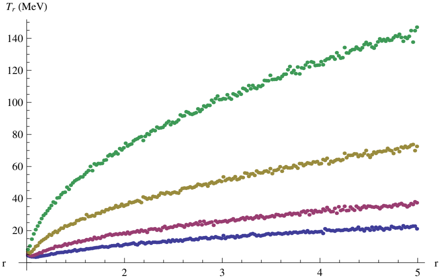


Figure: Geometric sequence configuration result showing $\Omega_{LSP}h^2$ and reheating temperature as a function of common ratio r, where points in blue, red, yellow, green are $N = 30, 50, 100, 200$ respectively

G_2MSSM, Reheating temperature and dark matter relics

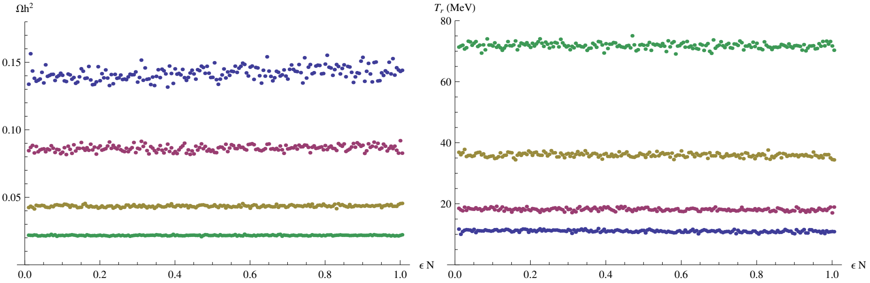


Figure: Double moduli dominated configuration result showing $\Omega_{LSP}h^2$ and reheating temperature as a function of common ratio r, where points in blue, red, yellow, green are $N = 30, 50, 100, 200$ respectively

N_{eff} calculation

- At the time of baryogenesis, the effective number of neutrinos is defined as

$$\rho_{rad} = \rho_{e^\pm} + \rho_\gamma + N_{eff}\rho_\nu$$

- The extra radiation part can be included inside the definition of N_{eff} as following.

$$\begin{aligned} \rho'_{rad} &= \rho_{e^\pm} + \rho_\gamma + N_{eff}\rho_\nu \left(1 + \frac{\rho_a}{\rho_{3\nu}}\right) \\ \frac{\Delta N_{eff}}{N_{eff}} &= \frac{\rho_a}{\rho_{3\nu}} \end{aligned}$$

- Since both neutrinos and axion are not in thermal equilibrium

$$\frac{\rho_a(\text{BBN})}{\rho_{3\nu}(\text{BBN})} = \frac{\rho_a(\nu\text{dec})}{\rho_{3\nu}(\nu\text{dec})}$$

where

$$\begin{aligned} \frac{\rho_{rad}}{\rho_{3\nu}} &= 1 + \frac{\rho_{e^\pm}}{\rho_{3\nu}} + \frac{\rho_\gamma}{\rho_{3\nu}} \\ &= \frac{43}{21} \end{aligned}$$

- Putting all relations together, we obtain

$$\Delta N_{eff} = \frac{43\rho_a(\nu\text{dec})}{7\rho_{rad}(\nu\text{dec})}$$

N_{eff} calculation

- To make prediction from moduli branching fraction, we need to relate this quantity to the time of reheating in moduli decay scenario. Firstly, because of its very weak coupling, axion has never been in thermal equilibrium. As a consequence, its energy density scale as $1/a^4$. We can write

$$\frac{\rho_a(\nu\text{dec})}{\rho_a(reheat)} = \frac{a^4(\text{reheat})}{a^4(\nu\text{dec})}$$

- Using the fact that comoving entropy is a conserved quantity $S \sim a^3 g_\star(T) T^3$ or $T \sim \frac{1}{a g_\star^{1/3}(T)}$, we get

$$\begin{aligned}
\rho_{rad} &\sim g_\star(T) T^4 \\
&\sim \frac{1}{a^4 g_\star^{1/3}(T)} \\
\frac{\rho_{rad}(\nu\text{dec})}{\rho_{rad}(\text{reheat})} &= \frac{a^4(\text{reheat}) g_\star^{1/3}(\text{reheat})}{a^4(\nu\text{dec}) g_\star^{1/3}(\nu\text{dec})}
\end{aligned}$$

- Substitute back, we obtain the effective number of neutrinos

$$\begin{aligned}
\Delta N_{eff} &= \frac{43}{7} \frac{\rho_a(\text{reheat})}{\rho_{rad}(\text{reheat})} \frac{g_\star^{1/3}(\nu\text{dec})}{g_\star^{1/3}(\text{reheat})} \\
&= \frac{43}{7} \frac{\text{Br}(X_i \to \text{axions})}{1 - \text{Br}(X_i \to \text{axions})} \frac{g_\star^{1/3}(\nu\text{dec})}{g_\star^{1/3}(\text{reheat})}
\end{aligned}$$

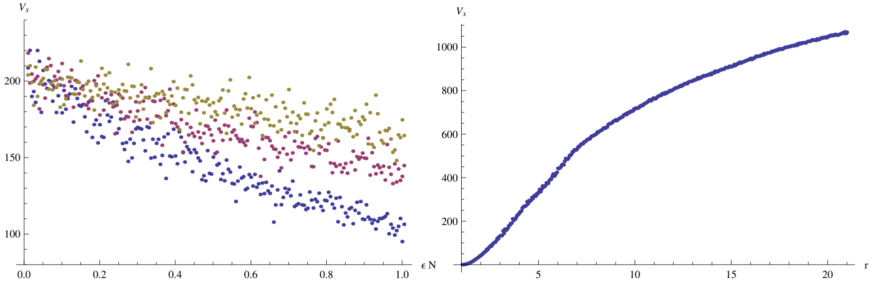
G_2MSSM, supergravity limits

- We can further test configurations a_i by GUT coupling constraint and supergravity limit.
- The volume function can be written by

$$V_X = \sum_{i=1}^{N} \langle s_i \rangle^{a_i} = \sum_{i=1}^{N} \left(\frac{a_i}{N_i^{sm}} \frac{3P_{eff}Q}{14\pi(Q-P)} \right)^{a_i}$$

- where we match N_i^{sm} with gauge coupling at GUT scale $\sum_{i=1}^{N} N_i^{sm} \langle s_i \rangle = \frac{1}{\alpha} \approx 25$

G_2MSSM, supergravity limits



(a) Double moduli dominated (b) Geometric sequence

Figure: Result from each configurations showing V_X as a function of parameter ϵN and common ratio r, where points in blue, red, yellow are $N = 20, 50, 100$ respectively