Turbulence at Weak and Strong Couplings in Quantum Field Theory

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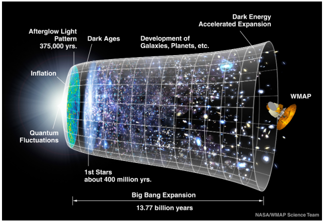
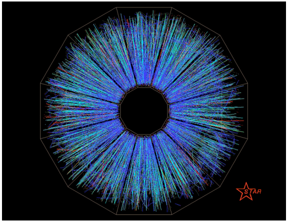
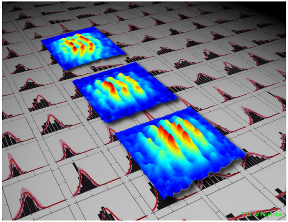
Physical Systems Far From Equilibrium



Inflationary Cosmology



Early Stages of
Heavy-Ion Collisions



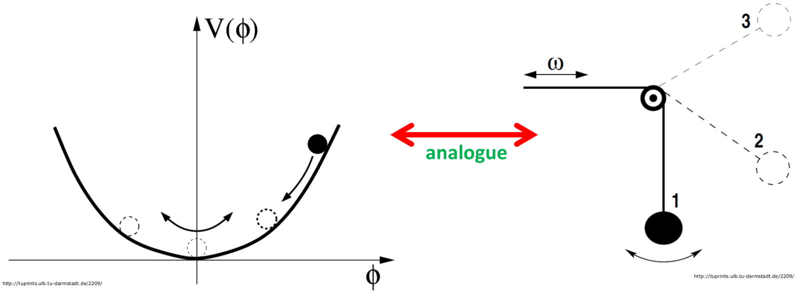
Ultracold Atom
Experiments

Very different energy scales: large $\longrightarrow$ small

BUT: Common properties!

Reheating after Inflation

- Transition from inflation to standard Big Bang cosmology.
- Inflaton field oscillates and decays $\longrightarrow$ massive particle production.
- Popular scenarios:
  - Tachyonic instability,
  - Parametric resonance:



$$\langle \Phi \rangle (t) \longleftrightarrow \omega, \qquad \langle \Phi\Phi \rangle (t) \longleftrightarrow x$$

$$\longrightarrow \text{ large over-occupation: } n \sim 1/\lambda$$

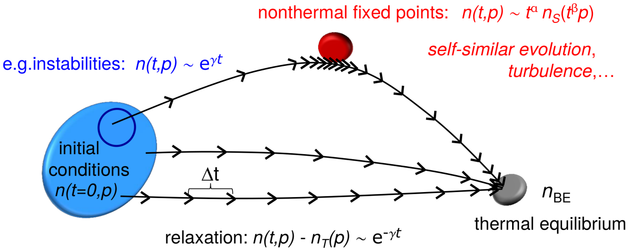
Kofman, Linde & Starobinsky

Thermalisation Process

Initial state:
Far from equilibrium

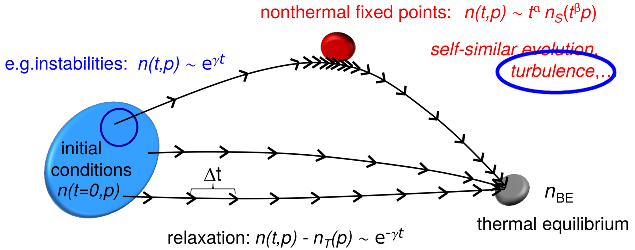
Non-equilibrium dynamics

Final state:
Thermal equilibrium



Berges

Thermalisation Process



Berges

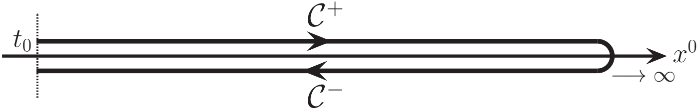
Outline

- Relativistic $O(N)$-symmetric real scalar field theory with quartic self-interaction in $d+1$-dimensional Minkowski space:

$$\mathcal{L}[\varphi] = \frac{1}{2}\partial_\mu \varphi_a(x)\partial^\mu \varphi_a(x) - \frac{1}{2}m^2\varphi_a(x)\varphi_a(x) - \frac{\lambda}{4!N}\big(\varphi_a(x)\varphi_a(x)\big)^2.$$

- Initial value problem!
  - Closed time-path (in-in formalism):

$$S[\varphi] = \int_{\mathcal{C}} \mathrm{d}x^0 \int \mathrm{d}^d x \mathcal{L}[\varphi] \equiv \int_{x,\mathcal{C}} \mathcal{L}[\varphi]. \qquad (1)$$



Schwinger; Keldysh

Formalism

2PI Effective Action

- Functional method to describe time evolution of quantum fields
- Two-point functions:

$$\rho(x,y) = \mathrm{i}\,\langle[\Phi(x),\Phi(y)]\rangle\,, \qquad F(x,y) = \frac{1}{2}\,\langle\{\Phi(x),\Phi(y)\}\rangle$$

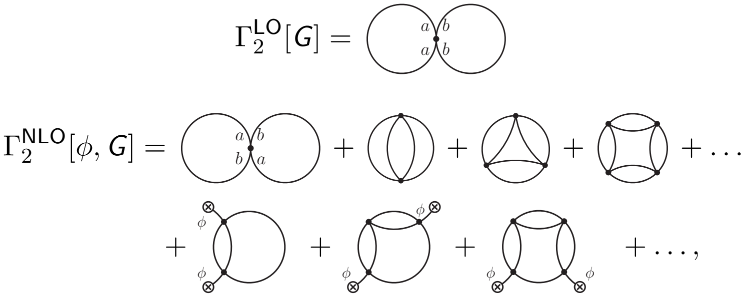
- Kadanoff-Baym equations of motion:

$$\begin{aligned}\left[\Box_x + M^2(x)\right] F(x,y) &= -\int_{t_0}^{x^0} \mathrm{d}z\, \Sigma^\rho(x,z) F(z,y) \\ &\quad + \int_{t_0}^{y_0} \mathrm{d}z\, \Sigma^F(x,z)\rho(z,y), \\ \left[\Box_x + M^2(x)\right] \rho(x,y) &= -\int_{y^0}^{x^0} \mathrm{d}z\, \Sigma^\rho(x,z)\rho(z,y),\end{aligned}$$

- Memory integrals!

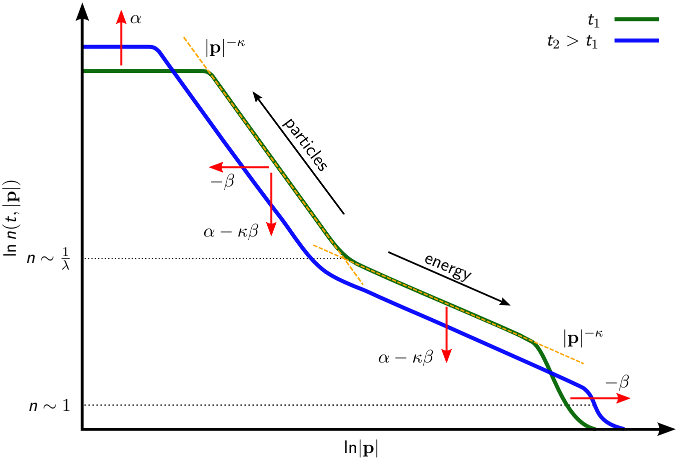
Berges

- $1/N$ expansion in the number of fields



Berges; Aarts et al.

Dual Cascade Picture

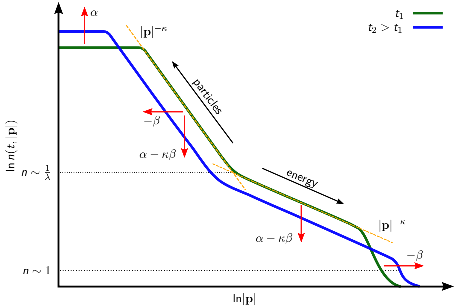


- Low momenta: inverse particle cascade
- High momenta: direct energy cascade
- Stationary exponent κ, dynamical exponents α, β

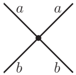
Micha & Tkachev; Berges, Rothkopf & Schmidt; Berges & Sexty; Orioli

Dual Cascade Picture

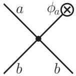
Theoretical Predictions from Kinetic Theory



- Symmetric phase ($\phi = 0$):
  - $\kappa_P = 4$ (non-rel.: 5)
  - $\kappa_E = 5/3$



- Broken phase ($\phi \neq 0$):
  - $\kappa_P = 1$
  - $\kappa_E = 3/2$

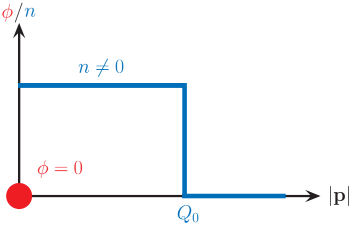


Micha & Tkachev; Berges, Rothkopf & Schmidt; Berges & Sexty; Orioli

Set-up and Initial Conditions

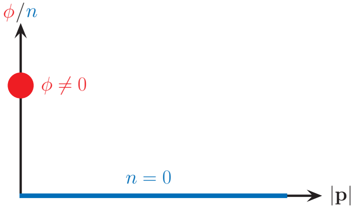
- Spatial homogeneity and isotropy in $3+1$ dimensions
- Calculations on isotropic momentum grid ($\rightarrow$ regularisation)
- Discretised time direction
- $N = 4$ scalar fields

Fluctuation-dominated initial condition



$$n(\mathbf{p}, t=0) = \frac{n_0}{\lambda}\Theta(Q_0 - |\mathbf{p}|)$$

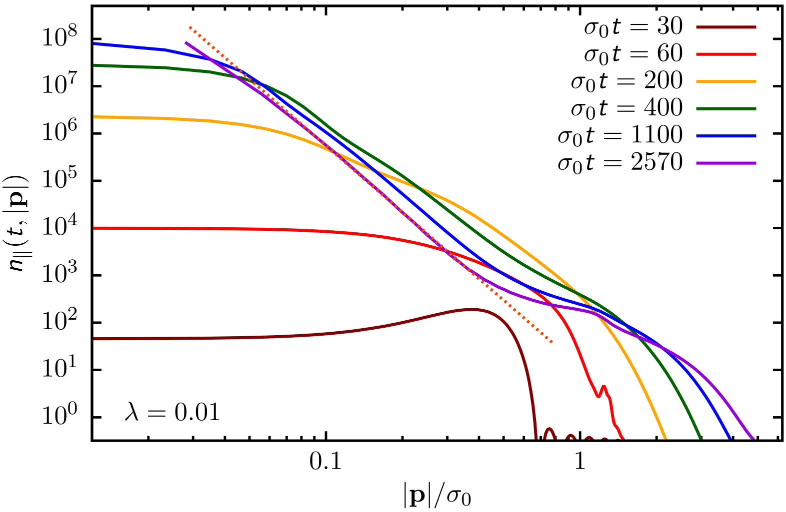
Macroscopic field-driven initial condition

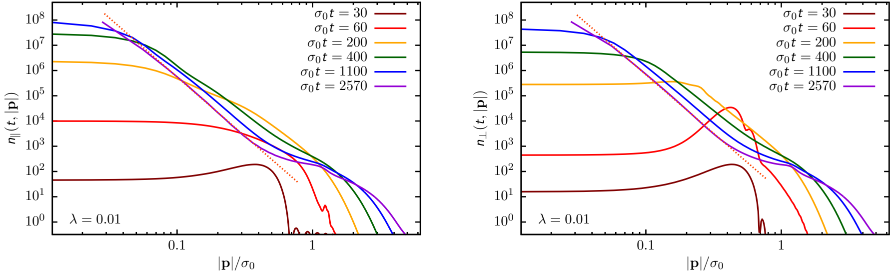


$$\phi(t=0) = \sqrt{\frac{6N}{\lambda}}\sigma_0$$

Turbulence at Weak Couplings

Parametric Resonance – Occupation Number

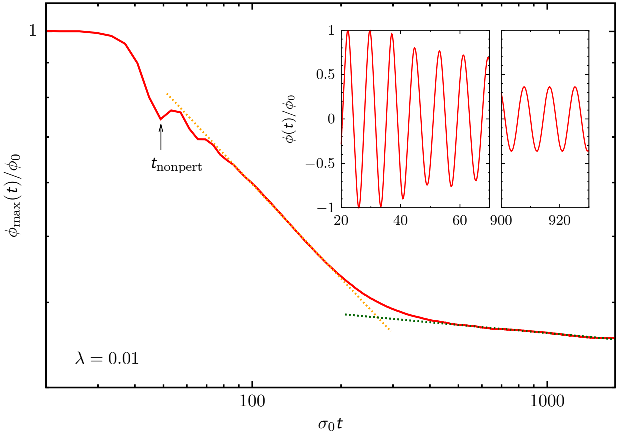




- Initial instability clearly visible.
- Evolution into **dual cascade picture** evident.
- Particle cascade: $\kappa \approx 4.7$ (still evolving).
- Reminder: theoretical prediction is $\kappa_{\mathrm{P}} = 5$ non-relativistically
- Initial difference vanishes.

Turbulence at Weak Couplings

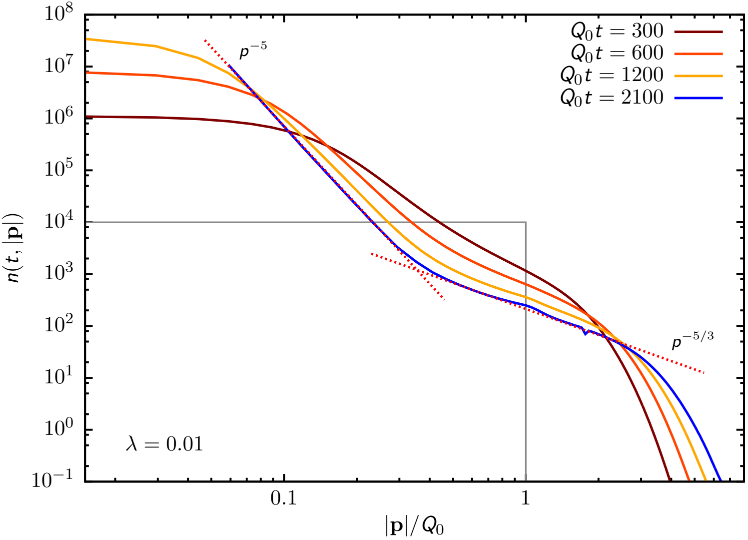
Parametric Resonance – Macroscopic Field Evolution



- Macroscopic field oscillates with $\omega_\phi \approx 0.6\sigma_0$ (mass!).
- Macroscopic field decays with $\sim t^{-0.5}$, then approximately constant.

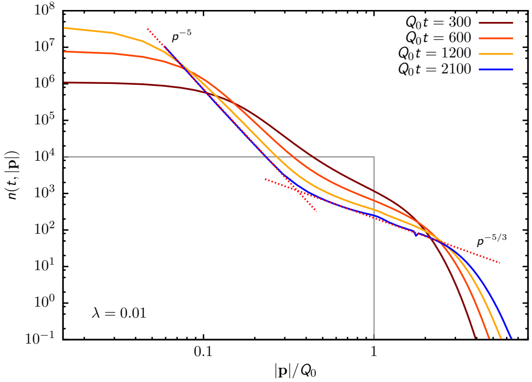
Turbulence at Weak Couplings

Fluctuation-Driven – Occupation Number



Turbulence at Weak Couplings

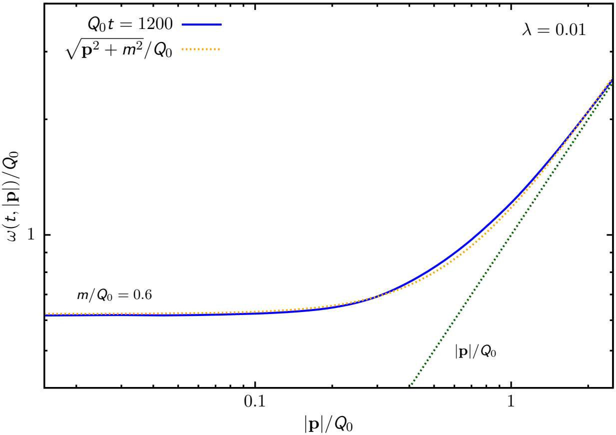
Fluctuation-Driven – Occupation Number



- Clear evolution into **dual cascade**.
- Particle cascade: $\kappa_P = 5$ (**non-relativistic** prediction!).
- Energy cascade: $\kappa_E = 5/3$ (as predicted!) – **first observation**!

Turbulence at Weak Couplings

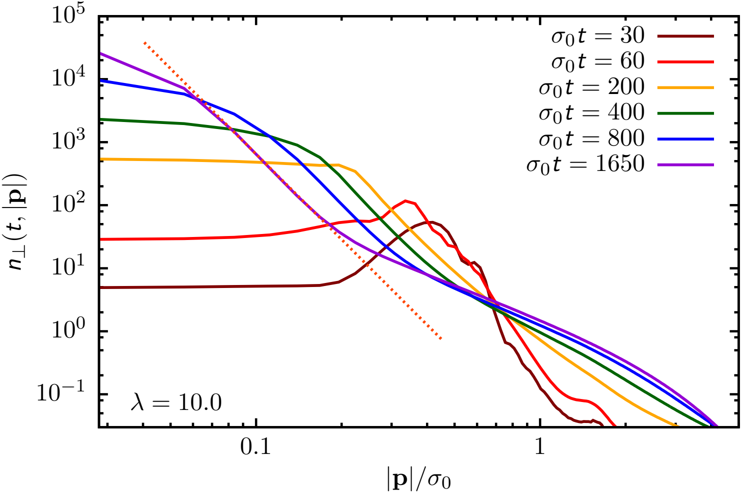
Fluctuation-Driven – Dispersion Relation



- Relativistic dispersion relation $\sqrt{\mathbf{p}^2 + m^2}$ with $m \approx 0.6 Q_0$.
- Explains non-relativistic behaviour in infrared!

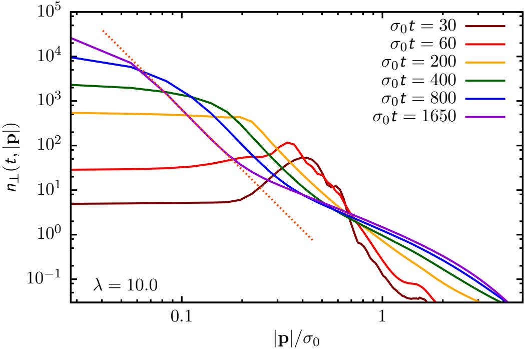
Turbulence at Strong Couplings

Parametric resonance



Turbulence at Strong Couplings

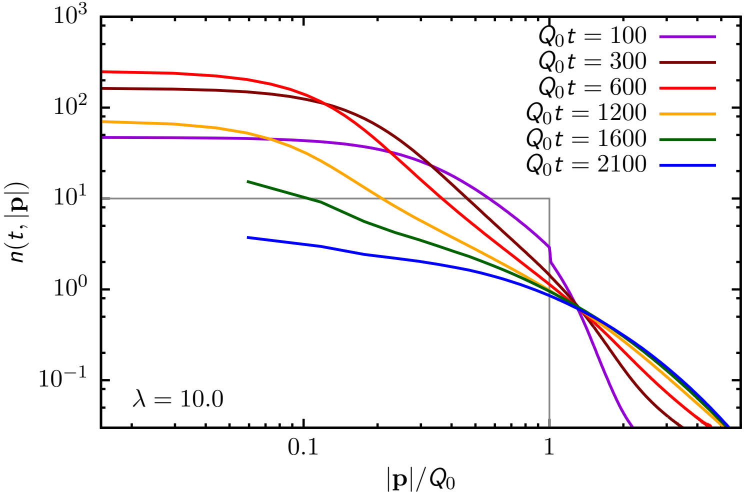
Parametric resonance



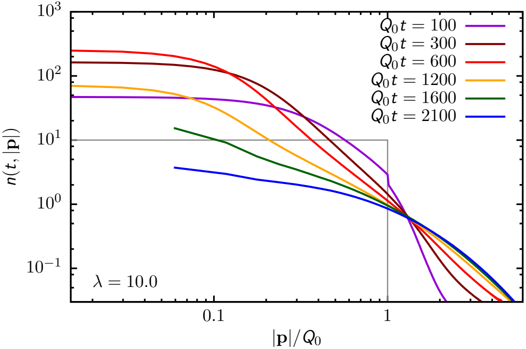
- Hybrid picture:
  - Turbulence in the infrared.
  - Approach to thermalisation in ultraviolet.

Turbulence at Strong Couplings

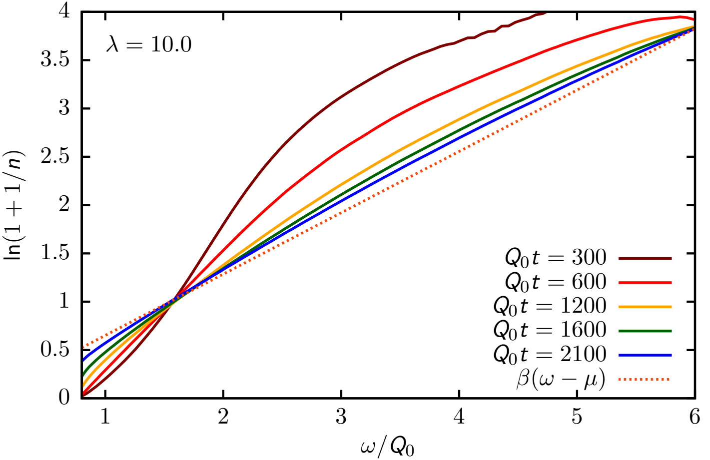
Fluctuation-Driven Initial Conditions



Turbulence at Strong Couplings
Fluctuation-Driven Initial Conditions



- Only onset of turbulence.
- Approach to quantum thermal equilibrium (Bose-Einstein distribution).



$$n_{\mathrm{B}}(\omega) = \frac{1}{\exp\{(\omega-\mu)/T\} - 1}, \qquad T \approx 0.6 Q_0, \mu \approx 0$$

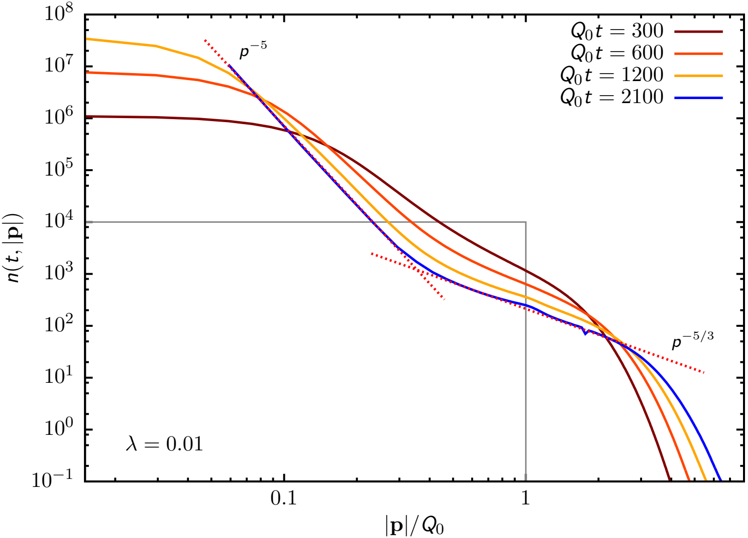
Conclusions and Outlook

- Study of turbulence in weakly- and strongly-coupled QFT,
- 2PI effective action truncated at NLO in $1/N$ expansion.

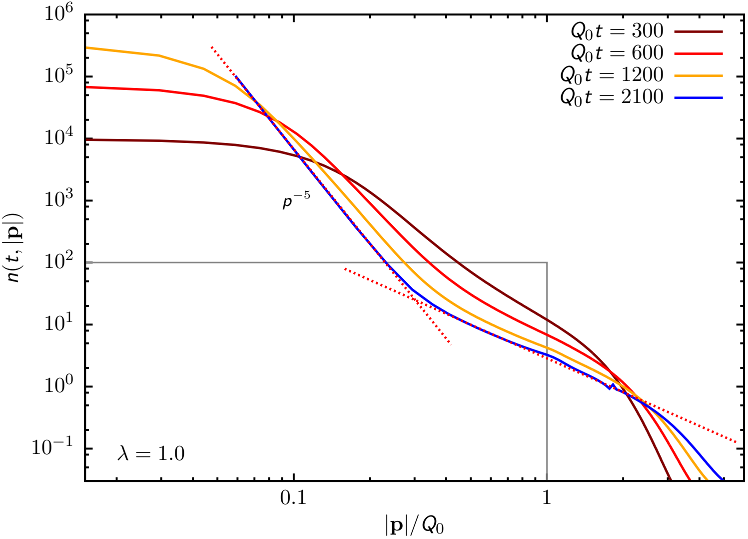
- Identification of direct energy cascade with $\kappa = 5/3$ in symmetric regime,
- Detection of turbulent cascades for strong couplings, regime where classical-statistical simulations have long broken down.

- Evolution to even later times,
- Study of approach to quantum thermal equilibrium,
- Inclusion of fermions,
- Calculation of reheating temperature from first principles.

Turbulence at Weak and Strong Couplings in QFT



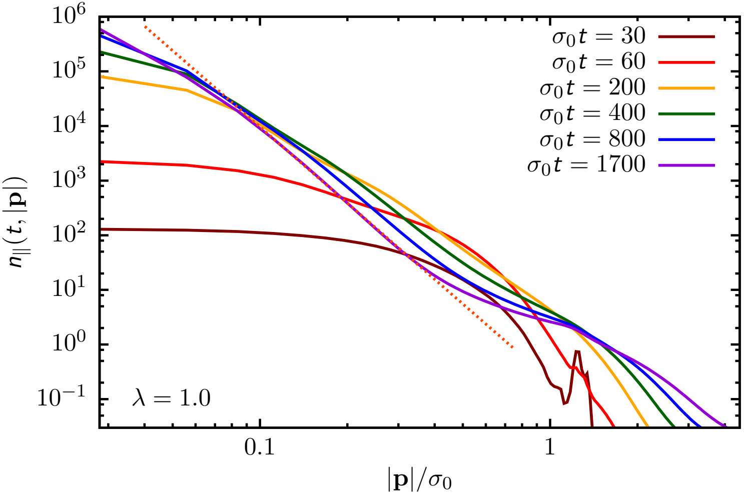
Backup Slides



10^6
10^5
10^4
10^3
10^2
10^1
10^0
10^{-1}
$n(t,|\mathbf{p}|)$
$Q_0 t = 300$
$Q_0 t = 600$
$Q_0 t = 1200$
$Q_0 t = 2100$
p^{-5}
$\lambda = 1.0$
0.1
1
$|\mathbf{p}|/Q_0$

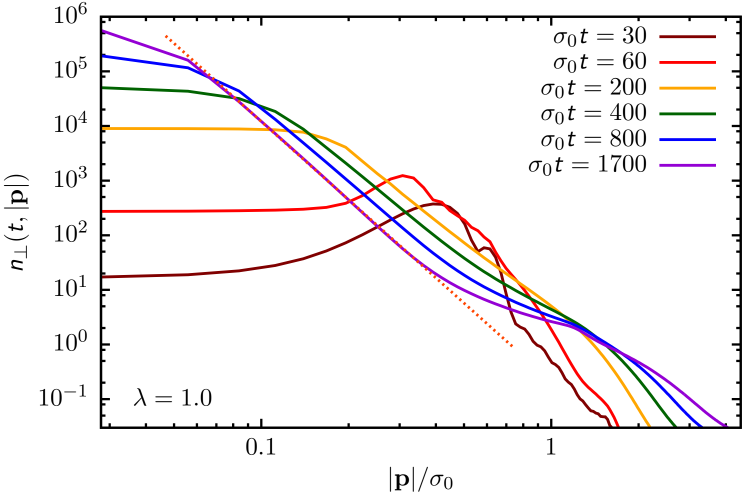
Turbulence at Strong Couplings

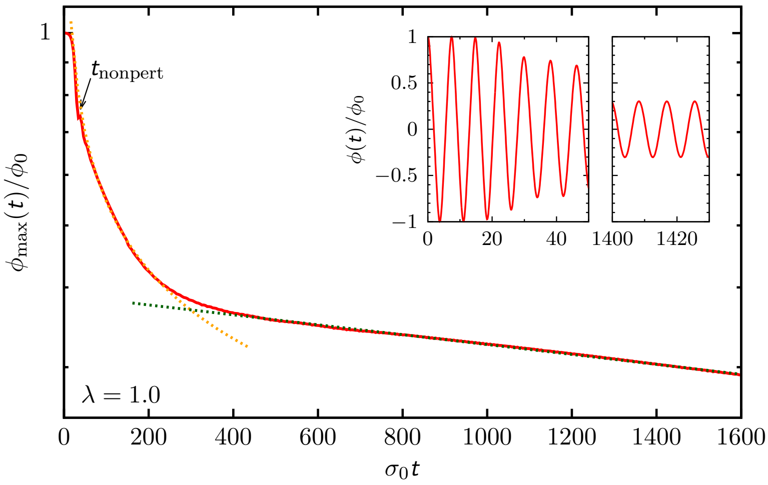
Parametric Resonance



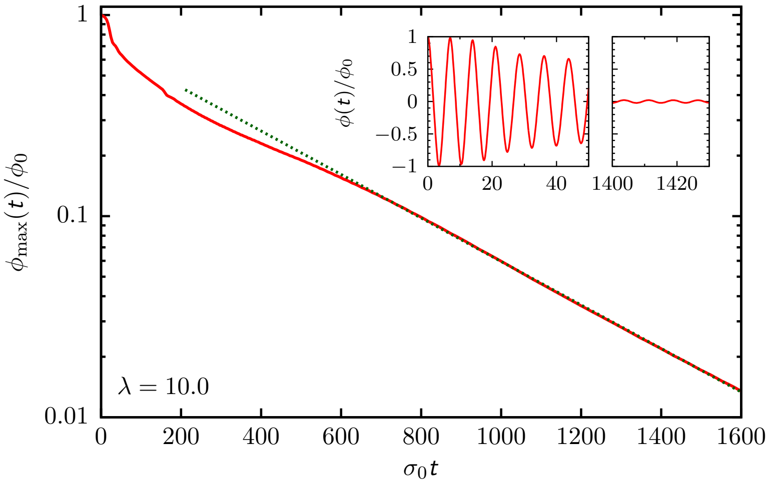
Turbulence at Strong Couplings

Parametric Resonance





1
t_{nonpert}
$\phi_{\max}(t)/\phi_0$
$\lambda = 1.0$
0
200
400
600
800
1000
1200
1400
1600
$\sigma_0 t$
$\phi(t)/\phi_0$
1
0.5
0
−0.5
−1
0
20
40
1400
1420



1
0.1
0.01
0
200
400
600
800
1000
1200
1400
1600
$\sigma_0 t$
$\phi_{\max}(t)/\phi_0$
$\lambda = 10.0$
$\phi(t)/\phi_0$
1
0.5
0
-0.5
-1
0
20
40
1400
1420