Lattice Phenomenology of Heavy Quarks Using Dynamical Fermions

Ava Khamseh

UKQCD Collaboration
Higgs Centre for Theoretical Physics, University of Edinburgh

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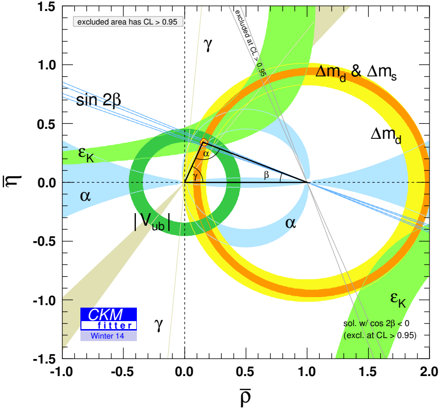
Outline

- Motivation and Introduction to Lattice QCD
- Field Correlators and Extracting Observables
- Meson Masses, Decay Constants, Bag Parameter
- Results
- Outlook

Motivation

- Flavor physics plays an important role in testing the limits of SM and constraining BSM theories
- Precision measurements at LHCb and other B-factories in searches for New Physics



Introduction

- Strong Interactions: QCD at low energy is confining $\rightarrow$ breakdown of perturbation theory
- Lattice QCD: Non-perturbative (numerical) method to solve the theory from first principles
- Discretise Euclidean space-time

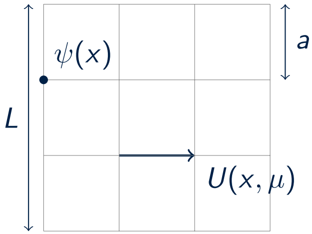
$$x_\mu = n_\mu a \quad n = 0, 1, ... \quad \mu = 1, 2, ..., D \quad \text{in D - dims}$$

for finite volume $n = 0, 1, ..., N - 1 \quad , \quad a =$ lattice spacing

$$\int d^D x \, f(x) \;\rightarrow\; a^D \sum_n f(na) \quad , \quad \int \mathcal{D}\phi \rightarrow \prod_n \phi_n$$

$$\nabla_\mu \phi(x) = \frac{\phi(x + a\hat{\mu}) - \phi(x)}{a} \quad , \quad \nabla^*_\mu \phi(x) = \frac{\phi(x) - \phi(x - a\hat{\mu})}{a}$$

Field Correlators



$$Z = \int \mathcal{D}U \mathcal{D}\psi \mathcal{D}\bar{\psi}\; e^{-S[\psi,\bar{\psi},U]}$$

$$S = \sum_{x,y\in\Lambda} \bar{\psi}(x)\, D(x,y;U)\, \psi(y) + S_g[U]$$

$$Z = \int \mathcal{D}U\; e^{-S_g[U]}\; (\det D[U])^{n_f}$$

Meson correlator, $O = \bar{q}'\Gamma q$ and $O^\dagger = \bar{q}\Gamma q'$

$$\langle O(n) O^\dagger(m)\rangle = \frac{1}{Z}\int \mathcal{D}[U] e^{-S_G[U]} \det[D_q] \det[D_q']$$
$$\times \operatorname{Tr}\left[\Gamma D_q^{-1}(n|m)\Gamma D_{q'}^{-1}(m|n)\right]$$

Gauge fields generated by Monte Carlo simulations. Performing $\det[D_q]$ (unquenched) is computationally expensive

Meson Two-Point Function

- Pseudoscalar $O = \bar{q}_1 \gamma_5 q_2$ and $O^\dagger = \bar{q}_2 \gamma_5 q_1$

$$C(x) = \langle O(x) O^\dagger(0) \rangle$$

- Inserting complete set of states for large T, gives

$$\begin{aligned} C_{PP}(t) &= \frac{1}{2m} \langle 0 | O(0) | n, \mathbf{p} = 0 \rangle \langle n, \mathbf{p} = 0 | O^\dagger(0) | 0 \rangle e^{-E_0 t} + \ldots \\ &= N_{PP} e^{-E_0 t} + \ldots \end{aligned}$$

- This can be done for different channels e.g. $O = \bar{d} \gamma_\mu \gamma_5 u$ then

$$C_{AP}(t) = N_{AP} e^{-E_0 t} + \ldots$$

Extracting Observables: Masses and Decay Constants

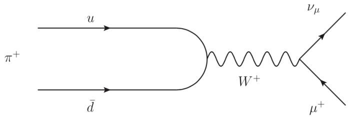
- For a finite volume lattice

$$C(t) = 2N_0 e^{-TE_0/2} \cosh((T/2 - t)E_0)$$

from which the meson ground state mass $M_{\mathrm{PS}} = E_0$ can be extracted.

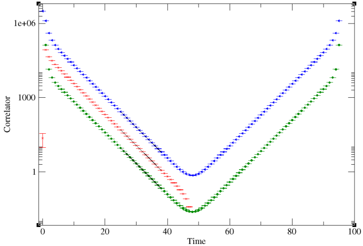
- Pseudoscalar decay constant

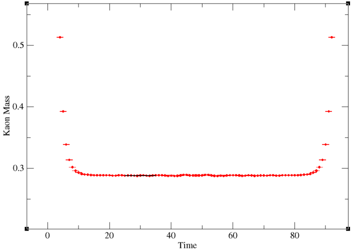
$$\langle 0|A_0(0)|\pi, \mathbf{p} = 0\rangle = f_{\mathrm{PS}} M_{\mathrm{PS}}$$



$$f_{\mathrm{PS}} = \sqrt{\frac{2N_{AA}}{M_{\mathrm{PS}}}} = \frac{\sqrt{2N_{AP}^2}}{\sqrt{M_{\mathrm{PS}} \times N_{PP}}}$$

Results 1: M_π, f_π, M_K, f_K





48^3 ($a^{-1} = 1.73$ GeV, 88 confgs) and

64^3 ($a^{-1} = 2.36$ GeV, 40 confgs) ensembles:

Meson	Mass 48^3 MeV	Mass 64^3 MeV	PDG Mass MeV
π	139.25(22)	139.33(26)	134.9766(6)
K	499.16(24)	507.80(94)	497.614(24)

After a short $\mathcal{O}(3)$% extrapolation agrees with the physical values:

$f_\pi = 130.19 \pm 0.89$ [130.4(0.04)(0.2)] MeV

$f_K = 155.51 \pm 0.83$ [156.2(0.2)(0.6)(0.3)] MeV

RBC/UKQCD, Nov 2014, http://arxiv.org/pdf/1411.7017v1.pdf

Results 2: M_{PS} and f_{PS} for Heavy Quarks

- Preliminary results for the masses and decay constants (MeV) of heavy-strange and heavy-light mesons (in progress)
- Extrapolate/interpolate to charm mass

Meson	Mass 48^3	Mass 64^3	Decay 48^3	Decay 64^3
Heavy-Strange	1526.59(21)	1600.60(26)	233.01(15)	238.64(19)
Heavy-Strange	1778.35(26)	2217.31(40)	238.57(25)	243.03(36)
Heavy-Light	1423.8(1.3)	1492.1(1.5)	201.41(92)	203.83(79)
Heavy-Light	1804.4(3.1)	2123.3(3.1)	206.1(2.7)	206.6(1.8)

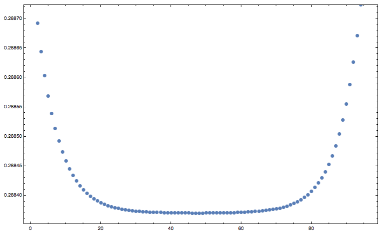
Excited States Contribution

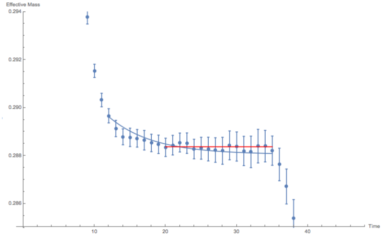
- It is possible to take the contribution of the first excited state into account, however, these become negligible at large times.

$$C(t) = c_0 h(t, 0, M_0) + c_1 h(x_0, 0, M_1) + \ldots$$

$$M_{\text{eff}} = M_0 \left\{ 1 + \frac{\epsilon(t) - \epsilon(t-a)}{\delta(t) - \delta(t-a)} + \ldots \right\}$$

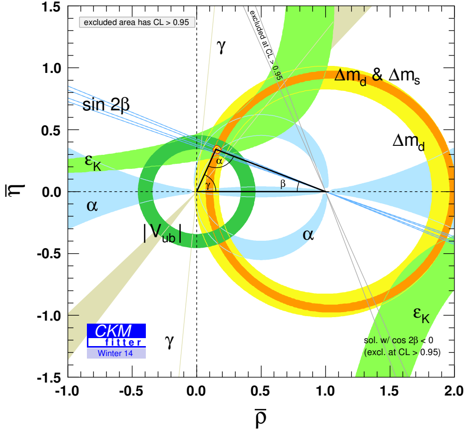
where $\epsilon(t)$ and $\delta(t)$ are functions of $h(t)$ defined before. M_0, M_1 and c_0/c_1 can be found as parameters of the fit.





Unitarity Triangle

$$\text{CKM matrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



Kaon Mixing and CP Violation

- CP is not an exact symmetry of weak interactions
- Experimental measure of indirect CP violation

$$\epsilon_K = \frac{A(K_L \to (\pi\pi))}{A(K_S \to (\pi\pi))}$$

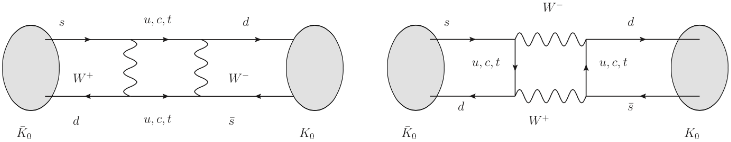
- Theoretically,

$$\epsilon_K \;\propto\; V^*_{xs} V_{xd} G_F^2 M_W^2 \times \langle K^0 | \underbrace{(\bar{s}\gamma_\mu d)(\bar{s}\gamma_\mu d) + (\bar{s}\gamma_5\gamma_\mu d)(\bar{s}\gamma_5\gamma_\mu d)}_{O_{VV+AA}} | \bar{K}^0 \rangle$$

non-perturbative hadronic matrix element is determined theoretically using Lattice. This imposes constraints on unitarity triangle through $V^*_{xs} V_{xd} \to$ green hyperbola

The Bag Parameter

- Dominant contribution comes from indirect CP violation through state-mixing, mediated by imaginary part of the the box diagram:

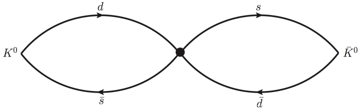


- Define "Bag Parameter" to parametrise $K^0 - \bar{K}^0$ mixing due to weak interactions:

$$B_K = \frac{\langle K^0 | O_{VV+AA} | \bar{K}^0 \rangle}{\frac{8}{3} f_K^2 M_K^2}$$

B_K on the Lattice: The 3-point Function

To determine the Bag Parameter, we integrate out M_W and heavy quark masses (EFT) and evaluate the following QCD diagram:



B_K can be found staring from the 3-point function on the lattice

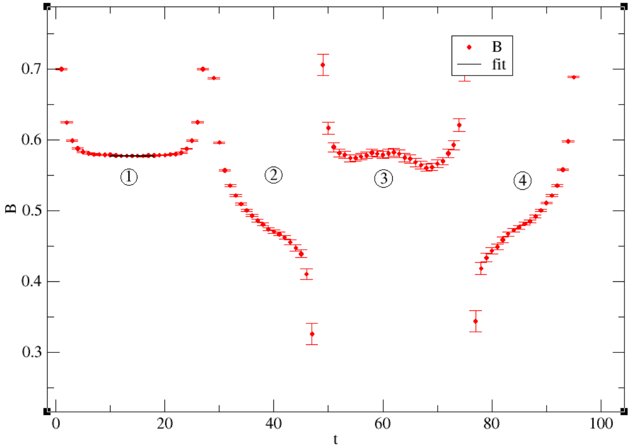
$$\langle Q(t_2) O_{VV+AA}(t_1) Q(0)\rangle = \langle 0|O|\bar{K}^0\rangle\langle\bar{K}^0|Q|K^0\rangle\langle K^0|O|0\rangle e^{-(t_y-t_x)E_{\bar{K}^0}} e^{-t_x E_{K^0}} + \dots$$

where the pseudo-scalar operator $Q = \bar{d}\gamma_5 s$ and $t_2 > t_1$. Time dependence is cancelled in this structure,

$$B_K^{\text{bare}} = \frac{\langle K^0(\Delta t)|O_{VV+AA}(t)|\bar{K}^0(0)\rangle}{\frac{8}{3}\langle K^0(\Delta t - t)|A_0(0)\rangle\langle A_0(t)|\bar{K}^0(0)\rangle}$$

Results 4: Fit Strategy and Excited States

- For the region where the time dependence cancels, B_0 can be found by fitting the plateau to a constant e.g. $B_{0K}^{\text{bare}} = 0.57748(60)$

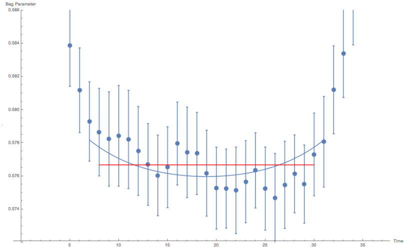


Results 4: Fit Strategy and Excited States

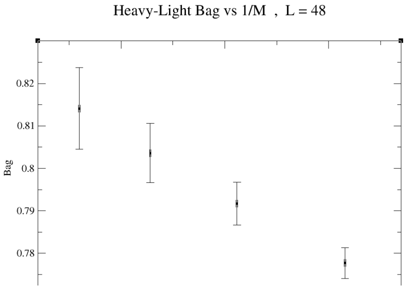
- In the same way as the 2-point function, the effect of the excited states for the Bag parameter can be taken into account. Then "effective" Bag parameter

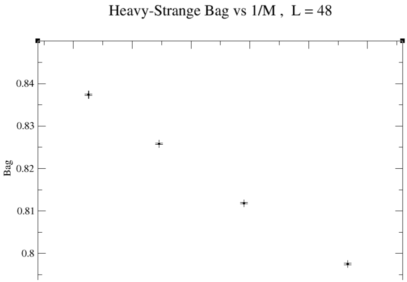
$$B_{\text{eff}} = B_0 \times$$
$$\left[1 + \alpha e^{-t_x(E_{K^0_1} - E_{K^0_0})} + \alpha e^{-(t_y - t_x)(E_{\bar{K}^0_1} - E_{\bar{K}^0_0})} - \frac{c_1}{c_0}\left(\epsilon_{\bar{K}}(t_y - t_x) + \epsilon_K t_x\right)\right] + \ldots$$

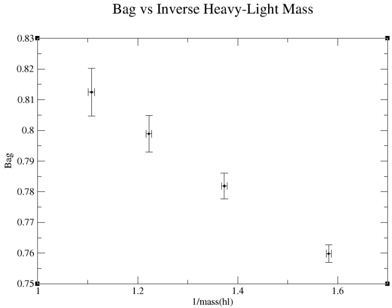
explaining the shape of the relevant region:

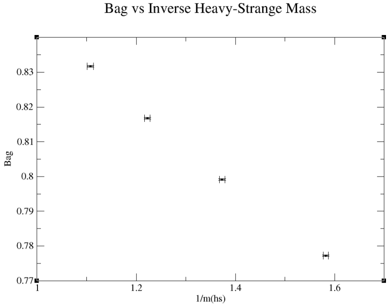


Results 5: Bag near Charm Mass

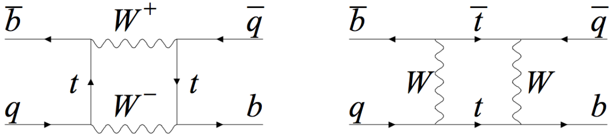








$B^0 - \bar{B}^0$ mixing and ξ Parameter



Light and Heavy mass eigenstates

$$|B_{L,H}\rangle = \alpha|B_q^0\rangle \pm \beta|\bar{B}_q^0\rangle \quad , \quad q = d, s$$

Mass difference $\Delta M_q = M_H - M_L$ is

$$\Delta M_q \propto |M_{12}^{(q)}| \propto G_F^2 M_W^2 |V_{tq}|^2 (B_{B_q} F_{B_q}^2)$$

where

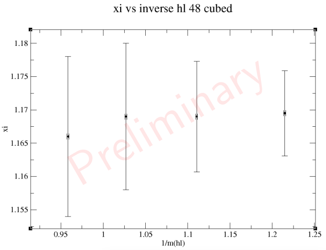
$$B_{B_q} = \langle B_q^0| \underbrace{(\bar{b}\gamma_\mu q)(\bar{b}\gamma_\mu q) + (\bar{b}\gamma_5\gamma_\mu q)(\bar{b}\gamma_5\gamma_\mu q)}_{Q_{VV+AA}} |\bar{B}_q^0\rangle$$

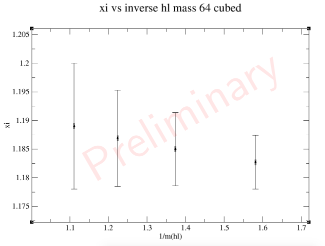
Results 6: ξ Parameter

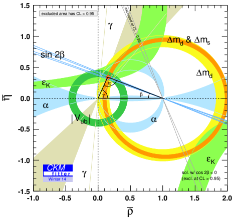
- $B^0 - \bar{B}^0$ mixing provides information on the $|\frac{V_{td}}{V_{ts}}|$ through the parameter

$$\xi = \frac{f_{hs}\sqrt{B_{hs}}}{f_{hl}\sqrt{B_{hl}}}$$

- This constrains the orange circle centred at $\bar{\rho} = 1$







Summary

- Simulation of pions near the physical mass $\rightarrow$ no large extrapolation required
- Simulation of heavy quarks near the charm mass
- Large lattice volumes 48^3 and 64^3 with two different UV cut-off scales
- Precise measurements of meson masses and decay constant
- Precise measurements of bag parameters and the ξ-parameter
- Use these measurements to contains the parameter space