Entropy Current for Non-Relativistic Fluids

based on [arXiv:1405.5687] (JHEP 1408 (2014) 037)

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December 18, 2014

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Young Theorists' Forum (YTF) 2014

Introduction

- System of interest: d (spatial) dimensional charged non-relativistic fluid to leading order.
- The respective relativistic system is well known.
- One can take a 'non-relativistic' limit ($v \ll c$) to get the non-relativistic counterpart. [Kaminski et al.'14]
- In [Rangamani et al.'08] an alternative approach to get (neutral) non-relativistic fluids was suggested – Light Cone Reduction (LCR), and later was extended to charged fluids by [Brattan '10].
- A goal of this work was to test this idea in presence of background electromagnetic fields.
- We were able to construct a NR entropy current, whose positive semi-definite nature constrains the fluid transport coefficients.

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Outline

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Relativistic Fluid Dynamics

- Fluids are near equilibrium limit of physical systems.
- State of a fluid is completely determined by a set of parameters like u^μ (four-velocity), T (temperature), M (chemical potential) etc. which are functions of space-time.

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Relativistic Fluid Dynamics

- Fluids are near equilibrium limit of physical systems.
- State of a fluid is completely determined by a set of parameters like u^μ (four-velocity), T (temperature), M (chemical potential) etc. which are functions of space-time.
- Dynamics of a fluid is governed by equations of energy-momentum and charge conservation:

$$\nabla_\mu T^{\mu\nu} = F^{\mu\nu} J_{Q\nu}, \qquad \nabla_\mu J^\mu_Q = 0. \tag{1}$$

- $T^{\mu\nu}$, J^μ_Q are in general determined in terms of fluid variables, external fields and their derivatives. These expressions are known as '*constitutive relations*' of a fluid.
- Constitutive relations specify a fluid system completely.

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- We use the 'near equilibrium' assumption of fluid, i.e. (space-time) derivatives of fluid parameters are fairly small and can be treated perturbatively.
- Constitutive relations can hence be expressed as a perturbative expansion in derivatives.

$$T^{\mu\nu} = T^{\mu\nu}_{(0)} + \Pi^{\mu\nu}_{(1)} + \dots, \qquad J^{\mu}_{Q} = J^{\mu}_{Q(0)} + \Upsilon^{\mu}_{(1)} + \dots. \tag{2}$$

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- At every order we put in all possible terms allowed by the symmetry. Every term comes with an arbitrary coefficient – a function of fluid thermodynamic variables T, M, known as '*transport coefficients*'.
- For ideal fluids we have:

$$T^{\mu\nu}_{(0)} = E(T,M)u^{\mu}u^{\nu} + P(T,M)\left(u^{\mu}u^{\nu} + g^{\mu\nu}\right), \tag{3}$$

$$J^{\mu}_{Q(0)} = Q(T,M)u^{\mu}. \tag{4}$$

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Dissipative Fluids

- Landau Gauge Condition:

$$u_\mu \Pi^{\mu\nu} = u_\mu \Upsilon^\mu = 0. \tag{5}$$

Use the projection operator: $P^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$.

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- Most generic symmetric tensors: $\nabla^{(\mu} u^{\nu)}$.
- Contribution to $T^{\mu\nu}$:

$$\Pi^{\mu\nu}_{(1)} = -2\eta\sigma^{\mu\nu} - \zeta\Theta P^{\mu\nu}. \tag{6}$$

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- Most generic vectors: $\nabla^\mu T$, $\nabla^\mu(M/T)$, $E^\mu = F^{\mu\nu}u_\nu$.
- Contribution to J^μ_Q:

$$\Upsilon^\mu_{(1)} = -\gamma P^{\mu\nu}\nabla_\nu T - \varrho P^{\mu\nu}\nabla_\nu\left(\frac{M}{T}\right) + \lambda E^\mu. \tag{7}$$

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Entropy Current

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- Since fluids are in local thermodynamic equilibrium, it asks for entropy to be created at every space-time point, or divergence of an entropy current should be positive semi-definite:

$$\nabla_\mu J_S^\mu \geq 0. \tag{8}$$

- The canonical form of entropy current is given by:

$$J_S^\mu = S u^\mu - \frac{M}{T}\Upsilon^\mu. \tag{9}$$

$$E + P = ST + QM, \qquad \mathrm{d}P = S\mathrm{d}T + Q\mathrm{d}M. \tag{10}$$

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- Entropy positivity gives some constraints on the transport coefficients coupling to derivative terms.

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- We get the constraints:

$$\Pi^{\mu\nu}_{(1)} = -2\eta\sigma^{\mu\nu} - \zeta\Theta P^{\mu\nu},$$
$$\eta \geq 0, \qquad \zeta \geq 0, \tag{11}$$

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$$\Upsilon^{\mu}_{(1)} = -\gamma P^{\mu\nu}\nabla_\nu T - \varrho P^{\mu\nu}\nabla_\nu \left(\frac{M}{T}\right) + \lambda E^{\mu},$$
$$\gamma = 0, \qquad \lambda = \frac{1}{T}\varrho \geq 0. \tag{12}$$

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Outline

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- Light-cone reduction is a prescription to reduce a relativistic algebra to a non-relativistic algebra in one lower dimension.

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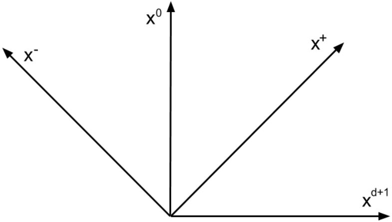
- We start with a $(d+1,1)$-dim relativistic theory, and undergo following coordinate transformation:

$$\{x^{\mu}\}_{\mu=0,1,\dots,d+1} \to \{x^{\pm}, x^{i}\}_{i=1,2,\dots,d}\,, \tag{13}$$

$$x^{\pm} = \frac{1}{\sqrt{2}}\left(x^{0} \pm x^{d+1}\right), \tag{14}$$



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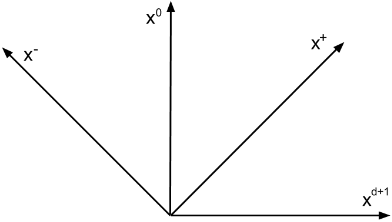

- Now we declare x^- to be a symmetry direction, $t \equiv x^+$ to be our new 'time' direction.
- The new theory is known to have non-relativistic symmetry, with coordinates $\{t = x^+, x^i\}_{i=1,2,\dots,d}$.

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- Generators of relativistic symmetry group under LCR reduces to non-relativistic symmetry group.

$$\left.\begin{array}{r}\text{translations}\\ \text{rotations}\\ \text{boosts}\end{array}\right\}\ \text{Poincaré}\quad\rightarrow\quad\text{Galilean}\ \left\{\begin{array}{l}\text{translations}\\ \text{rotations}\\ \text{Gal. boosts}\end{array}\right.$$

- Similarly 'Conformal Symmetry' in relativistic theories reduce to 'Schrödinger Symmetry' group.
- For more details consult: [Rangamani '09 - 'Holography for non-relativistic CFTs']

Outline

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LCR of Relativistic Fluid

- We start with reducing the conservation equations:

$$
\begin{aligned}
\nabla_+ T^{++} + \nabla_i T^{i+} &= F^{+\lambda} J_{Q\lambda} \\
\nabla_+ T^{+-} + \nabla_i T^{i-} &= F^{-\lambda} J_{Q\lambda} \\
\nabla_+ T^{+j} + \nabla_i T^{ij} &= F^{j\lambda} J_{Q\lambda} \\
\nabla_+ J_Q^{+} + \nabla_i J_Q^{i} &= 0
\end{aligned}
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$$\nabla_+ T^{++} + \nabla_i T^{i+} = F^{+\lambda} J_{Q\lambda} \quad \Rightarrow \quad \partial_t \rho + \partial_i (\rho v^i) = 0$$

$$\nabla_+ T^{+-} + \nabla_i T^{i-} = F^{-\lambda} J_{Q\lambda} \quad \Rightarrow \quad \partial_t (\epsilon + 1/2\ \rho \mathbf{v}^2) + \partial_i j^i = j_Q^i \epsilon_i$$

$$\nabla_+ T^{+j} + \nabla_i T^{ij} = F^{j\lambda} J_{Q\lambda} \quad \Rightarrow \quad \partial_t (\rho v^j) + \partial_i t^{ij} = q\epsilon^j - j_{Qk} \beta^{kj}$$

$$\nabla_+ J_Q^+ + \nabla_i J_Q^i = 0 \quad \Rightarrow \quad \partial_t q + \partial_i j_Q^i = 0$$

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$$\nabla_+ J^+_Q + \nabla_i J^i_Q = 0 \quad \Rightarrow \quad \partial_t q + \partial_i j^i_Q = 0$$

- The two sets will agree provided we identify:

$$v^i = \frac{u^i}{u^+} + \dots$$

$$\rho = (u^+)^2 (E + P) + \dots$$

$$\epsilon = \frac{1}{2}(E - P) + \dots$$

$$q = Q u^+ + \dots$$

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$$v^i = \frac{u^i}{u^+} + \dots \qquad A^+ = 0$$

$$\rho = (u^+)^2(E + P) + \dots \qquad \phi = A^-$$

$$\epsilon = \frac{1}{2}(E - P) + \dots \qquad a^i = A^i$$

$$q = Qu^+ + \dots \qquad \epsilon^i = -\partial^i \phi - \partial_t a^i$$

$$\beta^{ij} = \partial^i a^j - \partial^j a^i$$

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\nabla_+ T^{++} + \nabla_i T^{i+} &= F^{+\lambda} J_{Q\lambda} &\Rightarrow&\quad \partial_t \rho + \partial_i(\rho v^i) = 0 \\
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a^i &= A^i \\
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\beta^{ij} &= \partial^i a^j - \partial^j a^i
\end{aligned}
$$

$$
\begin{aligned}
p &= P + \dots \\
\tau &= \frac{T}{u^+} + \dots \\
\mu &= \frac{M}{u^+} + \dots
\end{aligned}
$$

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- We have the form of the currents:

$$t^{ij} = \rho v^i v^j + p g^{ij} - n\sigma^{ij} - z\delta^{ij}\partial_k v^k, \tag{15}$$

$$j^i = \left(\epsilon + p + \frac{1}{2}\rho \mathbf{v}^2\right) v^i - n\sigma^{ij} v_j - z\partial_k v^k v^i$$
$$-\kappa\partial^i \tau - \kappa\nabla^i\left(\frac{\mu}{\tau}\right) + \frac{\kappa}{\tau}(\epsilon^i - v_j\beta^{ji}), \tag{16}$$

$$j_Q^i = q v^i - \xi\nabla^i\tau - r\nabla^i\left(\frac{\mu}{\tau}\right) - m\nabla^i p + \sigma(\epsilon^i - v_k\beta^{ki}), \tag{17}$$

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- We do not get the 'most generic' constitutive relations for NR fluid. It is related to the choice of frame in relativistic theory.
- The fluid obeys Wiedemann-Franz Law for metals: $\kappa/\sigma = L\tau$.

$$L_{exp} = 2.45 \times 10^{-8}\, W\Omega K^{-2}, \qquad L_{th} = 6.68 \times 10^{-8}\, W\Omega K^{-2} \quad (18)$$

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LCR of Entropy Current

- Reduction of Entropy Current:

$$\nabla_+ J_S^+ + \nabla_i J_S^i \geq 0 \qquad \Rightarrow \qquad \partial_t s + \partial_i j_S^i \geq 0$$

- We get the identifications:

$$s = Su^+, \qquad j_S^i = sv^i - \frac{\mu}{\tau}\varsigma^i \tag{19}$$

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$$n \geq 0, \qquad z \geq 0, \qquad \kappa \geq 0 \tag{22}$$

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$$j_Q^i = qv^i - \xi\nabla^i\tau - r\nabla^i\left(\frac{\mu}{\tau}\right) - m\nabla^i p + \sigma(\epsilon^i - v_k\beta^{ki}), \tag{23}$$

$$m = 0, \qquad \xi \geq 0, \qquad \sigma = \frac{1}{\tau}r \geq 0. \tag{24}$$

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Summary of Parity-even Fluids

- We started with a relativistic charged fluid in electromagnetic background.
- Using Light Cone Reduction, we reached a consistent theory of charged non-relativistic fluids.
- We have constrained various transport coefficients of the non-relativistic theory using the demand of local entropy current positivity.
- LCR does not give the most generic non-relativistic fluid.

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Parity Odd Fluids

- Parity is not a symmetry of nature. So constitutive relations can be given parity-odd terms:

$$T^{\mu\nu} = T^{\mu\nu}_{(0)} + \Pi^{\mu\nu} + \tilde{\Pi}^{\mu\nu}, \qquad J^{\mu}_{Q} = J^{\mu}_{Q(0)} + \Upsilon^{\mu} + \tilde{\Upsilon}^{\mu} \tag{25}$$

- Only charge current gets parity odd terms at leading order:

$$\tilde{\Upsilon}^{\mu} = \left\{ \mho l^{\mu} + \tilde{\mho} B^{\mu} \right\}, \tag{26}$$

$l^{\mu} = \epsilon^{\mu\nu\rho\sigma} u_{\nu} \nabla_{\rho} u_{\sigma}$, $B^{\mu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} u_{\nu} F_{\rho\sigma}$.

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- In $(3+1)$-dim one can introduce an anomaly to the charge current of fluid such that:

$$\nabla_{\mu} J^{\mu}_{Q} = \{ C E^{\mu} B_{\mu} \}. \tag{27}$$

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- It was shown by [Son-Surówka '09] that canonical entropy current definition must be modified to include anomaly:

$$J^{\mu}_{S} = Su^{\mu} - \frac{M}{T}\left(\Upsilon^{\mu} + \tilde{\Upsilon}^{\mu}\right) + \left\{D l^{\mu} + \tilde{D} B^{\mu}\right\} \tag{28}$$

Entropy Current of Parity Odd Fluids

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- Demanding entropy positivity, all the coefficients: $\mho$, $\tilde{\mho}$, D, $\tilde{D}$ can be related to the anomaly coefficient C.

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LCR of Parity Odd Fluids

- Reduction of charge current will be modified:

$$\nabla_+ J_Q^+ + \nabla_i J_Q^i = \{C E^\mu B_\mu\} \qquad \Rightarrow \qquad \partial_t q + \partial_i j_Q^i = 0$$

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LCR of Parity Odd Fluids

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- LCR gives the parity-odd current:

$$\tilde{\varsigma}^i = \left\{ \bar{\kappa} \epsilon^{ij} \nabla_j \tau + \bar{\xi} \epsilon^{ij} \nabla_j \left(\frac{\mu}{\tau} \right) - \bar{m} \epsilon^{ij} \nabla_j p + \bar{\sigma} \epsilon^{ij} \left(\epsilon_j - v^k \beta_{kj} \right) \right\}, \tag{29}$$

The coefficients are determined in terms of $\omega = \mho (u^+)^2$, $\tilde{\omega} = \tilde{\mho} u^+$.

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The coefficients are determined in terms of $\omega = \mho (u^+)^2$, $\tilde{\omega} = \tilde{\mho} u^+$.

- Similarly we can reduce the entropy current. We need to add more terms to the entropy current for consistency of the theory.

$$j_S^i = s v^i - \frac{\mu}{\tau} \left(\varsigma^i + \tilde{\varsigma}^i \right) + \left\{ \mathfrak{b} \epsilon^{ij} \nabla_j \left(\frac{\mu}{\tau} \right) + \mathfrak{d} \epsilon^{ij} \left(\epsilon_j - v^k \beta_{kj} \right) \right\}. \qquad (30)$$

- We find that, ω and $\tilde{\omega}$ are left unconstrained by entropy current positivity *iff* fluid is 'incompressible' and is kept in 'constant magnetic field'. Otherwise they both are zero.

- The constraints are not consistent with the relativistic theory.

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Conclusions

- We were able to get a consistent theory describing non-relativistic (parity-violating) charged fluid, using the formalism of light cone reduction.
- Various transport coefficients appearing in the theory were constrained using the demand of local entropy positivity.
- Parity-odd transport coefficients (in $2+1$ dimensions) can only sustain if fluid is incompressible, and is subjected to constant magnetic field.
- In presence of anomalies, the constraints of relativistic and non-relativistic theories do not match.
- The theory gained by LCR is not most generic.

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Further Work

- Checking LCR for higher derivative fluids.
- Extending LCR of parity-odd sector to generic dimensions.

Further Work

- Checking LCR for higher derivative fluids.
- Extending LCR of parity-odd sector to generic dimensions.
- Extension is trivial for even to odd dimensional reduction. For odd to even, the entire first order of non-relativistic fluid is lost.

$$
\begin{aligned}
2n \text{ dimensions} \quad &\rightarrow \quad \text{first parity odd correction at } n-1 \text{ order} \\
2n-1 \text{ dimensions} \quad &\rightarrow \quad \text{first parity odd correction at } n-1 \text{ order}
\end{aligned}
$$

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Further Work

- Checking LCR for higher derivative fluids.
- Extending LCR of parity-odd sector to generic dimensions.
- Extension is trivial for even to odd dimensional reduction. For odd to even, the entire first order of non-relativistic fluid is lost.

$$\begin{aligned} 2n \text{ dimensions} \quad &\rightarrow \quad \text{first parity odd correction at } n-1 \text{ order} \\ 2n-1 \text{ dimensions} \quad &\rightarrow \quad \text{first parity odd correction at } n-1 \text{ order} \end{aligned}$$

- Perform LCR in generic fluid frames, and check if we get the most generic non-relativistic fluid.

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